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New soliton solutions for some important nonlinear partial differential equations using a generalized Bernoulli method

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Abstract
A generalized Bernoulli method is used for constructing new exact soliton solutions of nonlinear partial differential equations in a unified way. The main idea of this method is to take full advantage of the Bernoulli equation which has a simple exponential solution. Five important models in mathematical physics named, the nonlinear dispersive equation, the nonlinear Fisher-type equation, ZK-BBM equation, the general Burgers-Fisher equation and Drinfeld–Sokolov system are investigated. We successfully get new soliton solutions for these problems and recover some solutions that had been found by other methods for the same problems.

1. Introduction

In recent years, searching for exact solutions of nonlinear partial differential equations (NLPDEs) has become more attractive due to the availability of computer symbolic systems like Maple and Mathematica. These packages allow researchers to perform some complicated and tedious algebraic calculation on computer as well as help to find new exact solutions of NLPDEs. In this context, many efficient methods for obtaining analytical closed form traveling wave solutions have been presented. Some of these methods are sine-cosine method [22,23,26], generalized expansion method [17], F-expansion method[19]and (G'/G)- expansion method [1,2,22].

As more popular and efficient methods, we can also point to tanh-function method [16,21,24], extended tanh function method[10,27,28] and modified extended tanh-function method (METFM) [7,8] that are applied to solve a verity of NLPDEs.

Recently, the METFM combined with the random variable transformation technique(RVT)[14,18] have been proposed by A. Hussein and M.M.Selim[15] to solve the nonlinear stochastic generalized shallow water wave equation.

More recent, a new effective and straightforward method to construct exact soliton solutions to the NLPDEs is the generalized Bernoulli-method[3,4,5,11]. It has been implemented to get new traveling wave solutions for some special NLPDEs. The main idea of this method is to take full advantage of the nonlinear ODE of Bernoulli type, which has a simple exponential solution, to get new soliton solutions of NLPDEs.

In this paper, The Bernoulli method is applied to solve five important mathematical models that have a wide range of applications in many scientific fields, especially in plasma physics, plasma waves, fluid mechanics, solid state physics, capillary-gravity waves, and chemical physics. These models are the nonlinear dispersive equation, the general Burgers–Fisher equation, the generalized Drinfeld–Sokolovsystem, ZK-BBM (Benjamin-Bona–Mahony) equation and the nonlinear Fisher type equation. New exact soliton solutions for these NLPDEs are evaluated

2. Bernoulli Method

To illustrate the basic concepts of the Bernoulli method, consider a general (1+2) NLPDE in the following form

$$\Gamma(\phi,\phi_x,\phi_y,\phi_t,\phi_{tx},\phi_{ty},\phi_{xy},\phi_{xx},\phi_{yy},\phi_{xyt},.......)=0\,. \quad (1)$$

Considering the traveling solution $\Phi(\xi)=\phi(x,y,t)$, where $\xi=k(x+y\pm c\,t)$,Eq.(1) converts to the following ordinary differential equation (ODE)

$$F(\Phi,\Phi',\Phi'',\Phi''',...)=0\,, \quad (2)$$

where $':=d\,/d\,\xi$.

The next crucial step is that the solution for Eq. (2) is expressed in the following ansatz:

$$\Phi(\xi)=a_0+\sum_{i=1}^{M}a_iG^i(\xi)\,, \quad (3)$$

where a_0 *and* a_i $,i=1,2,3,.....M$, are constants to be determined later and $a_i\neq 0$. $G(\xi)$ is the solution of the following ODE of Bernoulli type

$$G'+\lambda G=\mu G^2\,, \quad (4)$$

where λ and μ are non-zero parameters to be determined. The Bernoulli ODE, Eq.(4), has the following general solution

$$G(\xi)=\frac{1}{\frac{\mu}{\lambda}+be^{\lambda\xi}}\,, \quad (5)$$

Where b is an arbitrary constant.

The parameter M can be found by homogeneous balance of the highest order linear term with the nonlinear term in Eq.(2) [7,8,9].

Substituting Eq.(3) into Eq.(2) and using Eq.(4), the left-hand side of Eq. (2) is converted to a polynomial in G. Equating each coefficient of this polynomial to zero, yields a system of nonlinear algebraic equations in a_0,a_i,k,c,μ and λ. Solving this system with the aid of Mathematica package and using the solution of Bernoulli equation, Eq. (5), we can construct the traveling wave solutions of the NLPDE(1) using Eq. (3).

3. Applications of Bernoulli Method

Herein, the Bernoulli method is used to solve five important NLPDEs.

3.1. The nonlinear Dispersive Equation

One of the forms of the nonlinear dispersive equation is [22]:

$$u^n\left(u^n\right)_t+a\left(u^{3n}\right)_x+d\,u^n\left(u^n\right)_{xxx}=0, \qquad n>0, \quad (6)$$

Where a and $d$are constants.

Setting $v=u^n$ into Eq. (6) and dividing both sides by v, we get

$$v_t+\frac{3}{2}a\left(v^2\right)_x+d\,v_{xxx}=0, \qquad n>0. \quad (7)$$

Using the wave variable $\xi=x-c\,t$ (c is the wave speed) and setting $V(\xi)=v(x,t)$, Eq. (7) converts to the following ODE

$$-c\,V'+\frac{3}{2}a\left(V^2\right)'+d\,V'''=0. \quad (8)$$

Integrating Eq. (8) once with respect to ξ and setting the constant of integration to zero, we obtain

$$-cV+\frac{3}{2}aV^2+dV''=0. \quad (9)$$

Balancing V'' with V^2 leads to $M=2$. Then, according to Eq.(3)

$$V(\xi)=a_0+a_1G(\xi)+a_2G^2(\xi), \quad (10)$$

where $G(\xi)$ is Given by Eq.(5).

Substituting from Eqs.(4) and (10) into Eq.(9) results an algebraic equation in powers of G. Equating the coefficients of G^i $(i=0,1,2,3,4)$ to zero yields a system of nonlinear algebraic equations in the parameters a_0,a_1,a_2, c, μ and λ. Solving this system with Mathematica gives the following two sets of solutions

$$a_0=0,\ a_1=\frac{4d\lambda\mu}{a},\ a_2=-\frac{4d\mu^2}{a},\ c=d\lambda^2. \quad (11)$$

$$a_0=-\frac{2d\lambda^2}{3a},a_1=\frac{4d\lambda\mu}{a},a_2=-\frac{4d\mu^2}{a},c=-d\lambda^2. \quad (12)$$

According to Eq. (11) the exact solution of Eq. (6) reads the following

$$u_1(x,t) = \left[-\frac{4d\mu^2}{a\left(be^{\lambda(x-dt\lambda^2)}+\frac{\mu}{\lambda}\right)^2} + \frac{4d\lambda\mu}{a\left(be^{\lambda(x-dt\lambda^2)}+\frac{\mu}{\lambda}\right)} \right]^{\frac{1}{n}}. \quad (13)$$

According to Eq.(12) the solution of Eq.(6) reads

$$u_2(x,t) = \left[-\frac{2d\lambda^2}{3a} - \frac{4d\mu^2}{a(be^{\lambda(x+dt\lambda^2)}+\frac{\mu}{\lambda})^2} + \frac{4d\lambda\mu}{a(be^{\lambda(x+dt\lambda^2)}+\frac{\mu}{\lambda})} \right]^{\frac{1}{n}}. \quad (14)$$

In the previous solutions $\lambda,\ \mu$ are arbitrary constants.

The solutions in Eqs. (13) and (14) are different from the others in [8] and [22]. So we can say that they are new exact soliton solutions for the problem. 3D graph in Fig.1 illustrates the behavior of the exact soliton solution, Eq. (13) and 2D graph in Fig. 2 shows the movement of the wave with time.

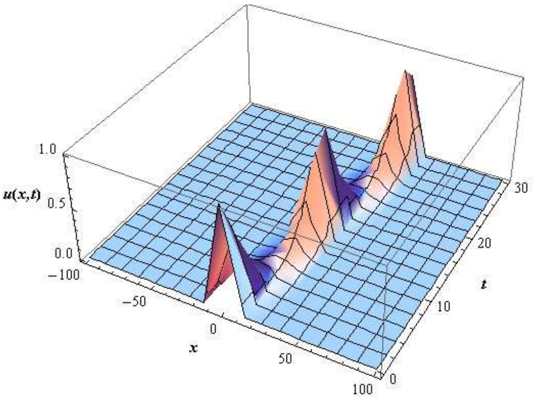

Fig1.The exact soliton solution, Eq. (13), with $a = 1,\ \mu = 1,\ \lambda = 1,\ d = 1,\ b = 1$ and $n = 2$.

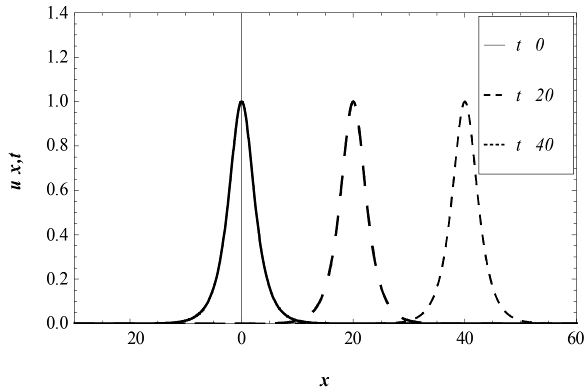

Fig2. The movement of the wave, Eq. (13), as time increases with $a = 1,\ \mu = 1,\ \lambda = 1,\ d = 1,\ b = 1$ and $n = 2$.

3.2. The General Burgers–Fisher Equation

A second illustrative example of the Bernoulli method is the solution of the general Burgers–Fisher equation given by [6]

$$u_t + p\,u^s u_x - u_{xx} - q\,u(1-u^s) = 0, \quad (15)$$

where *p, q* and *s* are some parameters. This NLPDE has a wide range of applications in plasma physics, fluid mechanics, capillary–gravity waves, nonlinear optics and chemical physics.

By setting $v = u^s$ into Eq. (15) we find

$$v v_t + p v^2 v_x - v v_{xx} + (1-\frac{1}{s}) v_x^2 - q s v^2 (1-v) = 0. \quad (16)$$

Using the wave variable $\xi = k(x - ct)$ (*k* and c are the wave number and the wave speed respectively) and setting $V(\xi) = v(x,t)$, Eq. (15) converts to

$$-kcVV' + pkV^2V' - k^2(1-\frac{1}{s})V'^2 - qs(1-V)V^2 = 0. \quad (17)$$

Balancing V^2V' with V'^2 gives $M = 1$ .Then, according to Eq.(3)

$$V(\xi) = a_0 + a_1 G. \quad (18)$$

Substitute Eq. (18) and Eq. (4) into Eq. (17) results an algebraic equation in powers of G. The coefficients of $G^i\,(i = 0,1,2,3,4)$ have to vanish. This leads to a system of nonlinear algebraic equations in the parameters $a_0,\ a_1, k, c, \mu$ and λ that has the following sets of solutions

$$a_0 = 1,\ \lambda = -\frac{ps}{k(1+s)},\ c = \frac{p^2+q+2qs+qs^2}{p(1+s)}, \mu = \frac{psa_1}{k(1+s)}. \quad (19)$$

$$a_0 = 0,\ \lambda = \frac{ps}{k(1+s)},\ c = \frac{p^2+q+2qs+qs^2}{p(1+s)},\ \mu = \frac{psa_1}{k(1+s)}. \quad (20)$$

$$a_0 = 0,\ c = p + \frac{q}{p},\ \lambda = \frac{qs}{kp},\ \mu = 0. \quad (21)$$

Using Eqs. (19), (20), and (21) together with Eq. (5) in Eq. (18) we obtain the following solutions for the Burgers-Fisher equation (15)

$$u_1(x,t) = \left[1 + \frac{a_1}{be^{-\frac{ps(-\frac{(p^2+q(1+s)^2)t}{p(1+s)}+x)}{1+s}} - a_1} \right]^{\frac{1}{s}}, \quad (22)$$

$$u_2(x,t) = \left[\frac{a_{u_2 1}}{be^{\frac{ps\left(-\frac{(p^2+q+2qs+qs^2)t}{p(1+s)}+x\right)}{1+s}} + a_1} \right]^{\frac{1}{s}}, \quad (23)$$

$$u_3(x,t) = \left[\frac{a_1}{b} e^{-\frac{qs(-(p+\frac{q}{p})t+x)}{p}} \right]^{\frac{1}{s}}. \quad (24)$$

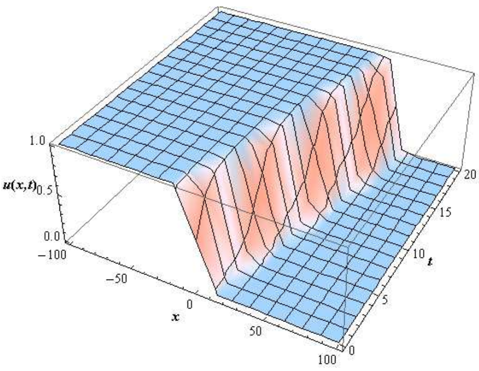

Fig3. *The exact soliton solution, Eq. (22), with* $a_1 = -1,\ p = 1,\ q = 1,\ b = 1$ *and* $s = 1$.

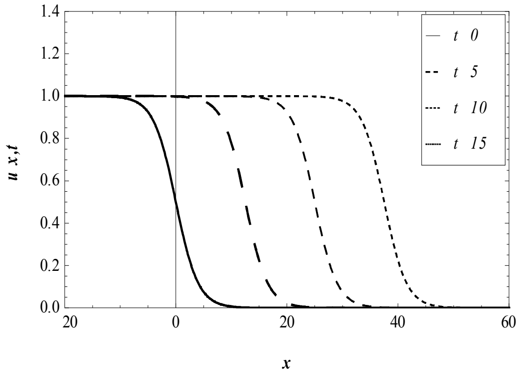

Fig4. *The movement of the wave, Eq. (22), as time increases with* $a = -1,\ p = 1,\ q = 1,\ b = 1$ *and* $s = 1$.

In the previous solutions, Eqs. (22-24), a_1 is arbitrary constants and k is a dummy constant. 3D graph in Fig.3 illustrates the behavior of the exact soliton solution, Eq. (22) and 2D graph in Fig.4 shows the movement of the wave along x-direction as time increases.

It is noted that the new solutions in equations (22-24) are not obtained by Wazwaz[24] using tanth method and also not obtained by El-wakil and Abdou[8] using METFM. Also these solutions are different from that obtained by Chen and H. Zhang [6].

3.3. The generalized Drinfeld–Sokolov System

The third problem is the generalized Drinfeld–Sokolov system of NLPDEs [8,26]:

$$u_t + (v^2)_x = 0, \tag{25a}$$

$$v_t - a v_{xxx} + 3d\, v u_x + 3k\, u\, v_x = 0, \tag{25b}$$

Where *a, d* and *k* are constants. This system was introduced by Drinfeld and Sokolov as an example of a system of nonlinear equations possessing Lax pairs of a special form[13].

Similarly as before we will use the transformation $\xi = x - c\,t$ with $U(\xi) = u(x,t)$, and $V(\xi) = v(x,t)$. This converts Eqs. (25) to a system of ordinary differential equation:

$$-cU' + (V^2)' = 0, \tag{26a}$$

$$-cV' - aV''' + 3d\,V\,U' + 3k\,U\,V' = 0. \tag{26b}$$

Integrating Eq. (26a) and neglecting the constant of integration we find

$$cU = V^2. \tag{27}$$

Inserting Eq. (27) into Eq. (26b) and integrating, with neglecting the constant of integration, we get

$$c^2 V - (2d+k)V^3 + a\,cV'' = 0. \tag{28}$$

Balancing highest order linear term, V'', with nonlinear term, V^3, in Eq.(28) gives $M = 1$. Then, according to Eq.(3)

$$V(\xi) = a_0 + a_1 G, \tag{29}$$

Substituting Eq.(29) into Eq.(28), using Eq.(4), and equating the coefficients of $G^i (i = 0,1,2,3)$ to zero givea system of nonlinear algebraic equations in the parameters a_0, a_1, c, μ and λ. The solutions of this system are found to be

$$\lambda = -\frac{\sqrt{2}(2d+k)^{3/4}a_0}{\sqrt{a(2d+k)a_0}},\ \mu = \frac{(2d+k)^{3/4}a_1}{\sqrt{2}\sqrt{a(2d+k)a_0}},\ c = \sqrt{2d+k}a_0, \tag{30}$$

$$\lambda = \frac{\sqrt{2}(2d+k)^{3/4}a_0}{\sqrt{a(2d+k)a_0}},\ \mu = -\frac{(2d+k)^{\frac{3}{4}}a_1}{\sqrt{2}\sqrt{a(2d+k)a_0}},\ c = \sqrt{2d+k}a_0, \tag{31}$$

$$\lambda = -\frac{i\sqrt{2}(2d+k)^{\frac{3}{4}}a_0}{\sqrt{a(2d+k)a_0}},\ \mu = \frac{i(2d+k)^{\frac{3}{4}}a_1}{\sqrt{2}\sqrt{a(2d+k)a_0}},\ c = -\sqrt{2d+k}a_0, \tag{32}$$

$$\lambda = \frac{i\sqrt{2}(2d+k)^{\frac{3}{4}}a_0}{\sqrt{a(2d+k)a_0}},\ \mu = -\frac{i(2d+k)^{\frac{3}{4}}a_1}{\sqrt{2}\sqrt{a(2d+k)a_0}},\ c = -\sqrt{2d+k}a_0. \tag{33}$$

Eqs. (30) to (33) together with Eq. (5) can be used in Eq.(29) to give the following solutions for the generalized Drinfeld–Sokolov system, Eqs. (25), respectively

$$v_1(x,t) = a_0 + \frac{a_1}{b e^{-\frac{\sqrt{2}(2d+k)^{1/4}\sqrt{a_0}(x-\sqrt{2d+k}ta_0)}{\sqrt{a}}} - \frac{a_1}{2a_0}}, \tag{34}$$

$$v_2(x,t) = a_0 + \frac{a_1}{b e^{\frac{\sqrt{2}(2d+k)^{1/4}\sqrt{a_0}(x-\sqrt{2d+k}ta_0)}{\sqrt{a}}} - \frac{a_1}{2a_0}}, \tag{35}$$

$$v_3(x,t) = a_0 + \frac{a_1}{b e^{-\frac{i\sqrt{2}(2d+k)^{1/4}\sqrt{a_0}(x+\sqrt{2d+k}ta_0)}{\sqrt{a}}} - \frac{a_1}{2a_0}}, \tag{36}$$

$$v_4(x,t) = a_0 + \frac{a_1}{be^{\frac{i\sqrt{2}(2d+k)^{1/4}\sqrt{a_0}(x+\sqrt{2d+k}ta_0)}{\sqrt{a}}} - \frac{a_1}{2a_0}}. \quad (37)$$

Remember that, according to Eq. (27)

$$u_i(x,t) = \frac{1}{c}[v_i(x,t)]^2, \qquad i = 1,2,3,4. \quad (38)$$

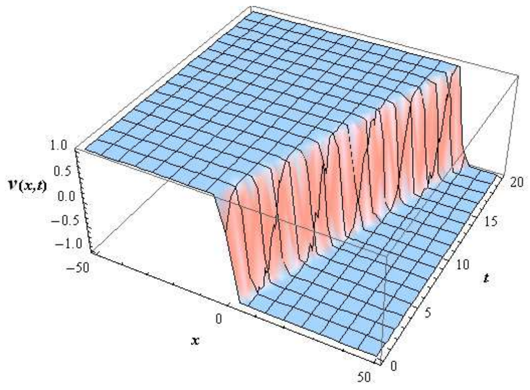

Fig5. *The exact soliton solution, Eq. (34), with* $a_0 = 1,\ a_1 = -1,\ a = 1,\ d = 1,\ k = 1$ *and* $b = 1$

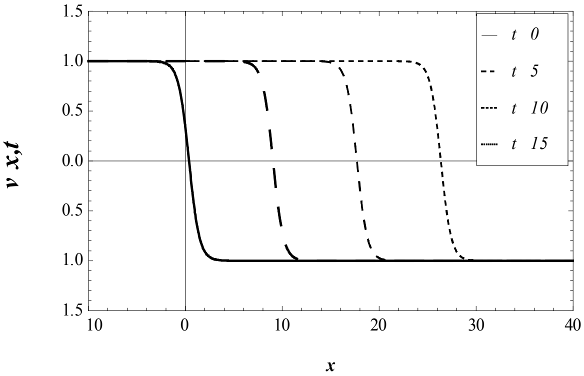

Fig6. *The movement of the wave, Eq. (34), as time increases with* $a_0 = 1,\ a_1 = -1,\ a = 1,\ d = 1,\ k = 1$ *and* $b = 1$

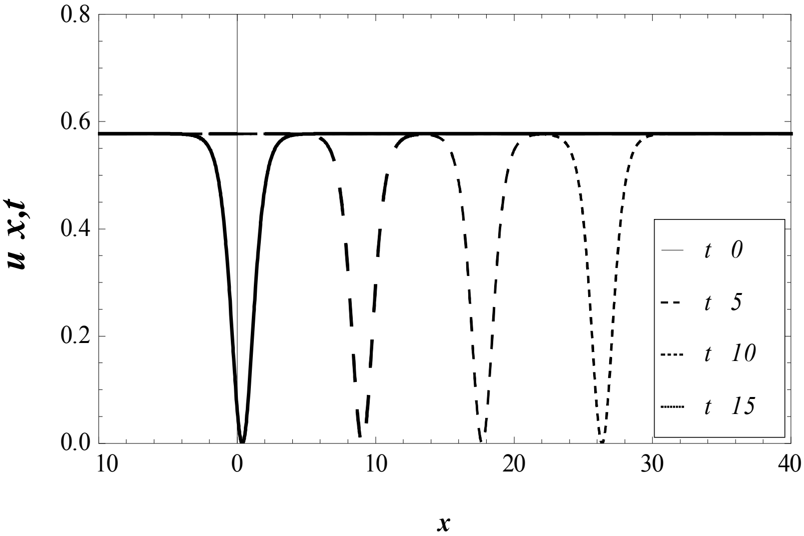

Fig7. *The movement of the wave, Eq. (38), as time increases with* $i = 1,\ a_0 = 1,\ a_1 = -1,\ a = 1,\ d = 1,\ k = 1$ *and* $b = 1$

In the previous solutions a_0 and a_1 are arbitrary constants. It is clear that the solutions v_1 and v_2 are real solutions while v_3 and v_4 are imaginary. The solitonsolution $v_1(x,t)$, given by Eq. (34), is illustrated in Fig.5 and Fig.6. Fig.7 shows the movement of the wave $u_1(x,t)$, as given in Eq. (38), along *x* axis as time increases.

The solutions obtained in (34-38) recover the solution in [11](for $a = 1, d = 1$ and $k = 1$) and gives new other solutions that not obtained in [8,11, 26].

3.4. (2+1)-Dimensional ZK-BBM (Benjamin–Bona–Mahony) Equation

The generalized model of ZK-BBM equation [25] is:

$$u_t + u_x + a(u^n)_x + d(u_{xt} + u_{yy})_x = 0, \quad (39)$$

Where *a,* and *d* are constants. Using the wave variable $\xi = x + y - ct$ with $U(\xi) = u(x,y,t)$, Eq. (39) coverts to

$$(1-c)U' + a(U^n)' + b(1-c)U''' = 0, \quad (40)$$

Integrating Eq. (40) and setting the constant of integration to zero we get

$$(1-c)U + aU^n + d(1-c)U'' = 0, \quad (41)$$

Balancing highest order linear term, U'', with nonlinear term, U^n, gives $M = \frac{2}{n-1}$.

It is suitable to consider *M* as a positive integer to get a closed analytical solution. If we take *n* = 3 then *M = 1* and

$$U(\xi) = a_0 + a_1 G \, , \tag{42}$$

Substituting Eq. (42) into Eq. (41) and equating the coefficients of $G^i (i = 0,1,2,3)$ to zero give a system of nonlinear algebraic equations in the parameters a_0, a_1, c, μ and λ. The solutions of this system are found to be

$$a_1 = \sqrt{2}\sqrt{d}\mu a_0, \;\; c = 1 + a a_0^2, \;\; \lambda = -\frac{\sqrt{2}}{\sqrt{d}}, \tag{43}$$

$$a_1 = -\sqrt{2}\sqrt{d}\mu a_0, \;\; c = 1 + a a_0^2, \;\; \lambda = \frac{\sqrt{2}}{\sqrt{d}}. \tag{44}$$

Insert the values of a_1, λ, and c from Eqs.(43) and (44) together with the value of G from Eq. (5) into Eq. (42) to give the following solutions for ZK-BBM Eq. (39) respectively

$$u_1(x,y,t) = a_0 + \frac{\sqrt{2}\sqrt{d}\mu a_0}{b e^{-\frac{\sqrt{2}(x+y-t(1+a a_0^2))}{\sqrt{d}}} - \frac{\sqrt{d}\mu}{\sqrt{2}}}, \tag{45}$$

$$u_2(x,y,t) = a_0 - \frac{\sqrt{2}\sqrt{d}\mu a_0}{b e^{\frac{\sqrt{2}(x+y-t(1+a a_0^2))}{\sqrt{d}}} + \frac{\sqrt{d}\mu}{\sqrt{2}}}. \tag{46}$$

In the previous solutions, a_0 and μ are arbitrary constants. Fig.8 shows the soliton solution, Eq. (45), at different times. The obtained solutions in Eqs. (45) and (46) are different from that obtained in [8, 25].

(a)

(b)

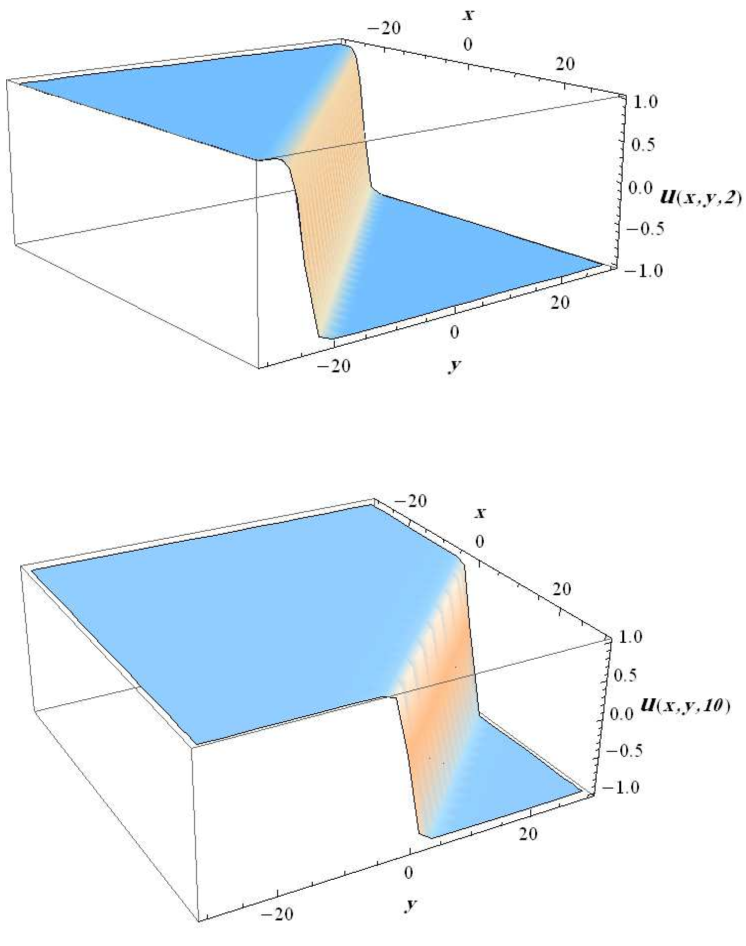

Fig 8. *The exact soliton solution, Eq. (45), at different times (a) t=2 and (b) t=10, with* $a_0 = 1$, $\mu = -1$, $a = 2$, $d = 1$ *and* $b = 1$

3.5. Nonlinear Fisher-Type Equation

The nonlinear Fisher-type equation takes the form:

$$u_t - u_{xx} - 2u(1-u^2) - \beta(1-u^2) = 0, \quad |\beta| < 1. \tag{47}$$

Eq. (47) is examined by Glasner [12] and arises in many contexts (ecology, flame front propagation, and dynamical phase transitions) and possesses diffuse interface solutions[21].

Using the wave variable, $\xi = x - ct$, will carry Eq. (47) into

$$cU' + U'' + 2U(1-U^2) + \beta(1-U^2) = 0, \quad |\beta| < 1, \tag{48}$$

Balancing highest order linear term, U'', with the nonlinear term, U^3, yields $M = 1$ and

$$U(\xi) = a_0 + a_1 G, \tag{49}$$

Substituting Eq. (49) into Eq. (48) and equating the coefficients of G^i $(i = 0,1,2,3)$ to zero gives a system of algebraic equations in the parameters a_0, a_1, c, μ and λ. Solving this system, the following sets of solutions are obtained

$$\text{Set1: } a_0 = -1,\ a_1 = -\mu,\ c = -\beta,\ \lambda = -2. \tag{50}$$

$$\text{Set2: } a_0 = -1,\ c = \beta,\ \lambda = 2,\ a_1 = \mu. \tag{51}$$

$$\text{Set3:} a_0 = 1,\ c = \beta,\ \lambda = -2,\ a_1 = \mu. \tag{52}$$

$$\text{Set4:} a_0 = 1,\ c = -\beta,\ \lambda = 2,\ a_1 = -\mu. \tag{53}$$

$$\text{Set5:} a_0 = -1,\ c = -\frac{6+\beta}{2},\ \lambda = \frac{2-\beta}{2},\ a_1 = \mu. \tag{54}$$

$$\text{Set6:} a_0 = -1,\ c = \frac{6+\beta}{2},\ \lambda = \frac{(-2+\beta)}{2},\ a_1 = -\mu. \tag{55}$$

$$\text{Set 7: } a_0 = 1,\ c = \frac{6-\beta}{2},\ \lambda = \frac{(-2-\beta)}{2},\ a_1 = \mu. \tag{56}$$

$$\text{Set8:} a_0 = 1,\ c = \frac{-6+\beta}{2},\ \lambda = \frac{(2+\beta)}{2},\ a_1 = -\mu. \tag{57}$$

$$\text{Set 9:} a_0 = -\frac{\beta}{2},\ b_1 = 0,\ c = \frac{(-6+\beta)}{2},\ \lambda = \frac{(-2-\beta)}{2},\ a_1 = -\mu. \tag{58}$$

$$\text{Set 10: } a_0 = -\frac{\beta}{2},\ c = \frac{6+\beta}{2},\ \lambda = \frac{2-\beta}{2},\ a_1 = -\mu. \tag{59}$$

$$\text{Set11:} a_0 = -\frac{\beta}{2},\ c = \frac{(-6-\beta)}{2},\ \lambda = \frac{(-2+\beta)}{2},\ a_1 = \mu. \tag{60}$$

$$\text{Set 12:} a_0 = -\frac{\beta}{2},\ c = \frac{6-\beta}{2},\ \lambda = \frac{2+\beta}{2},\ a_1 = \mu. \tag{61}$$

In view of Eqs. (50-61), the following solutions for nonlinear Fisher-type equation are obtained

$$u_1(x,t) = -1 - \frac{\mu}{be^{-2(x+t\beta)-\frac{\mu}{2}}}, \tag{62}$$

$$u_2(x,t) = -1 + \frac{\mu}{be^{2(x-t\beta)+\frac{\mu}{2}}}, \tag{63}$$

$$u_3(x,t) = 1 + \frac{\mu}{be^{-2(x-t\beta)-\frac{\mu}{2}}}, \tag{64}$$

$$u_4(x,t) = 1 - \frac{\mu}{be^{2(x+t\beta)+\frac{\mu}{2}}}, \tag{65}$$

$$u_5(x,t) = -1 + \frac{\mu}{be^{\frac{1}{2}\left(x-\frac{1}{2}t(-6-\beta)\right)(2-\beta)}+\frac{2\mu}{2-\beta}}, \tag{66}$$

$$u_6(x,t) = -1 - \frac{\mu}{be^{\frac{1}{2}\left(x-\frac{1}{2}t(6+\beta)\right)(-2+\beta)}+\frac{2\mu}{-2+\beta}}, \tag{67}$$

$$u_7(x,t) = 1 + \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(6-\beta))(-2-\beta)}+\frac{2\mu}{-2-\beta}}. \tag{68}$$

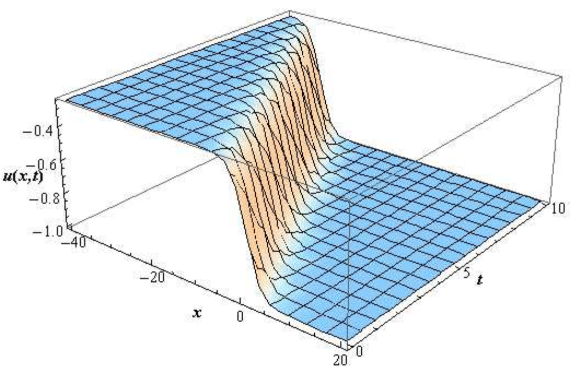

Fig9. *The exact soliton solution, Eq.(66), with* $b = 1$, $\beta = 0.5$ *and* $\mu = 1$.

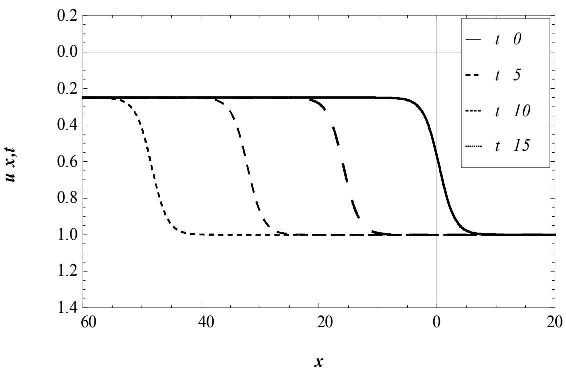

Fig10. *The movement of the wave, Eq.(66), at different times with* $b = 1$, $\beta = 0.5$ *and* $\mu = 1$.

$$u_8(x,t) = 1 - \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(-6+\beta))(2+\beta)}+\frac{2\mu}{2+\beta}} \tag{69}$$

$$u_9(x,t) = -\frac{\beta}{2} - \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(-6+\beta))(-2-\beta)}+\frac{2\mu}{-2-\beta}} \tag{70}$$

$$u_{10}(x,t) = -\frac{\beta}{2} - \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(6+\beta))(2-\beta)}+\frac{2\mu}{2-\beta}} \tag{71}$$

$$u_{11}(x,t) = -\frac{\beta}{2} + \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(-6-\beta))(-2+\beta)}+\frac{2\mu}{-2+\beta}} \tag{72}$$

$$u_{12}(x,t) = -\frac{\beta}{2} + \frac{\mu}{be^{\frac{1}{2}(x-\frac{1}{2}t(6-\beta))(2+\beta)}+\frac{2\mu}{2+\beta}} \tag{73}$$

In the previous solutions, μ is arbitrary constant.

The exact travelling wave solutions, Eqs. (62-73), are different from that obtained in [21]. Consequently, we can say that these are new exact travelling wave solution. The behavior of the travelling wave solution of Eq.(66) is shown in Fig.9 and Fig. 10 with a fixed value of $\beta = 0.5$.

5. Conclusions and Discussion

We have implemented the Bernoulli method to solve five NLPDEs. TheseNLPDEs are important models in mathematical physics namely, nonlinear dispersive equation, nonlinear Fisher-type equation, generalized ZK-BBM equation, general Burgers–Fisher equation and Drinfeld–Sokolov system. Applying Bernoulli method we have obtained new exact solutions compared to the published ones for the same problems.

In fact, this method is readily applicable to a large variety of NLPDs. We think that this method is simpler and more effective than other methods of solutions. Hence, we are investigating how to improve Bernoulli method to treat other complicated types of NLPDEs in the future.

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