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A solution by stochastic iteration method for nonlinear Black-Scholes equation with transaction cost and volatile portfolio risk in Hilbert space

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Abstract
We introduce a stochastic iteration method for the solution of a non-linear Black-Scholes equation which incorporates both the transaction cost and volatile portfolio risk measures. We first reduce the equation to a linear complementarity problem (LCP) and then propose an explicit steepest decent type stochastic method for the approximate solution of the LCP govern by a maximal monotone operator in Hilbert space. This scheme is shown to converge strongly to the non-zero solution of the LCP.

1. Introduction

In a complete financial market without transaction costs, the celebrated Black-Scholes no-arbitrage argument [1] provides not only a rational option pricing formula but also a hedging portfolio that replicates the contingent claim. However, the Black-Scholes hedging portfolio requires trading at all-time instants, and the total turnover of stock in the time interval [0, T] is infinite. Accordingly, when transactions cost – directly proportional to trading- is incorporated in the Black-Scholes model the resulting hedging portfolio is prohibitively expensive. It is therefore acceptable that in the continuous-timemodel with transaction costs, there is no portfolio that can replicate the European calloption with finite transaction costs. To proceed, the condition under which hedging can take place has to be relaxed such that the portfolio only dominates rather than replicates the value of the European call option at maturity. With this relaxation, there is always the trivial dominating hedging strategy of buying and holding one share of the stock on which the call is written. From arbitrage pricing theory, the price of an option should not be greater than the smallest initial capital that can support a dominating portfolio. Interesting results have evolved from this line of approach to pricing option without transaction cost, however, in the presence of constraints, in the presence of transaction costs.Soneret al [2] proved that the minimal hedging portfolio that dominates a European call option is the trivial one. In essence this suggests another way or technique to relaxing perfect hedging in models with transaction costs. Leland [3] used a relaxation with the effect that his model allowed transactions only at discrete times. By a formal δ - hedging argument, one can obtain a generalized option price that is equal to a Black- Scholes price but with an adjusted volatility of the form;

$\sigma^2 = \hat{\sigma}^2\left(1 - LeSgn(\partial_S^2 V)\right)$,

where$\sigma > 0$ is a constant historical volatility,$Le = \sqrt{\frac{2}{\pi}}\frac{C}{\hat{\sigma}^2\sqrt{\Delta t}}$ is the Leland number andΔt is time lag.

Assuming that inventor's preferences are characterized by an exponential utility function, Barles and Soner[4] derived a nonlinear Black- Scholes equation with volatility $\sigma = \sigma(\partial_S^2 V, S, t)$ given by

$$\sigma^2 = \hat{\sigma}^2\left(1 + \varphi\left(a^2 e^{r(T-t)} S^2 \partial_S^2 V\right)\right)^2,$$

Where $\varphi(X) \approx \left({}^3/_2\right)^{2/3} X^{1/3}$ for close to the originand$\hat{\sigma}^2$ is a constant.

Market models with transaction cost have been extensively dealt with (see for example Amster, et al [5], Avellanda and Paras [6]). A solution in Sobolev space which implies a weak solution of the nonlinear Black- Scholes equation has been obtained (see Osu and Olunkwa[7]). In a related paper,thesolution of a nonlinear Black-Scholes equation with the Crank-Nicholson scheme had also been obtained (see Mawah [8] and the references therein). The objective of this paper is to further incorporate volatile portfolio risk and show that a stochastic iteration method for the solution of a non-linear Black- Scholes equation which incorporates both the transaction cost and volatile portfolio risk measures exists and converges strongly to the nonzero solution.

2. The Model

Transaction costs as well as the volatile portfolio risk depend on the time –lag between two consecutive transactions. Minimizing their sum yields the optimal length of the hedge interval –time lag. This leads to a fully nonlinear parabolic PDE. If transaction costs are taken into account perfect replication of the contingent claim is no longer possible. Modeling the short rate $r = r(t)$ by a solution to a one factor stochastic differential equation.

$$dS = \mu(s,t)dt + \sigma(s,t)dw, \tag{1.1}$$

where$\mu(S,t)dt$ represent a trend or drift of the process and $\sigma(S,t)$ represents volatility part of the process, the risk adjusted Black-Scholes equation can be viewed as an equation with a variable volatility coefficient

$$\partial_t V + \frac{\sigma^2(s,t)}{2} S^2\left(1 - \mu(S\partial_S V)^{\frac{1}{3}}\right)\partial_S^2 V + rs\partial_S V - rV = 0, \tag{1.2}$$

where $\sigma^2(s,t)$ depends on a solution $V = V(s,t)$ and $\mu = 3\left(\frac{C^2 R}{2\pi}\right)^{\frac{1}{3}}$, since

$\hat{\sigma}^2(s,t) = \sigma^2(1 - \mu(S\partial_S^2 V(S,t))^{\frac{1}{3}}\cdot$

Incorporating both transaction costs and risk arising from a volatile portfolio into equation (1.2) we have the change in the value of portfolio to become.

$$\partial_t V + \frac{\hat{\sigma}^2(s,t)}{2} S^2 \partial_S^2 V + rS\partial_S V - rV = (r_{TC} + r_{VP})S$$

where

$r_{TC} = \frac{C|\Gamma|\hat{\sigma}S}{\sqrt{2\pi}}\frac{1}{\sqrt{\Delta t}}$is the transaction costs measure, $r_{Vp} = \frac{1}{2}R\hat{\sigma}^4 S^2\Gamma^2\Delta t$ is the volatile portfolio risk measure and $\Gamma = \partial_S^2 V$.

Minimizing the total risk with respect to the time lag Δtyields

$\min_{\Delta t}(r_{TC} + r_{VP}) = \frac{3}{2}\left(\frac{C^2 R}{2\pi}\right)^{\frac{1}{3}}\hat{\sigma}^2|S\partial_S^2 V|^{\frac{4}{3}}.$

They change in the value of the portfolio after minimizing the total risk with respect to time lag is given as

$$\partial_t V + \frac{\sigma^2(1-\mu(S\partial_S^2 v(s,t))^{\frac{1}{3}}}{2} S^2\partial_S^2 V + rS\partial_S V - rV - \frac{3}{2}\left(\frac{C^2 R}{2\pi}\right)^{\frac{1}{3}}\hat{\sigma}^2|S\partial_S^2 V|^{\frac{4}{3}} = 0\cdot \tag{1.3}$$

Let$f(S,t) = \frac{3}{2}\left(\frac{C^2 R}{2\pi}\right)^{\frac{1}{3}}\sigma^2|S\partial_S^2 V|^{\frac{4}{3}}$, and if we assume that there is no round trip transaction cost ,ie if we say there is no transaction cost by making $f(S,t) = 0$ then equation (1.3) becomes

$$\partial_t V + \frac{\sigma^2(1-\mu(S\partial_S^2 v(s,t))^{\frac{1}{3}}}{2} S^2\partial_S^2 V + rS\partial_S V - rV = 0\cdot \tag{1.4}$$

Equation (1.4) above is one form of Black-Scholes equation that incorporates both transaction costs and the volatile portfolio risk measures.

We are interested in the Black-Scholes equation of the form below that incorporates both the transaction costs measure and the volatile portfolio measure. That is a fully nonlinear parabolic equation of the form

$$\partial_t^v + \frac{1}{2}\hat{\sigma}^n s^2\left(1 \pm 3\left(\frac{C^2 R}{2\pi}\right)^{1/3}(s\partial_S^2 v)^{1/3}\right)\partial_S^2 v + rs\partial_S v - rv = 0, s > 0, t \in (0,T). \tag{1.5}$$

Note: (i) if R=0 or C=0, equation (1.5) reduces to the classical Black-Scholes equation.

(ii) Minus sign indicates Bid option price [9].

Denote $\alpha(F) = \frac{\sigma^2}{2}\left(1 - \mu F^{1/3}\right)F\mu = 3\left(\frac{C^2 R}{2\pi}\right)^{1/3}, \tau = T - t$ and $X = \ln\left({}^S/_E\right)(X \in R \to s > 0)$ with $F(X,T) = S\partial_S^2 v(s,t)$, then equation (1.5) can be transformed into a quasilinear equation of the form

$$\partial_x F = \partial_X^2\alpha(F) + \partial_x\alpha(F) + r\partial_X FT \in (0,T),\ X \in R, \tag{1.6}$$

with boundary conditions: $F(-\infty, T) = F(\infty, T) = 0$.

For $F_i^j \approx F(ih, jk), k = \frac{T}{m}, h = \frac{L}{n}$ we have (1.6) becoming;

$$\left(-\frac{k}{h^2}\alpha^1(F_{i-1}^{j-1}) + \frac{k}{h}r\right)F_{i-1}^{j-1}\left(1 + \frac{k}{h^2}\alpha^1(F_{i-1}^{j-1}) - \frac{k}{h}r - \frac{k}{h^2}\alpha^1(F_i^{j-1})\right)F_i^j - \frac{k}{h^2}\alpha^1(F_i^{j-1})F_{i-1}^j = F_i^{j-1} + \frac{k}{h}\left(\alpha(F_i^{j-1}) - \alpha(F_{i-1}^{j-1}\right) \quad (1.7)$$

For $i = -n+1, \dots, n-1$ and $j = 1, \dots, m, F_n^j = 0 = F_n^j$ and $F_0^0 = F(X_i, 0)$.

In matrix X form, we have

$$AF_i^j + b = 0. \quad (1.8)$$

Equation (1.8) in a linear complementarity problem (LCP) which is finding n-dimensional $F \in R^n$ such that

$$AF + b \geq 0, F \geq 0, F'(AF + b) = 0, \quad (1.9)$$

with

$$A = \begin{pmatrix} \left(r - \frac{n}{l}\omega\right)\frac{Tn}{mL} - \frac{Tn^2}{mL^2}\omega' & 0 & \cdots & 0 \\ 1 - r\frac{Tn}{mL} + \frac{Tn^2}{mL^2}(\omega - \omega') & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots 1 - r\frac{Tn}{mL} + \frac{Tn^2}{mL^2}(\omega - \omega') & \left(r - \frac{n}{l}\omega\right)\frac{Tn}{mL} - \frac{Tn^2}{mL^2}\omega' \end{pmatrix},$$

where $\omega = \alpha'(F_{i-1}^{j-1}), \omega' = \alpha'(F_i^{j-1})$ and

$$b = \begin{pmatrix} \frac{Tn}{mL}\varpi' \\ 0 \\ \frac{Tn}{mL}\varpi \end{pmatrix}.$$

Interest on LCP stems from the fact that many important Mathematical problems can be formulated as LCP (Cottle et al., [10]). This problem has been extensively studied by many authors including Murty[11].

We formulate (1.9) into an equivalent minimization problem:

$$\min\{F^T AF + Fb : x \geq 0, AF + b \geq 0\}. \quad (1.10)$$

We observe that from (1.10) that

$$f(F) = F^T AF + F^T b$$

has zero as a feasible minimize. Thus the problem reduces to that of searching for the global minimizer:

$$\{F \epsilon R^n : F \geq 0, b + AF \geq 0, F^T(AF + b) = 0\}.$$

But $F = 0, f(F) = 0$.

This suggests a reformation of the problem as a search for

$$V_F = \{F^* \epsilon R^n : F^* > 0 : \partial f(F^*) = 0\}, \quad (1.11)$$

where

$$\partial f(F^*) = \frac{\partial f(F^*)}{\partial F} = AF + b.$$

In this study stochastic gradient type recursive sequence is suggest:

$$F_{j+1} = F_j - \rho_j d_j, \quad (1.12)$$

where d_i is the estimate of $\partial f(F) = AF + b$ and $\{\rho_j\}$ is a sequence of positive scalars to be specified.

The procedure is a way of stochastically locating the set (1.11) when it exists. The iteration method described in this study differs from most iterative methods mainly in the way the search direction at each iteration and the starting point of the search algorithm are estimated to determine the optimum direction and provide maximum rate of decrease of $f(F)$.

Definition 1: Let H be a real Hilbert space with inner product (.,.) and norm $\|\cdot\|$. If T is a mapping with domain $D(T)$ in H, then T is said to be monotone if

$$\langle t_1^* - t_2^*, F_1 - F_2 \rangle \geq 0 \;\; \forall F_1, F_2 \in D(T), \forall t_1^* \in TF_1, \forall t_2^* \in TF_2 \quad (1.13)$$

We shall be interested in an important class of monotone operator which consists of the gradient of convex functions:Let f be a convex lower semi continuous function from H into $(-\infty, +\infty]$.

We assume that $f \neq +\infty$ and let $D(f) = \{F \in$ *H:fF<+∞* be the effective domain *f*.

For $F \in D(f)$, the set $\partial f(F) = \{t \in H : f(y) - f(F) \geq$ *t,y−F∀y∈DfF* is called the sub differential of *f* at *x*. The set $\partial f(F)$ is closed and convex.

We assume that H is a real separable Hilbert space with inner product (.,.) and norm $\|\cdot\|$. A random vector in H is a measurable mapping defined on a probability space $(\Omega, \Im, P)$ and taking values in H.

If u, v are random vectors in H and y is fixed vector in H, then $\|u\|, (u, v), (y, u)$ are real-valued random variables in the usual sense.

Let E denote the expectation operator. If $E\|u\| < \infty$, then Eu is defined by the requirement $E(y, u) = (y, Eu) \forall y \in H$.

Definition 2: Let $\{T_K(F_j)\}$ be a complete orthonormal basis of H associated with the data points $F_1, F_2, \dots, F_N$. Then $\hat{\partial} f$ is linear least square estimable in terms of some discrete function values computed from data point $F_1, F_2, \dots, F_N$ if the data points are suitably chosen such that:

$$\sum_{j=1}^N \{T_K(F_j), T_r(F_j)\} = \begin{cases} 0 \; if k \neq r \\ N if k = r \end{cases} \quad (1.14)$$

If H is $n-$dimensional Euclidean space a convenient basis for f, considered in (Okoroafor and Ekere,[12]) with concrete examples, is the set $\{t_j\}$ in R^n satisfying

$\sum_{j=0}^{N} t_{ij} = 0, i = 1,2,\dots,n, n+2 < N < \frac{n(n+1)}{2}$,

$\frac{1}{N}\sum_{j=1}^{N} t_{ij}^2 = 1.$

This yields the same result.

For the convex function f with

$$D(f) = \{F \in H: f(F) < \infty\} \neq \phi$$

Let $\hat{\partial} f$ be a single valued selection of ∂f. For every $u \in H$ and $\hat{\partial} f(u) \in \partial f(u)$, theTaylor theorem implies that

$$f(u+v) - f(u) = \langle v, \hat{\partial} f(u) + \tfrac{1}{2}\langle L(u)v, v\rangle + 0(\|v\|^2)\rangle, \quad (1.15)$$

for $v \in H$. Where $0(\cdot)$ indicates terms which can be ignored in the limit and $L(u)$ is the second derivative of f if it exists.

Remark 1: Where the second derivative of f does not exist in any sense, we consider the Taylor theorem of the form

$$f(u+v) - f(u) = \langle \hat{\partial} f(u), v\rangle + 0(\|v\|), \quad (1.16)$$

where, $0(\|.\|)$ indicates terms which can be ignored relative to v in the limit and ignore all second conditions since they have no influence on the convergence analysis of the method as we shall see in the sequel. For completeness assume the second derivative exists in some sense.Let

$y(F_j) = f\left(F^k + T(F_j)\right) - f(F^k), F^k \in D(f)$for a fixed k and $j = 1,2,\dots,N$.

Definition 3: The non-observable random errors of approximation on the data points $F_1, F_2, \dots, F_N$, is the sequence of random variables $\{e(F_j)\}$ satisfying $Ee(F_j) = 0$ for each j and $Ee(F_j)e(F_j) = \sigma^2\delta_{ij}$ where $0 < \sigma^2 < \infty$. A convenient basis for the function (1.15) is the complete orthonormal basis $\langle T_r\rangle$ in H, so that the approximation function is given by:

$$y(F_j) = \langle \partial f(F^*), T_r(F_j)\rangle + \tfrac{1}{2}\langle L(F^*), T_s(F_j)\rangle + e(F_j) \quad (1.17)$$

which is identifiable with (1.15).

The discrete function values $y(F_j)$, for each j, are real valued independent observable random variables performed on $x_j \in H$ whose distribution is that of $e(F_j)$. If at the point $F^k \in H$, for each k, the data points $F_1, F_2, \dots, F_N$ suitably chosen so that

$$F_j = F^k + T_r(F_j). \quad (1.18)$$

Then;

Theorem 1

A strong approximation of $\hat{\partial} f$ at F^k that is consistent is the random vector

$$d_k = \left[\sum_{j=1}^{N} T_r(F_j), T_r\right]^{-1} \sum_{j=1}^{N} T_r(F_j), y(F_j) \quad (1.19)$$

Which is the least square approximation computed from different data points $F_1, F_2, \dots, F_N$.

Proof: Assume

$$d_k = \left[\sum_{j=1}^{N} T_r(F_j), T_r\right]^{-1} \sum_{j=1}^{N} T_r(F_j), y(F_j)$$

$$\frac{1}{N}\sum_{j=1}^{N} t_{ij}^2 = 1$$

then

$$Ed_k = \frac{1}{N}\sum_{j=1}^{N} T_r(F_j), Ey(F_j)$$

$$= \hat{\partial} f_k.$$

So that

$$E\|d_k - \hat{\partial} f_k\| = E\|d_k - E\hat{\partial} f_k\|$$

$$= E\|T_r(F_j), e(F_j)\|$$

and

$$E\|d_k - \hat{\partial} f(F^k)\| = 0.$$

Moreover,

$$E\|d_k - \hat{\partial} f_k\|^2 = E\sum_{j=1}^{N} \langle T_r(F_j), e(F_j), T_r(F_j), e(F_j)\rangle$$

$= \frac{\sigma^2}{N}.$

Hence

$E\|d_k - \hat{\partial} f_k\|^2 \to 0$ as $N \to \infty$.

3. Getting the Domain of Attraction

Let $R_+^n - N(0)$ be partitioned into exclusive segments, $S_j, j = 1,2,\dots,t, n < t \leq 2^n$. Let F_j be chosen randomly in S_j, such that $f(F_j) > 0, \forall j$

Let $P_j = P(F_j = \alpha)$be the probability that $F_j = \alpha$ so that

$$P_j \geq 0, \sum_{j=1}^{t} P_j = 1. \quad (1.20)$$

Put

$P_j = \frac{f(F_j)}{\sum_{j=1}^{t} f(F_j)}$,

so that

$$\bar{F} = \sum_{j=1}^{t} F_j P_j = \sum_{j=1}^{t} \frac{F_j f(F_j)}{\sum_{j=1}^{t} f(F_j)}. \quad (1.21)$$

It is shown in (Okoroafor and Osu,[13]) that if

$$\hat{F} = \bar{F} - \rho d, \rho > 0, \quad (1.22)$$

where d is as in(1.19), then $f(\hat{F}) = \min\langle f(F_j): F_j \in s\rangle$. It follows that the segment S_T where $\hat{F} \in S_T$ contains $x > 0$ for which f(F) is minimum and hence we have $\varphi(V_{\bar{x}}) \subset S_T$ so that if $\{0\}$ is the attractor of the point $\bar{x}$ and $\varphi(\{0\}) \cap \varphi(V_{\bar{F}}) = \emptyset$ then $N(0) \cap N(V_{\hat{F}}) = \emptyset$ or else $N(0) = N(V_{\hat{F}})$ with global domain of attraction $\varphi(0) = \varphi(V_{\bar{F}})$. Where

$$V_{F^*} = \{F^* \in R^n: F^* > 0: \partial f(F^*) = 0\} \tag{1.23}$$

is a way of stochastically solving problem (1.11). Thus we have

Lemma 1: suppose that $V_{\hat{F}} \neq \phi$. thus there exists a neighborhood $N(V_{\hat{F}}) \subseteq D(\partial f)$ of $V_{\hat{F}}$ such that for any initial guess $\hat{F} \in \varphi(V_{\bar{F}})$, the non-negative minimizer $V_{\hat{F}}$ is obtained as the limit of iteratively constructed sequence $\{F_j\}_{j=1}^{\infty}$ generated form $\hat{F}$ by $F_{j+1} = F_j - \rho_j d_j$.

Then with $\hat{F}$ as our starting point we search for the minimizer of f as follows: starting at $\hat{F}$ as in Eq. (1.22).

A. Compute the d^k as in Eq. (1.19)

B. Compute the corresponding ρ as specified below

C. Compute $F_{j+1} = F_j - \rho_j d_j$.

Has the process converged? i.e., $\|F_{j+1} - F_j\| < \sigma, \sigma > 0$ if yes, then $F_{j+1} = F_j$ if no return to A.

Here we prove the strong convergence of the sequence to the solution of (1.22)

Theorem 1: Let $\{\rho_j$ be a real sequence such that

I. $\rho_0 = 1, 0 < \rho_j < 1 \forall j > 1$

II. $\sum_{j=0}^{\infty} \rho_j = \infty$

III. $\sum_{k=0}^{\infty} \rho_{2j} < \infty$

Then the sequence $\{F_j\}_{j=0}^{\infty}$ generated by $\hat{F} \in \varphi(V_{\hat{F}}) \subseteq D(\partial f)$ and defined iteratively by $F_{j+1} = F_j - \rho_j d_j$.remain in $D(\partial f)$ and converges strongly to $V_{\hat{F}}$.

Proof: Let $b_j = \rho_j \|d_j - \partial f_j\|$

Then $\{b_j\}_{j=1}^{\infty}$ is a sequence of independent random variable and from (1.18) $Eb_j = 0$ for each j.

Noticing that the sequence of partial sums $\{S_j\}_{j=1}^{\infty}$, $S_j = \sum_{j=1}^{k} b_j$, is a Martingale. Therefore,

$$ES_j^2 = \sum_{j=1}^{k} Eb_j^2 = \sum_{j=1}^{k} \rho_{2j}\, E\left\|\|d_j - \partial f_j\|\right\|^2$$

$= M^{-1}\sigma^2 \sum_{j=1}^{k} \rho_{2j}$.

And

$$\sum Eb_j^2 < \infty,\ since \sum_{j=1}^{k} \rho_{2j} < \infty$$

Hence by a version of Martingale convergence theorem (Whittle, [15]), we have

$$\lim_{k\to\infty} S_k = \sum_{j=1}^{\infty} b_j < \infty$$

So that

$$\lim_{j\to\infty} \rho_j = \|d_j - \partial f_j\| = 0$$

Noticing that in (1.22), A is positive definite so that $f(x)$ is convex and hence ∂f is monotone. But an earlier result in theory of monotone operators, due to (Chidume, [15]), shows that the sequence $\{F_j\}$ generated by $F_0 \in D(\partial f)$ and defined iteratively by:

$$F_{j+1} = F_j - \rho_j d_j.$$

remain in $D(\partial f)$ and converges strongly to $\{F^*: \partial f(F^*) = 0$. It follows from this result that our sequence converges strongly to V_F if $V_{F^*} \neq 0$.

4. Conclusion

Transaction costs as well as the volatile portfolio risk depend on the time-lag between two consecutive transactions. Minimizing their sum yields the optimal length of the hedge interval - time-lag, which leads to a fully nonlinear parabolic Black-Scholes PDE.We have used an implicit finite difference approximation and transform this PDE to the linear complementarity problem. We then constructed a steepest decent type stochastic sequence in a separable Hilbert space and show strong convergence to the solution of the LCP when exit.

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