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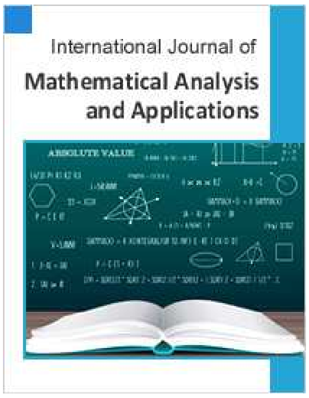

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Intuitionistic Fuzzy Set and Its Application in Selecting Specialization: A Case Study for Engineering Students

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Abstract
Intuitionistic fuzzy set (IFS) is useful in providing a flexible model to elaborate uncertainty and vagueness involved in decision making. In this paper, we reviewed the concept of IFS and proposed its application in selecting specialization. A case study for engineering students is explained in details. We give a method for computing degrees of membership and non-membership using given data. The suggested method can be applied in many similar real life situations.

1. Introduction

The concept of fuzzy sets (FS) introduced by L.A.Zadeh [12] has showed meaningful applications. Fuzzy set handles uncertainty and vagueness which Cantorian set could not address. The membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by K.T.Atanassov [9], [7] as intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin K.T.Atanassov [10],[8].E.Szmidt et al. [4] showed that intuitionistic fuzzy sets are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems too rough. Due to the flexibility of IFS in handling uncertainty, they are tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge E.Szmidt et al. [5]. S.K.De et al. [14] gave an intuitionistic fuzzy sets approach in medical diagnosis. So, Intuitionistic fuzzy set is used in modeling real life problems like sale analysis, new product marketing, financial services, decision making, psychological investigations and in many useful application in the life ….. Since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object E.Szmidt et al. [6], [4]. K.T.Atanassov [8], [11] carried out rigorous research based on applications of intuitionistic fuzzy sets. Many applications of IFS are carried out using distance measures approach. Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. Many distance measures between intuitionistic fuzzy sets have been proposed and researched in recent years E.Szmidt et al. [6], [3], W.Wang et al. [16] and used by E.Szmidt et al. [4], [5] in medical diagnosis. We show a novel application of intuitionistic fuzzy set in a more challenging area of decision making (i.e. department choice). An example of department determination will be presented, assuming there is a database (i.e. a description of a set of subjects 5, and a set of department D). We will describe the state of students knowing the results of their performance. The problem description uses the concept of IFS that makes it possible to render two important facts. First, values of each subject performance changes for each student. Second, in a department determination database describing department for different students, it should be taken into account that for different students aiming for the same department, values of the same subject performance can be different. We use the Hamming distance method given in E.Szmidt et al.[6], [3], [2] to measure the distance between each student and each department. The smallest obtained value, points out a proper department determination based on academic performance.

2. Concept of Intuitionistic Fuzzy Sets

Definition 1 L. A. Zadeh [12]: Let A be a nonempty set. A fuzzy set A drawn from X is defined as $A=\{\langle x,\mu_A(x)\rangle : x\in X\}$, where $\mu_A(x): X \rightarrow [0,\ 1]$ is the membership function of the fuzzy set A.

Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2 K. T. Atanassov [8]: Let X is a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where the functions $\mu_A(x),\nu_A(x):\ X\rightarrow[0,\ 1]$ define respectively, the degree of membership and degree of non-membership of the element $x\in X$ to the set A, which is a subset of X, and for every element $x\in X$, $0\le\mu_A(x)+\nu_A(x)\le 1$

Furthermore, we have $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x\in X$ to the IFS A and $\pi_A(x)\in[0,1]$ i.e., $\pi_A(x): X\rightarrow[0,1]$ and $0\le\pi_A\le 1$ for every $x\in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

For example, let A is an intuitionistic fuzzy set with $\mu_A(x)$ = 0.5 and $\nu_A(x)$ = 0.3 $\Rightarrow$ $\pi_A(x)$= 1- (0.5 + 0.3) = 0.2. It can be interpreted as "The degree that the object x belongs to IFS A is 0.5, the degree that the object does not belong to IFS A is 0.3 and the degree of hesitancy is 0.2".

3. Basic Relations and Operations on Intuitionistic Fuzzy Sets

IF A, B be IFS in X, then:

1. [inclusion] $A\subseteq B\leftrightarrow \mu_A(x)\le\mu_B(x)\,\text{and}\,\nu_A(x)\ge\nu_B(x)\forall x\in X$
2. [equality] $A=B\leftrightarrow \mu_A(x)=\mu_B(x)\,\text{and}\,\nu_A(x)=\nu_B(x)\forall x\in X$
3. [complement] $A^c=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$
4. [union] $A\cup B=\{\langle x,\max(\mu_A(x),\mu_B(x)),\min(\nu_A(x),\nu_B(x))\rangle : x\in X\}$
5. [intersection] $A\cap B=\{\langle x,\min(\mu_A(x),\mu_B(x)),\max(\nu_A(x),\nu_B(x))\rangle : x\in X\}$
6. [addition] $A\oplus B=\{\langle x,\mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x),\nu_A(x)\nu_B(x)\rangle : x\in X\}$
7. [multiplication] $A\otimes B=\{\langle x,\mu_A(x)\mu_B(x),\nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x)\rangle : x\in X\}$
8. [difference] $A-B=\{\langle x,\min(\mu_A(x),\nu_B(x)),\max(\nu_A(x),\mu_B(x))\rangle : x\in X\}$
9. [symmetric difference] $A\Delta B=\{\langle x,\max[\min(\mu_A,\nu_B),\min(\nu_A,\mu_B)],\min[\max(\nu_A,\mu_B),\max(\mu_A,\nu_B)]\rangle : x\in X\}$
10. [Cartesian product] $A\times B=\{\langle\mu_A(x)\mu_B(x),\nu_A(x)\nu_B(x)\rangle : x\in X\}$

Theorem 1: Let A and B be two IFS in a nonempty set X, then;

(i) A - B = $A\cap B^c$

(ii) A - B = B - A iff A = B

(iii) A - B = B^c - A^c.

Proof: (i) Let $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle x\in X\}$ and B= $\{\langle x,\mu_B(x),\nu_B(x)\rangle x\in X\}$ for A, B∈ X, then $A-B=\{\langle x,\min(\mu_A(x),\nu_B(x)),\max(\nu_A(x),\mu_B(x))\rangle : x\in X\}$ but $B^c=\{\langle x,\nu_B(x),\mu_B(x)\rangle x\in X\},\Rightarrow$

$A\cap B^c = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle : x\in X\}$ since

$A\cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle : x\in X\}$. The result follows.

(ii) $A-B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle : x\in X\}$

If A = B $\Rightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \forall x\in X$ from this, it is certain that B - A = A - B and the result follows.

(iii) $A-B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle : x\in X\}$

Given that, A^c = $\{\langle x, \nu_A(x), \mu_A(x)\rangle x\in X\}$ and B^c= $\{\langle x, \nu_B(x), \mu_B(x)\rangle x\in X\}$ it implies that, B^c - A^c = [x, min($\nu_B(x), \mu_A(x)), \max(\mu_B(x), \nu_A(x))\rangle x\in X\}$ and the result is straightforward.

Corollary1: Whenever A = B, A Δ B = B Δ A $\forall$ A, B$\in$ X.

Proof is straightforward from the proof of theorem 1 (ii).

Theorem 2: Let A and B be two IFS in a nonempty set X, then;

(i) A - A = ϕ (ii) A - ϕ = A (iii) A - B$\subseteq$ A (iv) A - B = ϕ iff A = B (v)

A-B = A iff B = ϕ (vi) A-B = A iff A$\cap$B=ϕ

It is easy to prove the above results.

Theorem 3: For IFS A,B,C in X and A $\subseteq$ B$\subseteq$ C, then we have; (i)B - A $\subseteq$C - A (ii)B Δ A $\subseteq$C Δ A

Proof: (i) Given that A, B, C$\in$ X and A $\subseteq$ B $\subseteq$ C $\Rightarrow$ "$\subseteq$" is transitive i.e. A$\subseteq$ B and B $\subseteq$C $\Rightarrow$ A $\subseteq$ C i.e. $\mu_A(x) \le \mu_B(x) \le \mu_C(x)$ and $\nu_A(x) \ge \nu_B(x) \ge \nu_C(x)$ $\forall$ x$\in$ X. Since A is the smallest of B and C, subtracting A from both side of B $\subseteq$ C means nothing, i.e. $\Rightarrow$ B - A $\subseteq$ C - A. The result follows.

Since "Δ" is the extension of "-", the result of (ii) follows.

Corollary 2: From the basic operations, we deduced the following relations:

a.AX B =BXA

b.(A X B) X C =A X (B X C)

c. A X (B$\cup$C) =(A X B)$\cup$ (A XC)

d.A X (B$\cap$C) = (A X B)$\cap$ (A X C)

f.A X (B$\oplus$C) = (A X B)$\oplus$(A X C)

e.AX (B$\otimes$C) = (A X B)$\otimes$(A X C)

4. Algebra Laws in Intuitionistic Fuzzy Sets

Let a, band c be IFS in x ,then the following algebra follow:

i. Complementary Law: $(.A^c)^c = A$

ii. Idempotent Laws: (i)A $\cup$A$^=$ A (ii)A$\cap$A = A

iii. Commutative Laws: (i) A$\cup$ B = B $\cup$ A (ii) A$\cap$ B = B$\cap$ A

(iii)A$\oplus$ B = B$\oplus$A (iv) A$\otimes$ B = B$\otimes$ A

i. Associative Laws:

(i) (A$\cup$B)$\cup$C = A$\cup$(B $\cup$C) (ii) (A$\cap$ B)$\cap$C = A$\cap$(B $\cap$C) (iii)A$\oplus$(B$\oplus$ C)=(A$\oplus$ B)$\oplus$C(iv) A$\otimes$(B$\otimes$C)= (A $\otimes$B)$\otimes$C

ii. Distributive Laws:

(i)A$\cup$(B$\cap$ C) = (A$\cup$B)$\cap$(A$\cup$C)

(ii) A$\cap$(B$\cup$ C)= (A$\cap$ B)$\cup$(A$\cap$ C)

(iii)A$\oplus$(B $\cup$ C) =(A$\oplus$ B)$\cup$(A$\oplus$ C)

(iv) A$\oplus$(B $\cap$ C) =(A$\oplus$ B)$\cap$ (A$\oplus$ C)

(v) A$\otimes$ (B $\cup$ C) =(A$\otimes$ B)$\cup$(A$\otimes$ C)

(vi) A$\otimes$ (B $\cap$ C) =(A$\otimes$ B)$\cap$ (A$\otimes$C)

vi. De Morgan's laws:

(i) $(A\cup B)^c = A^c \cap B^c$

(ii) $(A\cap B)^c = A^c \cup B^c$

(iii)$(A\oplus B)^c = A^c \otimes B^c$

(iv) $(A\otimes B)^c = A^c \oplus B^c$

vii. Absorption Laws: (i) A$\cap$ (A$\cup$ B) = A (ii) A$\cup$ (A$\cap$ B) = A

(i) ϕ^c=A (ii) Ac=ϕ (iii) A$\cup$$\phi$ =A (iv) A$\cap$$\phi$=$\phi$ (v) A $\cap$ Ac =ϕ

(i) A$\cup$ X =X (ii) A$\cup$ A^c =X (iii) A$\cap$ X =A

Note: Distributive Laws hold for both right and left hands. The proofs follow from the basic operations.

Theorem4: Let A, B ,C be IFS in X and B$\subseteq$C, then we have; (i)

A$\oplus$B $\subseteq$A$\oplus$ C (ii) A$\otimes$B$\subseteq$A$\otimes$ C (iii) A$\cup$ B $\subseteq$A$\cup$ C (iv) A$\cap$ B $\subseteq$A$\cap$ C.

Proof:(i) Given that A,B, C $\in$ X and B$\subseteq$C, it means $\mu_B(x) \le \mu_C(x)$ and $\nu_B(x) \ge \nu_C(x)$ for every x $\in$ X. If another IFSA$\in$X is added to B$\subseteq$C, it is certain that, A$\oplus$B $\subseteq$A$\oplus$ C and the result follows.

Results of (ii) - (iv) follow from the proof of (i)

Theorem 5: Let A and B be IFS in X, then (i) A$\cap$ B = A or A$\cap$ B = B, (ii) A$\cup$ B = A or A $\cup$ B = Biff A = B.

Proof: (i) For A, B, C$\in$X, it implies that A$\cap$ B $\in$ X. If A = B$\Rightarrow$ $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \forall$ x $\in$ X. Since A = B, from idempotent laws ,A$\cap$ B= A or A$\cap$ B= B. The result follows.

Result of (ii) follows from (i).

Definition 3 E. Szmidt [2]: The Hamming distance $d_{n-H}(A,B)$ between two IFS A and B is defined as

$$d_{n-H}(A,B) = \tfrac{1}{2n}\sum_{i=1}^{n} |\ \mu_A(x_i) - \mu_B(x_i)\ | + |\ \nu_A(x_i) - \nu_B(x_i)\ | + |\ \pi_A(x_i) - \pi_B(x_i)\ | \\ ,X = \{x_1, x_2, ..., x_n\} \text{ for } i = 1,2,....,n.$$

5. Application of Intuitionistic Fuzzy Sets In Selecting Specialization

In Egypt, For Example, IN Faculty Of Engineering, Tanta University, We suffer from Lack of specialization and How and Why choosing the department to each student where the famous in Egypt each student choose the department according to that its famous and ignoring that if this student eligible to this department or not. So, we need choosing the suitable department to each student by its degree of each object where each department needs to be excellent in specific objects.

For example:

* In electrical department need the student to be excellent in Mathematical, Physics and Computer (Logic).

* In Architecture department need the student to be excellent in drawing engineered and Computer (Logic).

* In Mechanics department need the student to be excellent in drawing engineered, mechanics and Computer (Logic).

* In civil department need the student to be excellent in drawing engineered and Computer (Logic).

We use intuitionistic fuzzy sets as tool since it incorporate the membership degree (i.e. the marks of the questions answered by the student),the non-membership degree (i.e. the marks of the questions the student failed) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

Let $S = \{s_1, s_2, s_3, s_4\}$ be the set of students, D= (architecture, electrical, civil, mechanics} be the set of department and Su= (logic, Mathematics, drawing engineered, physics, Physics, mechanics} be the set of subjects related to the departments. We assume the above students sit for examinations(i.e. over 100 marks total) on the above mentioned subjects to determine their department placements and choices. The table below shows departments and related subjects requirements.

Table 1. *Departments vs. Subjects.*

	Logic	Mathematics	Drawing engineered	Physics	Mechanics
Architecture	(0.8,0.1,0.1)	(0.5,0.3,0.2)	(0.9,0.1,0)	(0.5,0.5,0)	(0.5,0.4,0.1)
Electrical	(0.8,0.1,0.1)	(0.9,0.1,0)	(0.5,0.4,0.1)	(0.8,0.1,0.1)	(0.5,0.5,0)
Civil	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.1,0)	(0.5,0.2,0.3)	(0.6,0.3,0.1)
Mechanical	(0.6,0.3,0.1)	(0.5,0.5,0)	(0.8,0.1,0.1)	(0.5,0.5,0)	(0.9,0.1,0)

Each performance is described by three numbers i.e. membership, non-membership and hesitation margin. After the various examinations, the students obtained the following marks as shown in the table below.

Table 2. *Studentsvs Subjects.*

	Logic	Mathematics	Drawing engineered	Physics	mechanics
S1	(0.9,0.1,0)	(0.9,0.1,0)	(0.6,0.2,0.2)	(0.9,0.1,0)	(0.5,0.5,0)
S2	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.7,0.2,0.1)
S3	(0.7,0.1,0.2)	(0.6,0.3,0.1)	(0.7,0.1,0.2)	(0.5,0.4,0.1)	(0.4,0.5,0.1)
S4	(0.6,0.4,0.0)	(0.8,0.1,0.1)	(0.6,0.0,0.4)	(0.6,0.3,0.1)	(0.5,0.3,0.2)

Using Def. 3 above to calculate the distance between each studentand each specialization with reference to the subjects, we get the table below.

Table 3. *Students vs. Careers.*

	architecture	Electrical	Civil	Mechanics
S1	0.26	0.08	0.24	0.34
S2	0.22	0.28	0.24	0.2
S3	0.12	0.22	0.16	0.22
S4	0.26	0.24	0.22	0.28

From the above table, the shortest distance gives the proper department determination. S_1 is to read electrical (Electrical Engineer), $S_2$is to read mechanics (Mechanical Engineer), $S_3$is to read architecture (architecture Engineer), and$S_4$is to read civil (civil Engineer).

6. Conclusion

The case study presented in this mark can be applied in many real life applications. For example, medicine, political or social case.

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