International Journal of Mathematical Analysis and Applications
2014; 1(1): 9-19
Published online March 10, 2014 (http://www.aascit.org/journal/ijmaa)

Keywords
Exact Solution,
Kadomtsev-Petviashvili Equation,
Extended Tan-Cot Method,
Nonlinear Partial Differential Equations

Received: February 11, 2014
Revised: February 20, 2014
Accepted: February 21, 2014

Extended tan-cot method for the solitons solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation

Anwar Ja'afar Mohamad Jawad

Al-Rafidain University College , Baghdad, Iraq

Email address
anwar_jawad2001@yahoo.com

Citation
Anwar Ja'afar Mohamad Jawad. Extended tan-Cot Method for the Solitons Solutions to the (3+1)-Dimensional Kadomtsev-Petviashvili Equation. *International Journal of Mathematical Analysis and Applications.* Vol. 1, No. 1, 2014, pp. 9-19.

Abstract
The proposed extended tan-cot method is applied to obtain new exact travelling wave solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation and (2+1)-dimensional equation. The method is applicable to a large variety of nonlinear partial differential equations. The tan-cot method seems to be powerful tool in dealing with nonlinear physical models.

1. Introduction

Solitons are found in many physical phenomena. Solitons arise as the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems. Solitons are solitary waves with elastic scattering property. Due to dynamical balance between the nonlinear and dispersive effects these waves retain their shapes and speed to a stable waveform after colliding with each other. Onebasic expression of a solitary wave solution is of the form[1]:

$$u(x,t) = f(x - \lambda t) \tag{1}$$

where λ is the speed of wave propagation. For $\lambda > 0$, the wave moves in the positive x direction, whereas the wave moves in the negative x direction for $\lambda < 0$.

Travelling waves, whether their solution expressions are in explicit or implicit forms are very interesting from the point of view of applications.

These types of waves will not change their shapes during propagation and are thus easy to detect. Of particular interest are three types of travelling waves: the solitary waves, which are localized travelling waves, asymptotically zero at large distances, the periodic waves, which rise or descend from one asymptotic state to another. Recently, algebraic method, called the mapping method [2], is proposed to obtain exact travelling wave solutions for a large variety of nonlinear partial differential equations (PDEs). Other methods are proposed to obtain exact travelling wave solutions such as sine-cosine-function method[3], tanh-coth method[4-5], tan-cot-function method [6-7], sech method [8].

2. Description of Extended Tan-Cot Function Method

For a given nonlinear evolution equation, say, in four variables (3+1) - dimensional

$$P(u, u_t, u_x, u_y, u_z, u_{xx} \dots\dots\dots) = 0 \quad (2)$$

We seek a travelling wave solution of the form:

$$u(x, y, z, t) = U(\xi) \text{, and } \xi = kx + \alpha y + \beta z + \omega t + \theta_0 \quad (3)$$

Where $k, \alpha, \beta, \omega, \theta_0$ are considered constants. The following chain rule

$$\frac{\partial U}{\partial t} = \omega \frac{dU}{d\xi}, \quad \frac{\partial U}{\partial x} = k \frac{dU}{d\xi}, \quad \frac{\partial U}{\partial y} = \alpha \frac{dU}{d\xi}, \quad \frac{\partial U}{\partial z} = \beta \frac{dU}{d\xi}, \quad \frac{\partial^2 U}{\partial x^2} = k^2 \frac{d^2 U}{d\xi^2}$$

converted the PDE Eq.(2), to an ordinary differential equation ODE

$$Q(U, U', U'', U''', \dots\dots\dots .) = 0 \quad (4)$$

with Q being another polynomial form of their argument, which will be called the reduced ordinary differential equations of Eq.(4). Integrating Eq.(4) as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. However, the nonzero constants can be used and handled as well. Now finding the traveling wave solutions to Eq.(4) is equivalent to obtaining the solution to the reduced ordinary differential equation Eq.(4). introduce the ansatz, the new independent variable

$$T = \tan(\xi) \quad (5)$$

that leads to the change of variables:

$$\begin{aligned} \frac{dU}{d\xi} &= (1+T^2)\frac{dU}{dT} \\ \frac{d^2U}{d\xi^2} &= 2Y(1+T^2)\frac{dU}{dT} + (1+T^2)^2\frac{d^2U}{dT^2} \\ \frac{d^3U}{d\xi^3} &= 2(1+3T^2)(1+T^2)\frac{dU}{dT} + 6Y(1+T^2)^2\frac{d^2U}{dT^2} + (1+T^2)^3\frac{d^3U}{dT^3} \end{aligned} \quad (6)$$

The next step is that the solution is expressed in the form

$$U(\xi) = \sum_{i=0}^{m} a_i T^i + \sum_{i=1}^{m} b_i T^{-i} \quad (7)$$

where the parameter m can be found by balancing the highest-order linear term with the nonlinear terms in Eq.(4), and $k, \alpha, \beta, \omega, a_0, a_1, \dots, a_m, b_1, \dots, b_m$ are to be determined. Substituting Eq.(7) into Eq.(4) will yield a set of algebraic equations for $k, \alpha, \beta, \omega, a_0, a_1, \dots, a_m, b_1, \dots, b_m$ because all coefficients of T have to vanish. Having determined these parameters, knowing that m is positive integer in most cases, and using Eq.(7) we obtain analytic solutions $u(x,t)$,in a closed form.

The trigonometric functions can be extended to hyperbolic functions by using the complex form. So that a tanh-function expansion solution generates from a tan function expansion solution for $T = \tan(i\xi) = i\tanh(\xi)$, and a cot-function expansion solution generates from a coth function expansion solution for $T^{-1} = \cot(i\xi) = -i\coth(\xi)$.

3. Applications

In this section, we will bring to bear the new tan- cot method discussed in Section 2 to the (3+1)-dimensional KP equation and the (2+1)-dimensional equation which are very important in the field of nonlinear mathematical physics.

3.1. Exact Solutions to the (3+1)-Dimensional KP Equation

The (3 + 1)-dimensional KP-I equation is given by [9]:

$$(u_t + 6uu_x + u_{xxx})_x - 3(u_{yy} + u_{zz}) = 0 \quad (8)$$

This explains wave propagation in the field of plasma physics, fluid dynamics, etc. Soliton simulation studies for Eq.(8) have been done by Hasibun et al [10-11], Senatorski et al. [12],and Anwar et al. [13].

To study the travelling wave solutions to Eq.(8), substitute $u(x,y,z,t) = U(\xi)$, and $\xi = kx + \alpha y + \beta z + \omega t + \theta_0$ into Eq.(8) and integrating twice with zero constants, we have:

$$k^4 U'' + [k\omega - 3(\alpha^2 + \beta^2)]U + \frac{3}{2}k^2 U^2 = 0 \quad (9)$$

we postulate tan series , and the transformation given in Eq.(5), so that Eq.(9) reduces to:

$$k^4[2T(1+T^2)\frac{dU}{dT} + (1+T^2)^2\frac{d^2U}{dT^2}] + AU + \frac{3}{2}k^2U^2 = 0 \quad (10)$$

where:

$$A=[k\omega-3(\alpha^2+\beta^2)] \tag{11}$$

Now, to determine the parameterm, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq.(10) we balance U^2 with $U^{//}$, to obtain:m+2 = 2m , then m=2. The tan-cot method admits the use of the finite expansion for :

$$U=a_0+a_1T+a_2T^2+b_1T^{-1}+b_2T^{-2}$$

and

$$U^{/}=a_1+2a_2T-b_1T^{-2}-2b_2T^{-3}$$

and

$$U^{//}=2a_2+2b_1T^{-3}+6b_2T^{-4} \tag{12}$$

Substituting $U^{/}, U^{//}$ from Eq.(7) in Eq.(5),

$$\begin{aligned}&2k^4(a_1(T+T^3)+2a_2(T^2+T^4)-b_1(T^{-1}+T)-2b_2(T^{-2}+1))\\&+k^4(2a_2+2b_1T^{-3}+6b_2T^{-4}+4a_2T^2+4b_1T^{-1}+12b_2T^{-2}+2a_2T^4+2b_1T+6b_2)\\&+AU+\frac{3}{2}k^2(a_0(a_0+a_1T+a_2T^2+b_1T^{-1}+b_2T^{-2})+a_1(a_0T+a_1T^2+a_2T^3+b_1+b_2T^{-1})\\&+a_2(a_0T^2+a_1T^3+a_2T^4+b_1T+b_2)+b_1(a_0T^{-1}+a_1+a_2T+b_1T^{-2}+b_2T^{-3})\\&+b_2(a_0T^{-2}+a_1T^{-1}+a_2+b_1T^{-3}+b_2T^{-4}))=0\end{aligned}$$

then equating the coefficient of T^i , i= 0, 1, 2, 3, 4, -1, -2, -3, -4 leads to the following nonlinear system of algebraic equations:

$$\begin{aligned}&-4b_2k^4+k^4(2a_2+6b_2)+A(a_0)+\frac{3}{2}k^2(a_0a_0+a_1b_1+a_2b_2+b_1a_1+b_2a_2)=0\\&2k^4(a_1-b_1)+Aa_1+\frac{3}{2}k^2(a_0a_1+a_1a_0+a_2b_1+b_1a_2)=0\\&2k^4\,2a_2+k^4\,4a_2+Aa_2+\frac{3}{2}k^2(a_0a_2+a_1a_1+a_2a_0)=0\\&2k^4a_1+\frac{3}{2}k^2(a_1a_2+a_2a_1)=0\\&2k^4a_1\,2a_2+k^4\,2a_2+\frac{3}{2}k^2a_2a_2=0\\&-2k^4a_1b_1+4k^4b_1+Ab_1+\frac{3}{2}k^2(a_0b_1+a_1b_2+b_1a_0+b_2a_1)=0\\&-4k^4a_1b_2+k^4\,12b_2+Ab_2+\frac{3}{2}k^2(a_0b_2+b_1b_1+b_2a_0)=0\\&2k^4b_1+\frac{3}{2}k^2(b_1b_2+b_2b_1)=0\\&6k^4b_2+\frac{3}{2}k^2b_2b_2=0\end{aligned} \tag{13}$$

Solving the nonlinear systems of equations (13) we can get the following solutions,

$$u_j(x,y,z,t)=a_0+a_2\tan^2(\zeta)+b_2\cot^2(\zeta),\quad j=1,2,......,32 \qquad , \xi=kx+\alpha y+\beta z+\omega t+\theta_0 \tag{14}$$

There are two families of solutions:

Family 1

Where:

$$a_1=b_1=0\ ,\ a_2=-\frac{2}{3}k^2,\omega=\frac{A+3(\alpha^2+\beta^2)}{k}$$

With the following cases

Case 1

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k}, a_0 = -\frac{2}{3}(4+\sqrt{14})k^2, b_2 = -\frac{2}{3}k^2$$

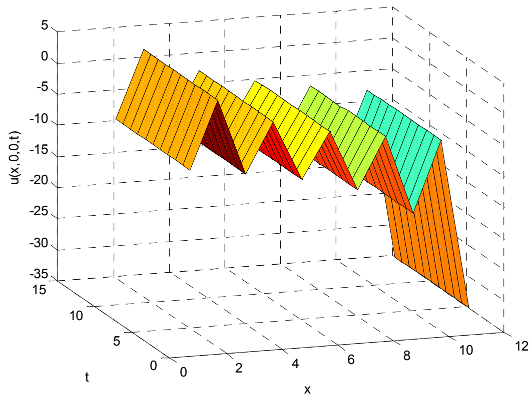
$$u_1(x,y,z,t) = -\frac{2}{3}k^2\{(4+\sqrt{14}) + (\tan^2(\zeta) + \cot^2(\zeta))\} \quad (15)$$

For $k=\alpha=\beta=1$, $\theta_0=0$, $\omega = 2\sqrt{14}+6$

$$u_1(x,y,z,t) = -\frac{2}{3}\{(4+\sqrt{14}) + (\tan^2(\zeta) + \cot^2(\zeta))\} \quad (16)$$

$$\xi = x + y + z + (2\sqrt{14}+6)t$$

Fig.(1). Represents the solitary soliton solution u(x,y,z,t) in Eq.(16) for $-5 \le x \le 5,\ 0 \le t \le 5,\ y = z = 0$



Fig(1).*Solitary soliton solution u(x,y,z,t) in Eq.(16) for* $-5 \le x \le 5,\ 0 \le t \le 5,\ y = z = 0$

Case 2

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k}\ a_0 = -\frac{2}{3}(4-\sqrt{14})k^2\ b_2 = -\frac{2}{3}k^2$$

$$u_2(x,y,z,t) = -\frac{2}{3}k^2\{(4-\sqrt{14}) + (\tan^2(\zeta) + \cot^2(\zeta))\} \quad (17)$$

Case 3

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k}\ a_0 = -\frac{2}{3}(4+\sqrt{14})k^2\ b_2 = -4k^2$$

$$u_3(x,y,z,t) = -\frac{2}{3}k^2\{(4+\sqrt{14}) + \tan^2(\zeta) + 6\cot^2(\zeta)\} \quad (18)$$

Case 4

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k}\ a_0 = -\frac{2}{3}(4-\sqrt{14})k^2\ b_2 = -4k^2$$

$$u_4(x,y,z,t) = -\frac{2}{3}k^2\{(4-\sqrt{14}) + \tan^2(\zeta) + 6\cot^2(\zeta)\} \quad (19)$$

Case 5

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2}{3}(4+\sqrt{14})k^2 \quad b_2 = -\frac{2}{3}k^2$$

$$u_5(x,y,z,t) = -\frac{2}{3}k^2\{(4+\sqrt{14}) + (\tan^2(\zeta) + \cot^2(\zeta))\} \tag{20}$$

Case 6

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2}{3}(4-\sqrt{14})k^2 \quad b_2 = -\frac{2}{3}k^2$$

$$u_4(x,y,z,t) = -\frac{2}{3}k^2\{(4-\sqrt{14}) + (\tan^2(\zeta) + \cot^2(\zeta))\} \tag{21}$$

Case 7

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2}{3}(4+\sqrt{14})k^2 \quad b_2 = -4k^2$$

$$u_7(x,y,z,t) = -\frac{2}{3}k^2\{(4+\sqrt{14}) + \tan^2(\zeta) + 6\cot^2(\zeta)\} \tag{22}$$

Case 8

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2}{3}(4-\sqrt{14})k^2 \quad b_2 = -4k^2$$

$$u_8(x,y,z,t) = -\frac{2}{3}k^2\{(4-\sqrt{14}) + \tan^2(\zeta) + 6\cot^2(\zeta)\} \tag{23}$$

Case 9

$$\omega = \frac{2\sqrt{34}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(6+\sqrt{34}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_9(x,y,z,t) = -\frac{2k^2}{3}\{(6+\sqrt{34}) + \tan^2(\zeta) + \cot^2(\zeta)\} \tag{24}$$

Case 10

$$\omega = \frac{2\sqrt{34}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(6-\sqrt{34}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{10}(x,y,z,t) = -\frac{2k^2}{3}\{(6-\sqrt{34}) + \tan^2(\zeta) + \cot^2(\zeta)\} \tag{25}$$

Case 11

$$\omega = \frac{-2\sqrt{34}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(6+\sqrt{34}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{11}(x,y,z,t) = -\frac{2k^2}{3}\{(6+\sqrt{34}) + \tan^2(\zeta) + \cot^2(\zeta)\} \tag{26}$$

Case 12

$$\omega = \frac{-2\sqrt{34}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(6-\sqrt{34}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{12}(x,y,z,t) = -\frac{2k^2}{3}\{(6-\sqrt{34}) + \tan^2(\zeta) + \cot^2(\zeta)\} \tag{27}$$

Case 13

$$\omega = \frac{\sqrt{190}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{k^2}{3}\left(12+\sqrt{190}\right) b_2 = -4k^2$$

$$u_{13}(x,y,z,t) = -\frac{k^2}{3}\{\left(12+\sqrt{190}\right) + 2\tan^2(\zeta) + 12\cot^2(\zeta)\} \tag{28}$$

Case 14

$$\omega = \frac{\sqrt{190}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{k^2}{3}\left(12-\sqrt{190}\right) b_2 = -4k^2$$

$$u_{14}(x,y,z,t) = -\frac{k^2}{3}\left[\left(12-\sqrt{190}\right) + 2\tan^2(\zeta) + 12\cot^2(\zeta)\right] \tag{29}$$

Case 15

$$\omega = \frac{-\sqrt{190}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{k^2}{3}\left(12+\sqrt{190}\right) b_2 = -4k^2$$

$$u_{15}(x,y,z,t) = -\frac{k^2}{3}\{\left(12+\sqrt{190}\right) + 2\tan^2(\zeta) + 12\cot^2(\zeta)\} \tag{30}$$

Case 16

$$\omega = \frac{-\sqrt{190}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{k^2}{3}\left(12-\sqrt{190}\right) b_2 = -4k^2$$

$$u_{16}(x,y,z,t) = -\frac{k^2}{3}\{\left(12-\sqrt{190}\right) + 2\tan^2(\zeta) + 12\cot^2(\zeta)\} \tag{31}$$

Family 2

Where:

$$a_1 = b_1 = 0 \;,\; a_2 = -\frac{4}{3}k^2 \;, \omega = \frac{A + 3(\alpha^2+\beta^2)}{k}$$

With the following cases:

Case 17

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(4 + \sqrt{14}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{17}(x, y, z, t) = -\frac{2k^2}{3}\{(4 + \sqrt{14}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \tag{32}$$

Case 18

$$\omega = \frac{2\sqrt{14}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(4 - \sqrt{14}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{18}(x, y, z, t) = -\frac{2k^2}{3}\{(4 - \sqrt{14}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \tag{33}$$

Case 19

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(4 + \sqrt{14}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{19}(x, y, z, t) = -\frac{2k^2}{3}\{(4 + \sqrt{14}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \tag{34}$$

Case 20

$$\omega = \frac{-2\sqrt{14}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}(4 - \sqrt{14}) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{20}(x, y, z, t) = -\frac{2k^2}{3}\{(4 - \sqrt{14}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \tag{35}$$

Case 21

$$\omega = \frac{4\sqrt{6}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{4}{3}(2 + \sqrt{6})k^2 \quad b_2 = -4k^2$$

$$u_{21}(x, y, z, t) = -\frac{4}{3}k^2\{(2 + \sqrt{6}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{36}$$

Case 22

$$\omega = \frac{4\sqrt{6}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{4}{3}(2 - \sqrt{6})k^2 \quad b_2 = -4k^2$$

$$u_{22}(x, y, z, t) = -\frac{4}{3}k^2\{(2 - \sqrt{6}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{37}$$

Case 23

$$\omega = \frac{-4\sqrt{6}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{4}{3}(2 + \sqrt{6})k^2 \quad b_2 = -4k^2$$

$$u_{23}(x,y,z,t) = -\frac{4}{3}k^2\{(2+\sqrt{6}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{38}$$

Case 24

$$\omega = \frac{-4\sqrt{6}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{4}{3}(2-\sqrt{6})k^2 \quad b_2 = -4k^2$$

$$u_{24}(x,y,z,t) = -\frac{4}{3}k^2\{(2-\sqrt{6}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{39}$$

Case 25

$$\omega = \frac{4\sqrt{11}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{4k^2}{3}(3+\sqrt{11}) \quad b_2 = -4k^2$$

$$u_{25}(x,y,z,t) = -\frac{4}{3}k^2\{(3+\sqrt{11}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{40}$$

Case 26

$$\omega = \frac{4\sqrt{11}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{4k^2}{3}(3\mp\sqrt{11}) \quad b_2 = -4k^2$$

$$u_{26}(x,y,z,t) = -\frac{4}{3}k^2\{(3-\sqrt{11}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{41}$$

Case 27

$$\omega = \frac{-4\sqrt{11}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{4k^2}{3}(3+\sqrt{11}) \quad b_2 = -4k^2$$

$$u_{27}(x,y,z,t) = -\frac{4}{3}k^2\{(3+\sqrt{11}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{42}$$

Case 28

$$\omega = \frac{-4\sqrt{11}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{4k^2}{3}(3-\sqrt{11}) \quad b_2 = -4k^2$$

$$u_{28}(x,y,z,t) = -\frac{4}{3}k^2\{(3-\sqrt{11}) + \tan^2(\zeta) + 3\cot^2(\zeta)\} \tag{43}$$

Case 29

$$\omega = \frac{2\sqrt{34}k^4 + 3(\alpha^2+\beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}\left(6+\sqrt{34}\right) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{29}(x,y,z,t) = -\frac{2}{3}k^2\{(6+\sqrt{34}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \tag{44}$$

Case 30

$$\omega = \frac{2\sqrt{34}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}\left(6 - \sqrt{34}\right) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{30}(x,y,z,t) = -\frac{2}{3}k^2\{(6-\sqrt{34}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \quad (45)$$

Case 31

$$\omega = \frac{-2\sqrt{34}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}\left(6 + \sqrt{34}\right) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{31}(x,y,z,t) = -\frac{2}{3}k^2\{(6+\sqrt{34}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \quad (46)$$

Case 32

$$\omega = \frac{-2\sqrt{34}k^4 + 3(\alpha^2 + \beta^2)}{k} \quad a_0 = -\frac{2k^2}{3}\left(6 - \sqrt{34}\right) \quad b_2 = -\frac{2}{3}k^2$$

$$u_{32}(x,y,z,t) = -\frac{2}{3}k^2\{(6-\sqrt{34}) + 2\tan^2(\zeta) + \cot^2(\zeta)\} \quad (47)$$

Results of solving (3+1)-dimensional KP in this paper are compatible with that results obtained by Anwar et al. [13].

3.2. Exact Solutions to the (2+1)-Dimensional Equation

The (2 + 1)-dimensional equation [13]:

$$u_{xt} - 4u_x u_{xy} - 2u_y u_{xx} + u_{xxxy} = 0 \quad (48)$$

Substituting $u(x, y, t) = U(\xi)$,and $\xi = kx + \alpha y + \omega t + \theta_0$ into Eq. (48) and integrating once with zero constant, we have:

$$k^2\alpha u^{///} + \omega u^{/} - 3k\alpha u^{/2} = 0 \quad (49)$$

we postulate the following tan-cot series , and the transformation given in Eq.(4),then Eq.(49) reduces to:

$$k^2\alpha[2(1+T^2)(3T^2+1)\frac{dU}{dT} + 6T(1+T^2)^2\frac{d^2U}{dT^2} + (1+T^2)^3\frac{d^3U}{dT^3}] + \omega[(1+T^2)\frac{dU}{dT}] - 3k\alpha[(1+T^2)\frac{dU}{dT}]^2 = 0 \quad (50)$$

Now, to determine the parameter m, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (50) we balance $U^{/2}$ with $U^{///}$, to obtain m+3 = 2m+2 , then m=1. The tan-cot method admits the use of the finite expansion for :

$U = a_0 + a_1 T + b_1 T^{-1}$,

$U^{/} = a_1 - b_1 T^{-2}$,

$U^{//} = 2b_1 T^{-3}$,

$U^{///} = -6b_1 T^{-4}$ (51)

Substituting $U^{/}, U^{//}, U^{///}$ from Eq.(51) in Eq. (50),

$$\begin{aligned}&2k^2\alpha(4a_1T^2-4b_1+a_1+a_1 3T^4-4b_1T^2)+12k^2\alpha b_1(T^{-2}+2+T^2)\\&-6k^2\alpha b_1(T^{-4}+3T^{-2}+3+T^2)+\omega[a_1-b_1T^{-2}+T^2a_1-b_1]\\&-3k\alpha a_1\left(a_1-b_1T^{-2}+T^2a_1-b_1\right)+3k\alpha b_1\left(a_1T^{-2}-b_1T^{-4}+a_1-T^{-2}b_1\right)\\&-3k\alpha a_1\left(a_1T^2-b_1+T^4a_1-T^2b_1\right)+\left(a_1-b_1T^{-2}+T^2a_1-b_1\right)3k\alpha b_1=0\end{aligned}\tag{52}$$

then equating the coefficient of T^i , i= 0,2,4, -2,-4 leads to the following nonlinear system of algebraic equations:

$$\begin{aligned}&2k^2\alpha a_1-2k^2\alpha b_1+[\omega-3k\alpha a_1+3k\alpha b_1](a_1-b_1)+6k\alpha b_1a_1=0\\&8k^2\alpha(a_1-b_1)+6k^2\alpha b_1+\omega a_1-3k\alpha a_1^2=0\\&6k^2\alpha a_1=0\\&-6k^2\alpha b_1-\omega b_1+3k\alpha a_1b_1=0\end{aligned}\tag{53}$$

Solving the nonlinear systems of equations (53) we can get:

$$\begin{aligned}&a_1=0, b_1=\frac{4k}{3}, \omega=-6k^2\alpha\\&u(x,y,t)=a_0+\frac{4k}{3}\cot\left(kx+\alpha y-6k^2\alpha t+\theta_0\right)\end{aligned}\tag{54}$$

For $a_0=1, k=\alpha=1, \theta_0=0$ the solitary solution in Eq.(54) is

$$u(x,y,t)=1+\frac{4}{3}\cot(x+y-6t)\tag{55}$$

Figure (2) solitary solution u(x, 1, t)for $y=1$, $-5\le x\le 5$, $0\le t\le 5$.

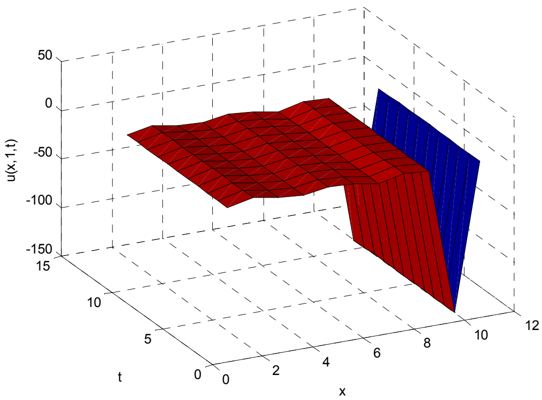


Figure (2).*solitary solution u(x,1,t) for* $-5\le x\le 5$, $0\le t\le 5$.

4. Conclusion

The exact travelling wave solutions to (3+1)-dimensional KP and (2+1)- dimensional equations have been studied by means of the extended tan-cot method. It can be easily seen that the implemented method used in this paper is powerful and applicable to a large variety of nonlinear partial differential equations.

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