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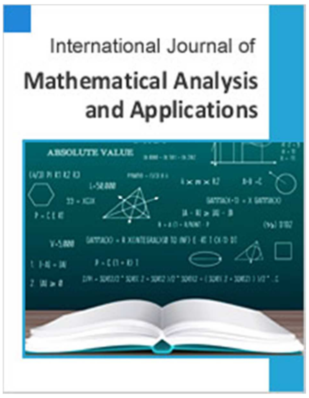

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The Behavior of Cauchy-Type Integral Near the Boundary of the Semicylindrical Domain

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Abstract
The purpose of this work is the elucidation of the behavior of Cauchy-type integrals near the boundary semicylindrical domain to $\theta(\delta)$- characteristics celebrated jordanovic of closed curves (in the case when $\theta(\delta)\sim\delta$, this class of curves is much wider class of piecewise-smooth class of curves for which the chord length relation to the pulling together arch are limited (K-curves), and also in it existence of cusps is allowed). The main characteristics for functions $f \in C_\Delta$ - the mixed and private modules of continuity which was proven continuously extendibility of n −multiple Cauchy-type integral to the border of the semicylindrical domain and the limit values of the types of Sokhoskiy's formulas.

1. Introduction

Let γ^k be a closed Jordan rectifiable curve (c.j.r.c.) with length l_k and diameter d_k in the complex planes of variables $z_k(\mathrm{k} = \overline{1,\mathrm{n}})$. A bounded domain D_k^+ with the bound γ^k we call an internal, the padding of $D_k^+ \cup \gamma^k$ we call an external and denote by D_k^-.

The contours $\gamma^1, \gamma^2, \ldots, \gamma^n$ defines in whole complex space of n variables 2^n various semicylindrical domains which are obtained by all possible combinations of characters in the topological multiplication

$$D_1^{\pm} \times D_2^{\pm} \times \ldots \times D_n^{\pm}.$$

Among them: one is of the type of $D_1^+ \times D_2^+ \times \ldots \times D_n^+$ ($D_1^- \times D_2^- \times \ldots \times D_n^-$), which we denote by D^+ (D^-); C_n^1 are domains of the type of $D_1^+ \times D_2^+ \times \ldots \times D_{p-1}^+ \times D_p^- \times D_{p+1}^+ \times \ldots \times D_n^+$ ($D_1^- \times D_2^- \times \ldots \times D_{p-1}^- \times D_p^+ \times D_{p+1}^- \times \ldots \times D_n^-$) which we denote by D_{-p}^+ (D_{+p}^-); similarly, C_n^2 are domains of the type of

$$D_1^+ \times D_2^+ \times \ldots \times D_{p-1}^+ \times D_p^- \times D_{p+1}^+ \times \ldots \times D_{q-1}^+ \times D_q^- \times D_{q+1}^+ \times \ldots \times D_n^+$$

$$(D_1^- \times D_2^- \times \ldots \times D_{p-1}^- \times D_p^+ \times D_{p+1}^- \times \ldots \times D_{q-1}^- \times D_q^+ \times D_{q+1}^- \times \ldots \times D_n^-)$$

which we denote by D_{-pq}^+ (D_{+pq}^-) and etc..

Borders of all these semicylindrical domains have the common part, namely, $\Delta = \gamma^1 \times \gamma^2 \times \ldots \times \gamma^n$ which is called *the core*.

If the function $\Phi(z) := \Phi(z_1, z_2, \ldots z_n)$ is defined in D^+ and for an arbitrary $t := (t_1, t_2, \ldots t_n) \in \Delta$ there exists

$$\Phi^+(t) := \lim_{D^+ \ni z \to t} \Phi(z),$$

we say $\Phi(z)$ is continuously extended up to the bound of D^+. Analogically, define continuously extendibility up to core of domains $D^-, D^{\pm}_{-p}\left(D^{-}_{+p}\right), D^{\pm}_{-pq}\left(D^{-}_{+p}\right)$, etc.. The corresponding limit values of the function $\Phi(z)$ is denoted by $\Phi^-(t), \Phi^{+}_{-p}(t)\left(\Phi^{-}_{+p}(t)\right), \Phi^{+}_{-pq}(t)\left(\Phi^{-}_{+p}(t)\right)$ etc., respectively.

If $\Phi(z)$ is continuously extended up to core from the domain D^{+}_{-} then we say that the function $\Phi(z)$ is continuously extendible up to core. We say that a function $\Phi(z)$ is continuously extended to the given boundary point of semicylindrical domain, if the function $\Phi(z)$ tends to a given boundary point along any path, while remaining at all times in this semicylindrical domain. The corresponding limits we call boundary values $\Phi(z)$ in this domain, and we denote them as well as the boundary values $\Phi(z)$ on the core Δ, with the replacement t core of Δ corresponding boundary point. It is easily seen that if the function $\Phi(z)$ is continuously extended to the core Δ from every 2^n semicylindrical domain which boundaries have a common core of Δ, then it will continuously be extended to any boundary point of each of these semicylindrical domains.

Let us consider n - multiple integral of Cauchy-type

$$\Phi(z_1, z_2, \dots, z_n) = \frac{1}{(2\pi i)^n} \int_\Delta \frac{f(\tau_1, \tau_2, \dots, \tau_n)}{\prod_{k=1}^n (\tau_k - z_k)} d\tau_1 d\tau_2 \dots d\tau_n, \quad (1)$$

where $f(\tau_1, \tau_2, \dots, \tau_n) \in C_\Delta, C_\Delta$- is the space of continuous functions on Δ.

In the paper [1] was studied the behavior of integral (1) for smooth contours and functions of Holder's class and in papers [2], [3] and [4] (for *n=2*) under some assumptions on the curves γ^k and the function $f(\tau)$ the continuity up to core of the integral was investigated. In [5] and [6] (for *n=2*) the investigation of the integral (1) was extended to a case of summable density. The papers [7], [8], [9] and [12] are focused on study the integral near a bicylindrical fields and [13-16] contain results the behavior of integral Martinelli-Bochner, which (1) turns into a Cauchy-type integral for n = 1.

In the current work the behavior of n -multiple integral (1) on the border of semicylindrical domain in terms of the continuity modulus and $\theta(\delta)$ characteristic curve γ^k $(k = \overline{1,n})$ is studied under the most common assumptions concerning function f and curves γ^k (it was first given in [7], and then generalized in [10], [15-17].) The paper is organized as follows: in the next section are presented some results and notations which will be used in the formulation of the main theorems. In Section 3 we give our main results and their proofs.

2. Preliminaries

For the brevity of the writing we introduce the following notations as in [1]

$$\tau = (\tau_1, \tau_2, \dots \tau_n), \;\; t = (t_1, t_2, \dots, t_n), z = (z_1, z_2, \dots z_n),$$

$$\tau_{t_p} = \left(\tau_1, \tau_2, \dots \tau_{p-1}, t_p, \tau_{p+1}, \dots \tau_n\right),$$

$$\left[t_{\tau_p} = \left(t_1, t_2, \dots, t_{p-1}, \tau_p, t_{p+1}, \dots, t_n\right)\right],$$

$$\tau_{t_{pq}} = \left(\tau_1, \tau_2, \dots \tau_{p-1}, t_p, \tau_{p+1}, \dots \tau_{q-1}, t_q, \tau_{q+1}, \dots \tau_n\right),$$

$$\left[t_{\tau_{pq}} = \left(t_1, t_2, \dots, t_{p-1}, \tau_p, t_{p+1}, \dots, t_{q-1}, \tau_q, t_{p+1}, \dots, t_n\right)\right],$$

$$N = \{1,2 \dots, n\}, N_{[p]} = \{1,2, \dots p-1, p+1, \dots, n\},$$

$$N_{[pq]} = \{1,2, \dots, p-1, p+1, \dots, q-1, q+1, \dots, n\} \text{ etc..}$$

$$t_{[p]} = (t_1, t_2, \dots, t_{p-1}, t_{p+1}, \dots, t_n),$$

$$t_{[pq]} = (t_1, t_2, \dots, t_{p-1}, t_{p+1}, \dots, t_{q-1}, t_{q+1}, \dots, t_n), \text{ etc..}$$

$$d\tau = d\tau_1 d\tau_2 \dots d\tau_n, \; d\tau_{[p]} = d\tau_1 d\tau_2 \dots d\tau_{p-1} d\tau_{p+1} \dots d\tau_n,$$

$$d\tau_{pq} = d\tau_1 d\tau_2 \dots d\tau_{p-1}, d\tau_{p+1} \dots d\tau_{q-1} d\tau_{q+1} \dots d\tau_n \text{ etc..}$$

$$\Delta_{[p]} = \gamma^1 \times \gamma^2 \times \gamma^{p-1} \times \gamma^{p+1} \times \dots \times \gamma^n,$$

$$\Delta_{[pq]} = \gamma^1 \times \gamma^2 \times \dots \times \gamma^{p-1} \times \gamma^{p+1} \times \dots \times \gamma^{q-1} \times \gamma^{q+1} \times \dots \times \gamma^n,$$

$$\eta^2 = \eta_1^2 \eta_2^2 \dots \eta_n^2, \;\; \eta_{[p]}^2 = \eta_1^2 \eta_2^2 \dots \eta_{p-1}^2 \eta_{p+1}^2 \dots \eta_n^2,$$

$$\eta_{[pq]}^2 = \eta_1^2 \eta_2^2 \dots \eta_{p-1}^2 \; \eta_{p+1}^2 \dots \eta_{q-1}^2 \eta_{q+1}^2 \dots \eta_n^2, \text{ etc..}$$

$$\tau - z = \prod_{k=1}^{n} (\tau_k - z_k), \qquad (\tau - z)_{[p]} = \prod_{\substack{k=1 \\ k \neq p, p=\overline{1,n}}}^{n} (\tau_k - z_k),$$

$$(\tau - z)_{\substack{[pq] \\ p<q}} = \prod_{\substack{k=1 \\ k \neq p,q \\ p,q=\overline{1,n}}}^{n} (\tau_k - z_k) \text{ etc..}$$

Then be these notations (1) takes form

$$\Phi(z) = \frac{1}{(2\pi i)^n} \int_\Delta \frac{f(\tau)}{\tau - z} d\tau.$$

Let us denote by $\Delta f(\tau; t)$ the following

$$\Delta f(\tau; t) := f(\tau) - \sum_{p=1}^{n} f\left(\tau_{t_p}\right) + \sum_{\substack{p=1 \\ p<q}}^{n} \sum_{q=1}^{n} f\left(\tau_{t_{pq}}\right) - \dots +$$

$$+(-1)^{n-2}\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n} f(\tau_{pq})+(-1)^{n-1}\sum_{p=1}^{n} f\left(t_{\tau_p}\right)+(-1)^{n} f(t).$$

It is easy to verify that holds the identity

$$f(\tau)=f(\tau;t)+\sum_{p=1}^{n}\Delta f\left(\tau_{t_p};t\right)+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\Delta f\left(\tau_{t_{pq}};t\right)+\cdots+$$

$$+\sum_{p=1}^{n}\sum_{\substack{q=1\\p<q}}^{n}\Delta f\left(\tau_{t_p};t\right)+\sum_{p=1}^{n}\Delta f\left(t_{\tau_p};t\right)+f(t)$$

holds. Using this and (1) we have

$$\Phi(z)=\frac{1}{(2\pi i)^n}\int_{\Delta}\frac{\Delta f(\tau;t)}{\tau-z}d\tau+\sum_{p=1}^{n}\frac{\varkappa_p(z_p)}{(2\pi i)^{n-1}}\int_{\Delta[p]}\frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau-z)_{[p]}}d\tau_{[p]}+$$

$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\frac{\varkappa_p(z_p)\varkappa_q(z_p)}{(2\pi i)^{n-2}}\int_{\Delta[pq]}\frac{\Delta f\left(\tau_{t_{qp}};t\right)}{(\tau-z)_{[pq]}}d\tau_{[pq]}+\cdots+$$

$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\frac{\varkappa(z)_{[pq]}}{(2\pi i)^2}\int_{\gamma^p}\int_{\gamma_q}\frac{\Delta f\left(t_{\tau_{pq}};t\right)}{(\tau_p-z_p)(\tau_q-z_q)}d\tau_p d\tau_q+$$

$$+\sum_{p=1}^{n}\frac{\varkappa(z)_{[p]}}{2\pi i}\int_{\gamma^p}\frac{\Delta f\left(t_{\tau_p};t\right)}{\tau_p-z_p}d\tau_p+\varkappa(z)f(t)=$$

$$=\Psi^n(z)+\Psi_p^{n-1}\left(z_{t_p}\right)+\Psi_{pq}^{n-2}\left(z_{t_{pq}}\right)+\cdots+\Psi_{pq}^{2}\left(t_{z_{pq}}\right)+\Psi_p^{1}\left(t_{z_p}\right)+\varkappa(z)f(t), \qquad (2)$$

where $\varkappa(z):=\varkappa_1(z_1)\varkappa_2(z_2)\ldots\varkappa_n(z_n)$, $\varkappa_i(z_i)=\frac{1}{2\pi i}\int_{\gamma^i}\frac{d\tau_i}{\tau_i-z_i}, i=\overline{1,n}.$

The integrals on the right hand side in (2) we consequently denote by

$$\Psi^n(z),\ \Psi_p^{n-1}\left(z_{t_p}\right),\ \Psi_{pq}^{n-2}\left(z_{t_{pq}}\right),\ldots\ldots,\ \Psi_{pq}^{2}\left(t_{z_{pq}}\right),\ \Psi_p^{1}\left(t_{z_p}\right),$$

which will be further used.

Let γ^k be a closed rectiable Jordan curve (c.r.j.c.) $t_k=t(s_k)$, $(0\le s_k\le l_k)$, l_k be the length of the curve and γ^k be an equation of the curve in arc coordinates $k=\overline{1,n}$. Let us denote

$$\theta(t_k,\delta)=mes\{\tau\epsilon\gamma^k:\left|\tau_{t_k}-\tau\right|\le\delta,\delta\in(0,d_k]\},$$

$$d_k=\sup_{\tau_k,t_k\epsilon\gamma^k}|\tau_k-t_k|,\ \theta_k(\delta)=\sup_{t_k\epsilon\gamma^k}\theta(t_k,\delta),k=\overline{1,n}.$$

The function $\Theta_k(\delta)$ is chosen as the main characteristics of the curve γ^k. Monotonically increasing function $\theta_k^v(\delta)$ defined by

$$\theta_k^v(\delta):=sup\{y:\Theta_k(y)\le\delta,\delta\epsilon(0,d_k]\}$$

is called a generalized inverse with respect to $\theta_k(\delta)$. The concept of generalized inverse function is introduced and studied in [8]. To investigate the behavior of integral (1) on the boundary of semicylindrical domain appears the following main characteristics to function $f\in C_\Delta$:

1). mixed continuity module (for the case $n=2$ was given in [11])

$$\omega_f^N(\delta):=\omega_f^N(\delta_1,\delta_2,\ldots,\delta_n)=\delta_1\delta_2\ldots\delta_n\sup_{\xi\ge\delta_1,\ldots,\xi_n\ge\delta_n}\frac{\omega^N(f;\xi_1,\xi_2,\ldots,\xi_n)}{\xi_1\xi_2\cdots\xi_n}:=\delta\sup_{\xi\ge\delta}\frac{\omega^N(f,\xi)}{\xi},$$

where

$$\delta_i>0,i=\overline{1,n},\ \omega^N(f;\delta)=\sup_{|\tau_1-t_1|\le\delta_1,\ldots,|\tau_n-t_n|\le\delta_n}|\Delta f(\tau;t)|:=\sup_{|\tau-t|\le\delta}|\Delta f(\tau;t)|$$

2). private continuity modules

$$\omega_f^{N[p]}(\delta) := \delta_{[p]} \sup_{\xi_{[p]} \geq \delta_{[p]}} \frac{\omega^{N[p]}(f;\xi_{[p]})}{\xi_{[p]}}$$

where

$$\omega_f^{N[p]}(\delta) = \sup_{t_p} \sup_{|\tau_{t_p} - t| \leq \delta_{[p]}} \left|\Delta f\left(\tau_{t_p}; t\right)\right|, p = \overline{1,n}$$

$$\omega_{\substack{f \\ p<q}}^{N[pq]}(\delta_{[pq]}) := \delta_{[pq]} \sup_{\xi_{[pq]} \geq \xi_{[pq]}} \frac{\omega^{N[pq]}(f;\xi_{[pq]})}{\xi_{[pq]}}$$

where

$$\omega_{p<q}^{N[pq]}(f;\delta_{[pq]}) = \sup_{t_p, t_q} \sup_{|\tau_{t_{pq}} - t| \leq \delta_{[pq]}} \left|\Delta f\left(\tau_{t_{pq}}; t\right)\right| \mathrm{p} = \overline{1,\mathrm{n}}$$

$$\omega_f^{pq}(\delta_p, \delta_q) = \delta_p \delta_q \sup_{\xi_p \geq \delta_p, \xi_q \geq \delta_q} \frac{\omega^{pq}(f, \xi_p, \xi_q)}{\xi_p \xi_q},$$

$$\omega_{p<q}^{pq}(f, \delta_p, \delta_q) = \sup_{t_{[pq]}} \sup_{|\tau_p - t_p| \leq \delta_p, |\tau_q - t_q| \leq \delta_q} \left|\Delta f\left(t_{\tau_{pq}}; t\right)\right|,$$

$$\omega_f^p(\delta_p) = \delta_p \sup_{\xi_p \geq \delta_p} \frac{\omega^p(f;\xi_p)}{\xi_p},$$

$$\omega^p(f; \delta_p) = \sup_{t_{[p]}} \sup_{|\tau_p - t_p| \leq \delta_p} \left|\Delta f\left(t_{\tau_p}; t\right)\right|, p = \overline{1,n}.$$

Let us denote by $\Phi^1(0,d]$ a multiple nonnegative monoton increasing function $\varphi(\delta)$ on $(0,d]$ such that $\lim_{\delta\to 0}\varphi(\delta) = 0$, and $\frac{\varphi(\delta)}{\delta}$ monoton decreases. By

$$\Phi_{\prod_{k=1}^{n}(0,d_k]} \coloneqq \Phi_{(0,d]}$$

denote a set of functions $\omega(\delta_1, \delta_2, \dots, \delta_n) \coloneqq \omega(\delta)$ defined on $(0,d]$ and lying in Φ^1 on each argument, i.e., $\omega(\delta)\epsilon\Phi^1(0,d]$ by δ_p at fixed $\delta_i; i = \overline{1,n}; i \neq p$. It is clear that

$$\omega_f^N(\delta)\epsilon\Phi_{(0,d]}.$$

Lemma 1 ([7], [10]).

1). Let $g(\xi)$ be a nonincreasing function on $(0,d]$. Then the following

$$\int_{\gamma_{\varepsilon''}(t)\setminus\gamma_{\varepsilon'}(t)} g(|\xi - t|)|d\xi| = \int_{\varepsilon'}^{\varepsilon''} g(\xi)d\theta_t(\xi)$$

holds for arbitrary $\varepsilon', \varepsilon''\epsilon(0,d], \varepsilon' < \varepsilon''$.

2). Let $g(\xi)$ be a nonincreasing function on $(0,d]$ and $\mu_k(\delta)$ satisfy the conditions

$$\mu_k(\delta) > 0, \mu_k(\delta) \uparrow, \mu_k(\delta) \to 0, k = 1,2, \ \mu_1(\delta) \leq \mu_2(\delta),$$

then

$$\int_0^d g(y)d_{\mu_1}(y) \leq \int_0^d g(y)d_{\mu_2}(y);$$

3). Let $g(\xi)$ be nonincreasing function on $(0,d]$. Then

$$\int_0^d g(y)d_{\theta_k}(y) \leq \int_0^d g(y)d_\theta(y).$$

Let us emphasize

$$f\epsilon J_0(\Delta,\theta) := \{f \in C_\Delta : \int_0^d \frac{\omega_f^N(\xi)}{\xi} d\theta(\xi) < \infty,$$

$$\int_0^{d_{[p]}} \frac{\omega_f^{N[p]}(\xi_{[p]})}{\xi_{[p]}} d\theta(\xi)_{[p]} < \infty,$$

$$\int_0^{d_{[pq]}} \frac{\omega_f^{N[pq]}(\xi_{[pq]})}{\xi_{[Pq]}} d\theta(\xi)_{[pq]} < \infty, \cdots$$

$$\int_0^{d_p}\int_0^{d_q} \frac{\omega_f^{pq}(\xi_p,\xi_q)}{\xi_p\xi_q} d\theta_p(\xi_p)d\theta_q(\xi_q) < \infty, \int_0^{d_p} \frac{\omega_f^p(\xi_p)}{\xi_p} d\theta_p(\xi_p) < \infty\}.$$
$$p < q; \ p,q = \overline{1,n}.$$

3. Main Results

Theorem 3.1. Let γ^k be a closed rectiable Jordan curve and $f\epsilon J_0(\Delta,\theta)$. Then for arbitrary $z = (z_1, z_2, \dots, z_n) \overline{\in} \Delta, t = (t_1, t_2, \dots, t_n) \in \Delta, |z_k - t_k| \leq d_k$ the following estimate holds

$$|\Psi^n(z) - \Psi^n(t)| = \left|\frac{1}{(2\pi i)^n}\int_\Delta \frac{\Delta f(\tau;t)}{\tau - z} d\tau - \frac{1}{(2\pi i)^n}\int_\Delta \frac{\Delta f(r;t)}{r - t} dr\right| \leq$$

$$\leq C_n[Z(\omega_f^N; |z-t|, \theta) + \sum_{p=1}^{n} Z(\omega_f^N; |z-t|_{[p]}, d_p, \theta) +$$
$$+ \sum_{P=1}^{n}\sum_{\substack{q=1 \\ p<q}}^{n} Z(\omega_f^N; |z-t|_{[pq]}, d_p, d_q, \theta) + \cdots + \quad (3)$$
$$+ \sum_{P=1}^{n}\sum_{q=1}^{n} Z(\omega_f^N; |z_p - t_p|, |z_q - t_q|, d_{[pq]}, \theta) + \sum_{p=1}^{n} Z(\omega_f^N; |z_p - -t_p|, d_{[p]}, \theta)],$$

$$\left|\Psi_p^{n-1}(z_{t_p}) - \Psi_p^{n-1}(t)\right| = \left|\frac{1}{(2\pi i)^n}\left(\int_{\Delta_{[p]}} \frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau - z)_{[p]}} d\tau_{[p]} - \int_{\Delta_{[p]}} \frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau - t)_{[p]}} d\tau_{[p]}\right)\right| \leq$$

$$\leq C_{n-1}[Z\left(\omega_f^{N[p]} \ |z-t|_{[p]}, \theta_{[p]}\right) + \sum_{\substack{q=1 \\ q\neq p}}^{n} Z\left(\omega_f^{N[p]}; |z-t|_{[pq]}, d_q, \theta_{[p]}\right) +$$

$$+\sum_{\substack{q=1\\q<r;\\q,r\neq p}}^{n}\sum_{r=1}^{n} Z\left(\omega_f^{N[p]};|z-t|_{[pqr]},d_q,d_r,\theta_{[p]}\right)+\cdots+$$

$$+\sum_{\substack{q=1\\q<r;\\q,r\neq p}}^{n}\sum_{r=1}^{n} Z\left(\omega_f^{N[p]};|z_q-z_q|,|z_r-t_r|,d_{[qr]},\theta_{[p]}\right)+\sum_{\substack{q=1\\q\neq p}}^{n} Z(\omega_f^{N[p]};|z_q-t_q|,d_{[pq]},\theta_{[p]})], \quad (4)$$

$$\left|\Psi_{pq}^2(t_{z_{pq}})-\Psi_{pq}^2(t)\right| = |\frac{1}{(2\pi i)^2}(\int_{\gamma^p}\int_{\gamma^q}\frac{\Delta f\left(t_{\tau_{pq}};t\right)}{(\tau_p-z_p)(\tau_q-z_q)}d\tau_p d\tau_q -$$

$$-\int_{\gamma^p}\int_{\gamma^q}\frac{\Delta f\left(t_{\tau_{pq}};t\right)}{(\tau_p-z_p)(\tau_q-z_q)}d\tau_p d\tau_q| \le C_2[Z(\omega_f^{pq};|z_p-t_p|,|z_q-t_q|,\theta_p,\theta_q)+$$

$$+Z(\omega_f^{pq};|z_p-t_p|,d_q,\theta_p,\theta_q)+Z(\omega_f^{pq};|z_q-t_q|,d_q,\theta_p,\theta_q)], \quad (5)$$

$$\left|\Psi_p^1(t_{z_p})-\Psi_p^1(t)\right| = \left|\frac{1}{2\pi i}\int_{\gamma^p}\frac{\Delta f(\tau_{t_p};t)}{\tau_p-z_p}d\tau_p-\frac{1}{2\pi i}\int_{\gamma^p}\frac{\Delta f(\tau_{t_p};t)}{\tau_p-t_p}d\tau_p\right| \le C_1 Z(\omega_f^p;|z_p-t_p|,\theta_p), \quad (6)$$

where

$$Z(\omega_f^N;\delta_1,\delta_2,\dots,\delta_n,\theta_1,\theta_2,\dots,\theta_n)\overset{\text{def}}{=}Z(\omega_f^N;\delta,\theta)=\int_0^{\delta}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)+\sum_{p=1}^{n}\delta_p\int_0^{\delta_{[p]}}\int_{\delta_p}^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[p]}\xi_p^2}d\theta(\xi)+$$

$$+\sum_{p=1}^{n}\sum_{\substack{q=1\\p<q}}^{n}\delta_p\delta_q\int_0^{\delta_{[pq]}}\int_{\delta_p}^{d_p}\int_{\delta_q}^{d_q}\frac{\omega_f^N(\xi)}{\xi_{[pq]}\xi_p^2\xi_q^2}d\theta(\xi)+\cdots+$$

$$+\sum_{p=1}^{n}\sum_{\substack{q=1\\p<q}}^{n}\delta_{[pq]}\int_0^{\delta_p}\int_0^{\delta_q}\int_{\delta_{[pq]}}^{d_{[pq]}}\frac{\omega_f^N(\xi)}{\xi^2{}_{[pq]}\xi_p\xi_q}d\theta(\xi)+\sum_{p=1}^{n}\delta_{[p]}\int_0^{\delta_q}\int_{\delta_{[p]}}^{d_{[p]}}\frac{\omega_f^N(\xi)}{\xi_{[p]}^2\xi_p}d\theta(\xi)+\delta\int_{\delta}^{d}\frac{\omega_f^N(\xi)}{\xi^2}d\theta(\xi),$$

$$C_i(i=\overline{1,n}) - \text{constant}.$$

Proof. Let us denote by $t_{z_p}(p=\overline{i,n})$ an arbitrary point of the border γ^p, such that $|z_p-t_{z_p}|=inf_{\tau_p\in\gamma^p}|z_p-\tau_p|=\rho(z_p,\gamma^p)$. Then the identity takes the place

$$\Delta f(\tau;t)=\Delta f(t_z;t)-\sum_{p=1}^{n}\Delta f\left(t_z;t_{\tau_p}\right)+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\Delta f\left(t_z;t_{\tau_{pq}}\right)+\cdots+$$

$$+(-1)^{n-2}\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\Delta f\left(t_z;\tau_{t_{pq}}\right)+(-1)^{n-1}\sum_{p=1}^{n}\Delta f\left(t_z;\tau_{t_p}\right)+\ +(-1)^n\Delta f(t_z;\tau), \quad (7)$$

the validity of which is easily shown by direct calculations of items. It is easy to see also that

$$\Delta f(\tau;t)=(-1)^n\,\Delta f(t;\tau). \quad (8)$$

Let us consider the difference

$$\Psi^n(z)-\Psi^n(t)=\frac{1}{(2\pi i)^n}\left(\int_{\Delta}\frac{\Delta f(\tau;t)}{\tau-z}d\tau-\int_{\Delta}\frac{\Delta f(\tau;t)}{\tau-t}d\tau\right). \quad (9)$$

Using (8) and (7) in (9) we obtain

$$\Psi^n(z)-\Psi^n(t)=\frac{1}{(2\pi i)^n}\left(\int\limits_{\Delta}\frac{\Delta f(\tau;\mathrm{t_z})}{\tau-z}d\tau-\int\limits_{\Delta}\frac{\Delta f(\tau;\mathrm{t_z})}{\tau-\mathrm{t_z}}d\tau\right)+$$

$$+(-1)^{n-1}\frac{\varkappa_p(z_p)}{(2\pi i)^{n-1}}\sum_{p=1}^{n}\int\limits_{\Delta_{[p]}}\frac{\Delta \mathrm{f}\left(\mathrm{t_z};\tau_{\mathrm{t_p}}\right)}{(\tau-z)_{[\mathrm{p}]}}d\tau_{[\mathrm{p}]}+$$

$$+(-1)^{n-2}\frac{\varkappa_p(z_p)\varkappa_q(z_q)}{(2\pi i)^{n-2}}\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\int\limits_{\Delta_{[\mathrm{pq}]}}\frac{\Delta \mathrm{f}\left(\mathrm{t_z};\tau_{\mathrm{t_{pq}}}\right)}{(\tau-z)_{[\mathrm{pq}]}}d\tau_{[\mathrm{pq}]}+$$

$$+\cdots+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\frac{\varkappa(z_p)_{[pq]}}{(2\pi i)^2}\int\limits_{\gamma^p}\int\limits_{\gamma^{\mathrm{q}}}\frac{\Delta \mathrm{f}\left(\mathrm{t_z};\mathrm{t}_{\tau_{\mathrm{pq}}}\right)}{(\tau_p-z_p)(\tau_q-z_q)}d\tau_p d\tau_q-$$

$$-\sum_{p=1}^{n}\frac{\varkappa(z_p)_{[p]}}{2\pi i}\int_{\gamma^p}\frac{\Delta \mathrm{f}\left(\mathrm{t_z};\tau_{\mathrm{t_p}}\right)}{\tau_p-z_p}\mathrm{d}\tau_{\mathrm{p}}+\Delta f(t_z;t)\varkappa(z)+\frac{1}{(2\pi i)^n}\left(\int_{\Delta}\frac{\Delta f(\tau;t_z)}{\tau-z}d\tau-\int_{\Delta}\frac{\Delta f(\tau;t\,)}{\tau-t}d\tau\right). \tag{10}$$

Let us first denote items on the right hand side of equality in (10) by $J_n, J_{n-1}, \ldots, J_2, J_1, J_0, i_n$, , respectively, and we estimate each of them separately.

Before proceed to assess J_n remind the is an identity

$$\frac{1}{\tau-z}-\frac{1}{\tau-t_z}=\frac{1}{\prod_{k=1}^{n}(\tau_k-z_k)}-\frac{1}{\prod_{k=1}^{n}(\tau_k-t_{z_k})}=$$

$$=\frac{1}{\prod_{k=1}^{n}(\tau_k-z_k)}\left(\prod_{k=1}^{n}\frac{z_k-t_{z_k}}{\tau_k-z_k}+\sum_{k=1}^{n}\prod_{\substack{p=1\\p\neq k}}^{n}\frac{z_p-t_{z_p}}{\tau_p-z_p}+\right.$$

$$+\sum_{\substack{k=1\\k<r}}^{n}\sum_{r=1}^{n}\prod_{\substack{p=1\\p\neq k,r}}^{n}\frac{z_p-t_{z_p}}{\tau_p-z_p}+\ldots+\sum_{\substack{k=1\\k<r}}^{n}\sum_{r=1}^{n}\frac{z_k-t_{z_k}}{\tau_k-z_k}\frac{z_r-t_{z_r}}{\tau_r-z_r}+$$

$$\left.\sum_{k=1}^{n}\frac{z_k-t_{z_k}}{\tau_k-z_k}\right).$$

Let us consider

$$J_n=\frac{1}{(2\pi i)^n}\left(\int_{\Delta}\frac{\Delta f(\tau;t_z)}{\tau-z}d\tau-\int_{\Delta}\frac{\Delta f(\tau;t_z)}{\tau-t_z}d\tau\right). \tag{11}$$

The difference $\frac{1}{\tau-z}-\frac{1}{\tau-t_z}$ standing under the integral in J_n, we replace with a right member of identity (11). Then we have

$$J_n=\frac{1}{(2\pi i)^n}\int_{\Delta}\left[\prod_{k=1}^{n}\frac{z_k-\mathrm{t_{z_k}}}{\tau_k-z_k}+\sum_{\substack{k=1\\k<r}}^{n}\prod_{\substack{p=1\\p\neq k}}^{n}\frac{z_p-\mathrm{t_{z_p}}}{\tau_p-z_p}+\sum_{\substack{k=1\\k<r}}^{n}\sum_{r=1}^{n}\prod_{\substack{p=1\\p\neq k,r}}^{n}\frac{z_p-\mathrm{t_{z_p}}}{\tau_p-z_p}+\cdots+\right.$$

$$\left.+\sum_{\substack{k=1\\k<r}}^{n}\sum_{r=1}^{n}\frac{z_k-\mathrm{t_{z_k}}}{\tau_k-z_k}\frac{z_r-\mathrm{t_{z_r}}}{\tau_r-z_r}+\sum_{k=1}^{n}\frac{z_k-\mathrm{t_{z_k}}}{\tau_k-z_k}\right]\frac{\Delta f(\tau;t_z)}{\prod_{k=1}^{n}\left(z_k-\mathrm{t_{z_k}}\right)}\mathrm{d}\tau\,. \tag{12}$$

Let ε_k be an arbitrary number from $(0,d_k]$ $(k=\overline{1,n})$,

$$\gamma_{\varepsilon_k}^k\left(\mathrm{t_{z_k}}\right)=\left\{\tau\in\gamma^k,\left|\mathrm{t_{z_k}}-\tau\right|<\varepsilon_k\right\}.$$

The integral (12) is represent table a type of the sum of two integrals of J_n' and J_n'', taken, on Δ_ε and $\Delta\backslash\Delta_\varepsilon$, respectively, where

$$\Delta_\varepsilon=\gamma_{\varepsilon_1}^1\left(t_{z_1}\right)\times\gamma_{\varepsilon_2}^2\left(t_{z_2}\right)\times\ldots\times\gamma_{\varepsilon_n}^n\left(t_{z_n}\right),$$

$$\Delta\backslash\Delta_\varepsilon=\sum_{p=1}^{n}D_p^{(n-1)}+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}D_{pq}^{(n-2)}+\cdots+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}D_{pq}^{(2)}+\sum_{p=1}^{n}D_p^{(1)}+D_\varepsilon,$$

$$\underset{p=\overline{1,n}}{D_p^{(n-1)}}=\gamma_{\varepsilon_1}^1\left(t_{z_1}\right)\times\ldots\times\gamma_{\varepsilon_{p-1}}^{p-1}\left(t_{z_{p-1}}\right)\times\gamma^p\mid\gamma_{\varepsilon_p}^p\left(t_{z_p}\right)\times\gamma_{\varepsilon_{p+1}}^{p+1}\left(t_{z_{p+1}}\right)\times\ldots\times\gamma_{\varepsilon_n}^n\left(t_{z_n}\right),$$

$$\underset{p<q;\,p,q=\overline{1,n}}{D_{pq}^{(n-1)}}=\gamma_{\varepsilon_1}^1\left(t_{z_1}\right)\times\ldots\times\gamma_{\varepsilon_{p-1}}^{p-1}\left(t_{z_{p-1}}\right)\times\gamma^p\mid\gamma_{\varepsilon_p}^p\left(t_{z_p}\right)\times\gamma_{\varepsilon_{p+1}}^{p+1}\left(t_{z_{p+1}}\right)\times$$

$$\times\ldots\times\gamma_{\varepsilon_{q-1}}^{q-1}\left(t_{z_{q-1}}\right)\gamma^q\mid\gamma_{\varepsilon_q}^q\left(t_{z_q}\right)\times\gamma_{\varepsilon_{q+1}}^{q+1}\left(t_{z_{q+1}}\right)\times\ldots\times\gamma_{\varepsilon_n}^n\left(t_{z_n}\right),$$

$$D_{pq}^{(2)}=\gamma^1|\gamma_{\varepsilon_1}^1\left(t_{z_1}\right)\times\ldots\times\gamma^{p-1}|\gamma_{\varepsilon_{p-1}}^{p-1}\left(t_{z_{p-1}}\right)\times\gamma_{\varepsilon_p}^p\left(t_{z_p}\right)\times$$

$$\times \gamma^{p+1} \backslash \gamma_{\varepsilon_{p+1}}^{p+1}\left(t_{z_{p+1}}\right) \times \ldots \times \gamma^{q-1} | \gamma_{\varepsilon_{q-1}}^{q-1}\left(t_{z_{q-1}}\right) \times \gamma_{\varepsilon_q}^{q}\left(t_{z_q}\right) \times$$

$$\times \gamma^{q+1} \backslash \gamma_{\varepsilon_{q+1}}^{q+1}\left(t_{z_{q+1}}\right) \times \ldots \times \gamma^{n} \backslash \gamma_{\varepsilon_n}^{n}\left(t_{z_n}\right),$$

$$D_p^{(1)} = \gamma^1 | \gamma_{\varepsilon_1}^1\left(t_{z_1}\right) \times \gamma^2 | \gamma_{\varepsilon_2}^2\left(t_{z_2}\right) \times \ldots \times \gamma^{p-1} | \gamma_{\varepsilon_{p-1}}^{p-1}\left(t_{z_{p-1}}\right) \times \gamma_{\varepsilon_p}^{p}\left(t_{z_p}\right) \times$$

$$\times \gamma^{p+1} | \gamma_{\varepsilon_{p+1}}^{p+1}\left(t_{z_{p+1}}\right) \times \ldots \times \gamma^{n} | \gamma_{\varepsilon_n}^{n}\left(t_{z_n}\right),$$

$$D_\varepsilon = \gamma^1 | \gamma_{\varepsilon_1}^1\left(t_{z_1}\right) \times \gamma^2 | \gamma_{\varepsilon_2}^2\left(t_{z_2}\right) \times \ldots \times \gamma^{n} | \gamma_{\varepsilon_n}^{n}\left(t_{z_n}\right).$$

Let us denote by

$$Q_p^{n-1}(z, t_z), Q_{pq}^{n-2}(z, t_z), \ldots, Q_{pq}^{2}(z; t_z), Q_p^{1}(z; t_z) \text{ and } Q_\varepsilon(z; t_z)$$

the integrals taken piecemeal cores: $D_p^{(n-1)}, D_{pq}^{(n-2)}, \ldots, D_{pq}^{(2)}, D_p^{(1)}, D_{\varepsilon.}$ respectively. As for every

$$\tau_k \in \gamma^k \left| \frac{z_k - t_{z_k}}{\tau_k - z_k} \right| \leq M_k < 1, \quad k = \overline{1, n}, \tag{13}$$

we have

$$|J_n'| \leq \frac{1}{(2\pi)^n} \int\limits_{\Delta_\varepsilon} \left[\prod_{k=1}^{n} M_k + \sum_{k=1}^{n} \prod_{\substack{p=1 \\ p \neq k}}^{n} M_p + \sum_{\substack{m=1 \\ k<m}}^{n} \sum_{k=1}^{n} \prod_{\substack{p=1 \\ p \neq k,m}}^{n} M_p + \cdots + \sum_{k=1}^{n} \sum_{\substack{m=1 \\ k<m}}^{n} M_k M_m + \sum_{k=1}^{n} M_k \right] \frac{\omega_f^N\left(|\tau_k - t_k|\right)}{|\tau - t_z|} |d\tau| \leq$$

$$\leq M \int\limits_{\gamma_{\varepsilon_1}^1(t_{z_1})} \frac{|d\tau_1|}{\left|\tau_1 - t_{z_1}\right|} \int\limits_{\gamma_{\varepsilon_2}^2(t_{z_2})} \frac{|d\tau_2|}{\left|\tau_2 - t_{z_2}\right|} \cdots \int\limits_{\gamma_{\varepsilon_n}^n(t_{z_n})} \frac{\omega_f^N\left(\prod_{k=1}^{n}\left|\tau_k - \tau_{z_k}\right|\right)}{|\tau_n - t_{2n}|} |d\tau_n|,$$

where

$$M = \prod_{k=1}^{n} M_k + \sum_{k=1}^{n} \prod_{\substack{p=1 \\ p \neq k}}^{n} M_p + \sum_{\substack{m=1 \\ k<m}}^{n} \sum_{k=1}^{n} \prod_{\substack{p=1 \\ p \neq k,m}}^{n} M_p + \cdots + \sum_{k=1}^{n} \sum_{\substack{m=1 \\ k<m}}^{n} M_k M_m + \sum_{k=1}^{n} M_{k.}$$

Consequently applying 1), 2), 3) of Lemma 2.1 and choosing $\varepsilon_k = |z_k - t_k|, k = \overline{1, n}$, we obtain

$$|J_n'| \leq M \int\limits_{\gamma_{\varepsilon_1}^1(t_{z_1})} \frac{|d\tau_1|}{\left|\tau_1 - t_{z_1}\right|} \int\limits_{\gamma_{\varepsilon_2}^2(t_{z_2})} \frac{|d\tau_z|}{\left|\tau_2 - t_{z_2}\right|} \cdots \int\limits_{\gamma_{\varepsilon_{n-1}}^{n-1}(t_{z_{n-1}})} \frac{|d\tau_{n-1}|}{\left|\tau_{n-1} - t_{z_{n-1}}\right|} x$$

$$x \int\limits_{O}^{\varepsilon_n} \frac{\omega_f^N \begin{pmatrix} n-1 \\ \prod \left|\tau_k - t_{z_k}\right|, \xi_n \\ k=1 \end{pmatrix}}{\xi_n} d\theta_n(\xi_n) \leq M \int\limits_{\gamma_{\varepsilon_1}^1(t_{z_1})} \frac{|d\tau_1|}{\left|\tau_1 - t_{z_1}\right|} x$$

$$x \int\limits_{\gamma_{\varepsilon_{n-2}}^{n-2}(t_{z_{n-2}})} \frac{|d\tau_2|}{\left|\tau_{n-2} - t_{z_{n-2}}\right|} \int\limits_{0}^{\varepsilon_{n-1}} \int\limits_{O}^{\varepsilon_n} \frac{\omega_f^N \begin{pmatrix} n-2 \\ \prod \left|\tau_k - t_{z_k}\right|, \xi_{n-1}, \xi_n \\ k=1 \end{pmatrix}}{\xi_{n-1} - \xi_n} d\theta_{n-1}(\xi_{n-1}) d\theta_n(\xi_n) \leq$$

$$\leq M \int_0^{\varepsilon_1} \int_0^{\varepsilon_2} \ldots \int_0^{\varepsilon_n} \frac{\omega_f^N(\xi_1, \xi_2, \ldots, \xi_n)}{\xi_1 \xi_2 \ldots \xi_n} d\theta_1(\xi_1) d\theta_n(\xi_n) \stackrel{\text{def}}{=} M \int_0^{\varepsilon} \frac{\omega_f^N(\xi)}{\xi} d\theta(\xi) \,.$$

Thus,

$$|J_n'| \leq M \int_0^{\varepsilon} \frac{\omega_f^N(\xi)}{\xi} d\theta(\xi). \tag{14}$$

Now we estimate J_n''. For this aim we first estimate integral $Q_p^{n-1}(z,t_z)$:
Now we estimate the integral $Q_p^{n-1}(z,t_z)$ as follows

$$\left|Q_p^{n-1}(z,t_z)\right| \leq \frac{1}{(2\pi i)^n}\int_{D_p^{n-1}}\Bigg[\left|\frac{z_p-t_{z_p}}{\tau_p-z_p}\right|\Bigg(\prod_{\substack{i=1\\k\neq p}}^{n}\left|\frac{z_k-t_{z_k}}{\tau_k-z_k}\right|+$$

$$+\sum_{\substack{k=1\\k\neq p}}^{n}\prod_{\substack{i=1\\i\neq k,p}}^{n}\left|\frac{z_i-t_{z_i}}{\tau_i-z_i}\right|+\sum_{\substack{k=1\\k<m\\k,m\neq p}}^{n}\sum_{m=1}^{n}\prod_{\substack{i=1\\i\neq k,m,p}}^{n}\left|\frac{z_i-t_{z_i}}{\tau_i-z_i}\right|+\dots+\sum_{k=1}^{n}\left|\frac{z_k-t_{z_k}}{\tau_k-z_z}\right|+1\Bigg)+$$

$$+\prod_{\substack{k=1\\k\neq p}}^{n}\left|\frac{z_k-t_{z_k}}{\tau_k-z_z}\right|+\sum_{\substack{k=1\\k\neq p}}^{n}\prod_{\substack{i=1\\i\neq k,p}}^{n}\left|\frac{z_i-t_{z_i}}{\tau_i-z_i}\right|+\sum_{\substack{k=1\\k<m\\k,m\neq p}}^{n}\sum_{m=1}^{n}\prod_{\substack{i=1\\i\neq k,m,p}}^{n}\left|\frac{z_i-t_{z_i}}{\tau_i-z_i}\right|+\dots+$$

$$+\sum_{\substack{k=1\\k<m\\k,m\neq p}}^{n}\sum_{m=1}^{n}\left|\frac{z_k-t_{z_k}}{\tau_k-z_z}\right|\cdot\left|\frac{z_m-t_{z_m}}{\tau_m-z_m}\right|+$$

$$+\sum_{\substack{k=1\\k\neq p}}^{n}\left|\frac{z_k-t_{z_k}}{\tau_k-z_k}\right|\Bigg]\frac{\omega_f^N\left(\prod_{k=1}^{n}|\tau_k-t_{z_k}|\right)}{\prod_{k=1}^{n}|\tau_k-t_{z_k}|}|d\tau|\,.$$

From this and (13) we obtain the following

$$\left|Q^{n-1}_{\substack{p\\p=\overline{1,n}}}(z,t_z)\right|\leq (M_{[p]}+1)(|z_p-t_p|\times$$

$$\times\int_0^{|z-t|_{[p]}}\int_{|z_p-t_p|}^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[p]}\xi_p^2}d\theta(\xi)+\int_0^{\varepsilon_{[p]}}\int_0^{d_p}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi))$$

for every $\tau_k\in\gamma^k$ such $\left|\frac{\tau_k-t_{z_k}}{\tau_k-z_k}\right|\leq 2$. Taking into account that $|z_k-t_{z_k}|\leq|z_k-t_k|$ and $\varepsilon_k=|z_k-t_k|, k=\overline{1,n}$, from the last estimate we have

$$\left|Q^{n-1}_{\substack{p\\p=\overline{1,n}}}(z,t_z)\right|\leq (M_{[p]}+1)(|z_p-t_p|\times$$

$$\times\int_0^{|z-t|_{[p]}}\int_{|z_p-t_p|}^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[p]}\xi_p^2}d\theta(\xi)+\int_0^{|z-t|_{[p]}}\int_0^{d_p}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)). \tag{15}$$

Taking similar transforms which were used in the estimate of (15), we have for the integral $Q_{pq}^{n-2}(z,t_z)$ the following:

$$\left|Q^{n-2}_{\substack{p\\p=\overline{1,n}}}(z,t_z)\right|\leq (M_{[pq]}+1)(|z_p-t_p||z_q-t_q|\times$$

$$\times\int_0^{\varepsilon_{[pq]}}\int_{|z_p-t_p|}^{d_p}\int_{|z_q-t_q|}^{d_q}\frac{\omega_f^N(\xi)}{\xi_{[pq]}\xi_p^2}d\theta(\xi)+|z_p-t_p|\int_0^{|z-t|_{[pq]}}\int_{|z_p-t_p|}^{d_q}\int_0^{d_p}\frac{\omega_f^N(\xi)}{\xi_p^2\xi_{[p]}}d\theta(\xi)+$$

$$+|z_q-t_q|\int_0^{|z-t|_{[pq]}}\int_{|z_q-t_q|}^{d_q}\int_0^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[q]}\xi_q^2}d\theta(\xi)+\int_0^{d_p}\int_0^{d_q}\int_0^{|z-t|_{[pq]}}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)\Bigg), \tag{16}$$

$$\left|Q^2{}_{\substack{pq\\p<q\\p,q=\overline{1,n}}}(z;t_z)\right|\leq\left(M_p+M_q+M_p+M_q++1\right)x$$

$$x\left[|z-t|_{[pq]}\int_{|z-t|_{[pq]}}^{d_{[pq]}}\int_0^{|z_p-t_p|}\int_0^{|z_q-t_q|}\frac{\omega_f^N(\xi)}{\xi_{[pq]}^2\xi_p\xi_q}d\theta(\xi)+\right.$$

$$+\sum_{\substack{r=1\\ r\neq p,q}}^{n}|z-t|_{[pqr]}\int_{|z-t|_{[pqr]}}^{d_{[pqr]}}\int_0^{d_r}\int_0^{[z_p-t_p]}\int_0^{[z_q-t_q]}\frac{\omega_f^N(\xi)}{\xi_{[pqr]}^2\xi_p\xi_q\xi_r\xi_s}d\theta(\xi)+$$

$$+\sum_{\substack{r=1\\ r<s\\ r,s\neq p,q}}^{n}\sum_{s=1}^{n}|z-|_{[pqr]}\int_{|z-t|_{[pqrs]}}^{d_{[pqrs]}}\int_0^{d_r}\int_0^{d_s}\int_0^{|z_p-t_p|}\int_0^{|z_q-t_q|}\frac{\omega_f^N(\xi)}{\xi_{[pqr]}^2\xi_p\xi_q\xi_r\xi_s}d\theta(\xi)+$$

$$+\sum_{\substack{r=1\\ r<s\\ r,s\neq p,q}}^{n}\sum_{s=1}^{n}\int_0^{d_{[pqrs]}}|z_r-t_r||z_s-t_s|\int_{|z_r-t_r|}^{d_r}x$$

$$x\int_{|z_s-t_s|}^{d_s}\int_0^{|z_p-t_p|}\int_0^{|z_q-t_q|}\frac{\omega_f^N(\xi)}{\xi_{[rs]}\xi_r^2\xi_s^2}d\theta(\xi)+$$

$$+\sum_{\substack{r=1\\ r\neq p,q}}^{n}\int_0^{d_{[pqr]}}|z_r-t_r|\int_{|z_r-t_r|}^{d_r}\int_0^{|z_p-t_p|}\int_0^{|z_q-t_q|}\frac{\omega_f^N(\xi)}{\xi_r^2\xi_{[r]}}d\theta(\xi)+$$

$$\left.+\int_0^{d_{[pq]}}\int_0^{|z_p-t_p|}\int_0^{|z_q-t_q|}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)+\right], \tag{17}$$

$$\left|Q^1{}_{\substack{p\\ p=\overline{1,n}}}(z;t_z)\right|\leq(M_p+1)\left((|z-t|_{[p]}\int_{|z-t|_{[p]}}^{d_{[p]}}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi_{[p]}^2\xi_p}d\theta(\xi)+\right.$$

$$+\sum_{\substack{q=1\\ q\neq p}}^{n}|z_r-t_r|_{[pq]}\int_{|z-t|_{pq}}^{d_{[pq]}}\int_0^{d_q}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi_{[pq]}^2\xi_p\xi_q}d\theta(\xi)+$$

$$+\sum_{q=1}^{n}\sum_{r=1}^{n}|z-t|_{[pqr]}\int_{|z-t|_{[pqr]}}^{d_{[pqr]}}\int_0^{d_q}\int_0^{d_r}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi_{[pqr]}^2\xi_p\xi_q\xi_r}d\theta(\xi)+\cdots+$$

$$+\sum_{\substack{q=1\\ q<r}}^{n}\sum_{r=1}^{n}\int_0^{d_{[pqr]}}|z_q-t_q|\int_{|z_q-t_q|}^{d_q}|z_r-t_r|\int_{|z_r-t_r|}^{d_r}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi_{[qr]}\xi_q^2\xi_r^2}d\theta(\xi)+$$

$$+\sum_{\substack{r=1\\ q\neq p}}^{n}\int_0^{d_{[pq]}}|z_q-t_q|\int_{|z_q-t_q|}^{d_q}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi_r^2\xi_{[q]}}d\theta(\xi)+$$

$$\left.+\int_0^{d_{[p]}}\int_0^{|z_p-t_p|}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)\right), \tag{18}$$

$$|Q_\varepsilon(z;t_z)| \le (|z-t|)\int\limits_{|z-t|}^{d}\frac{\omega_f^N(\xi)}{\xi^2}d\theta(\xi)+$$

$$+\sum_{p=1}^{n}|z-t|_{[p]}\int\limits_{|z-t|_{[p]}}^{d_{[p]}}\int\limits_{0}^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[p]}^2\xi_p}d\theta(\xi)+$$

$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}|z-t|_{[pq]}\int\limits_{|z-t|_{[pq]}}^{d_{[pq]}}\int\limits_{0}^{d_p}\int\limits_{0}^{d_q}\frac{\omega_f^N(\xi)}{\xi^2{}_{[pq]}\xi_p\xi_q}d\theta(\xi)+\cdots+$$

$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\int\limits_{0}^{d_{[pq]}}|z_p-t_p|\,|z_q-t_q|\int\limits_{|z_p-t_p|}^{d_p}\int\limits_{|z_q-t_q|}^{d_q}\frac{\omega_f^N(\xi)}{\xi_{[pq]}\xi_p{}^2\xi_q{}^2}d\theta(\xi)+$$

$$+\sum_{p=1}^{n}\int_0^{d_{[p]}}|z_p-t_p|\int_{|z_p-t_p|}^{d_p}\frac{\omega_f^N(\xi)}{\xi_{[p]}\xi_p{}^2}d\theta(\xi). \tag{19}$$

Summarizing the obtained estimates in $(15)-(19)$, we get an estimate for J_n''.

Now we estimate J_{n-1}. As $|t_{z_k}-t_k|\le 2|z_k-t_k|$, $|\tau_k-t_{z_k}|\le 2|\tau_k-t_k|$, $k=\overline{1,n}$, applying items 1) and 2) of Lemma 2.1 we have

$$|J_{n-1}|\le\frac{1}{(2\pi)^{n-1}}\sum_{p=1}^{n}\int\limits_{\Delta_{[p]}}\frac{\omega_f^N\left(\prod_{\substack{k=1\\k\neq p}}^{n}\left|t_{z_p}-t_k\right|\right)}{\prod_{k=1}^{n}|\tau_k-t_k|}\,|d\tau|_{[p]}\le$$

$$\le\frac{1}{\pi^{n-1}}\sum_{p=1}^{n}\int\limits_{0}^{d_{[p]}}\frac{\omega_f^N\left(\xi_{[p]},|z_p-t_p|\right)}{\xi_{[p]}}d\theta(\xi)_{[p]}.$$

Owing to a lack of growth $\frac{\omega_f^N(\delta_{[p]},\delta_p)}{\delta_p}$ on δ_p $(p=\overline{1,n})$,

$$\int\limits_0^{|z_p-t_p|}\int\limits_0^{d_p}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi)\le\int\limits_0^{d_{[p]}}\frac{\omega_f^N\left(\xi_{[p]},|z_p-t_p|\right)|z_p-t_p|}{|z_p-t_p|\,\xi_{[p]}}d\theta(\xi)_{[p]}=$$

$$=\int\limits_0^{d_{[p]}}\frac{\omega_f^N\left(\xi_{[p]},|z_p-t_p|\right)}{\xi_{[p]}}d\theta(\xi)_{[p]}.$$

Therefore,

$$|J_{n-1}|\le C_{n-1}\sum_{p=1}^{n}\int_0^{|z_p-t_p|}\int_0^{d_{[p]}}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi). \tag{20}$$

Similarly estimating the rest integrals we have

$$|J_{n-2}|\le C_{n-1}\sum_{p=1}^{n}\int_0^{|z_p-t_p|}\int_0^{|z_p-t_p|}\int_0^{d_{[p,q]}}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi), \tag{21}$$

$$|J_1|\le C_1\sum_{p=1}^{n}\int_0^{|z_p-t_p|}\int_0^{d_p}\frac{\omega_f^N(\xi)}{\xi}d\theta(\xi), \tag{22}$$

$$|J_0| \le C_1 \sum_{p=1}^{n} \int_0^{|z_p-t_p|} \frac{\omega_f^N(\xi)}{\xi} d\theta(\xi). \tag{23}$$

For get an estimate for the integral i_n we apply Theorem 2 in [18]. Summarizing all estimates for integrals J_n', J_n'' and i_n , and estimates (14), $(15) - (19)$ and (20)-(23) we finally obtain the required estimate (3).

To estimate the following difference

$$\Psi_p^{n-1}\left(z_{t_p}\right) - \Psi_p^{n-1}(t) = \frac{1}{(2\pi i)^{n-1}}\left(\int\limits_{\Delta_{[p]}} \frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau-z)_{[p]}} d\tau_{[p]} - \int\limits_{\Delta_{[p]}} \frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau-t)_{[p]}} d\tau_{[p]}\right)$$

we use the following identity

$$\Delta f\left(\tau_{t_p};t\right) = \Delta f\left(t_{z_{t_p}};t\right) - \sum_{\substack{q=1\\ q\neq p}}^{n} \Delta f\left(t_{z_{t_p}};t_{\tau_q}\right) + \sum_{\substack{q=1\\ q<r;}}^{n}\sum_{\substack{r=1\\ q,r\neq p}}^{n} \Delta f\left(t_{z_{t_p}};t_{\tau_{qr}}\right) + \cdots + (-1)^{n-3} \sum_{\substack{q=1\\ q<r;\\ q,r\neq p}}^{n}\sum_{r=1}^{n} \Delta f\left(t_{z_{t_p}};\tau_{t_{pqr}}\right) +$$

$$+(-1)^{n-2}\sum_{\substack{q=1\\ q\neq p}}^{n} f\left(t_{z_{t_p}};\tau_{t_{pq}}\right) + (-1)^{n-1}\Delta f\left(t_{z_{t_p}};\tau_{t_p}\right)$$

the validity of which follows from (7). Then $\Psi_p^{n-1}(z_{t_p}) - \Psi_p^{n-1}(t)$ is represented in a form of a difference (10) and is estimated also as estimates for $J_n, J_{n-1}, \ldots, J_1, J_o$ and i_n the distinction consists only among integrals. Therefore, we obtain

$$\left|\Psi_p^{n-1}(z_{t_p}) - \Psi_p^{n-1}(t)\right| \le C_{n-1}\{Z\left(\omega_f^{N[p]};|z-t|_{[p]},\Theta_{[p]}\right) +$$

$$+\sum_{\substack{q=i\\ q\neq p}}^{n} Z\left(\omega_f^{N[p]};|z-t|_{[pq]},d_q,\Theta_{[p]}\right) + \sum_{q=1}^{n}\sum_{r=1}^{n} Z\left(\omega_f^{N[p]};|z-t|_{[pqr]},d_q,d_r,\Theta_{[p]}\right) + \cdots +$$

$$+\sum_{\substack{q=1\\ q<r;q,r\neq p}}^{n}\sum_{r=1}^{n} Z\left(\omega_f^{N[p]};\delta_{[pqr]},|z_q-t_q|,|z_r-t_r|,Q_{[p]}\right) +$$

$$+\sum_{\substack{q=1\\ q\neq p}}^{n} Z\left(\omega_f^{N[p]};\delta_{[pq]},|z_q-t_q|,Q_{[p]}\right)\}.$$

Similarly, we have

$$\left|\Psi_{pq}^{n-2}(z_{t_{pq}}) - \Psi_{pq}^{n-2}(t)\right| \le C_{n-2}\left(Z\left(\omega_f^{N[pq]};|z-t|_{[pq]},Q_{[pq]}\right)\right) +$$

$$+\sum_{r=1}^{n} Z\left(\omega_f^{N[pq]};|z-t|_{[pqr]},d_r,Q_{[pq]}\right) +$$

$$+\sum_{\substack{r=1\\ r<m;\, m,r\neq p,q}}^{n}\sum_{m=1}^{n} Z\left(\omega_f^{N[pq]};|z-t|_{[pqrm]},d_r,d_m,Q_{[pq]}\right) + \cdots +$$

$$+\sum_{\substack{r=1\\ r<m;\, m,r\neq p,q}}^{n}\sum_{m=1}^{n} Z\left(\omega_f^{N[pq]};d_{[pqrm]},|z_r-t_r|,|z_m-t_m|,Q_{[pq]}\right) +$$

$$+\sum_{\substack{r=1\\ r\neq p,q}}^{n} Z\left(\omega_f^{N[pq]}; d_{[pqr]}, |z_r - t_r|, Q_{[pq]}\right).$$

By continuing this process we show estimates for the differences

$$\Psi_{pq}^2(z_{t_{pq}}) - \Psi_{pq}^2(t),\ \Psi_{pq}^1(z_{t_{pq}}) - \Psi_{pq}^1(t)$$

have the forms:

$$\left|\Psi_{pq}^2\left(z_{t_{pq}}\right) - \Psi_{pq}^2(t)\right|_{p<q,p,q=\overline{1,n}} \le C_2(Z(\omega_f^{pq}; |z_p - t_p|, |z_q - t_q|, Q_p, Q_q) +$$

$$+Z(\omega_f^{pq}; |z_p - t_p|, d_q, \theta_p, \theta_q) + Z(\omega_f^{pq}; |z_q - t_q|, d_p, \theta_p, \theta_q)),$$

$$\left|\Psi_{pq}^1\left(z_{t_{pq}}\right) - \Psi_{pq}^1(t)\right| \le C_1 Z(\omega_f^p; |z_p - t_p|, \theta_p).$$

These prove the theorem.
From the theorem immediately follow the following equalities

$$\Psi_p^1(t) = \Psi_{pq}^{1,\pm}(t), \Psi_{pq}^2(t) = \Psi_{pq}^{2,\pm\pm}(t), \dots, \Psi_{pq}^n(t) = \Psi_{pq}^{n,\pm\pm\cdots\pm}(t).$$

Theorem 3.2. If $f \in J_o(\Delta,\theta)$, then for every $z \notin \Delta$, and $t \in \Delta$ such $|z_{k-}t_k| < d_k, k = \overline{1,n}$, the following estimate

$$|\Phi(z) - (\frac{1}{(2\pi i)^n}\int_{\Delta} \frac{\Delta f(\tau;t)}{\tau - t} d\tau + \sum_{p=1}^{n} \frac{\varkappa_p(z_p)}{(2\pi i)^{n-1}} \int_{\Delta_{[p]}} \frac{\Delta f\left(\tau_{t_p};t\right)}{(\tau - t)_{[p]}} d\tau_{[p]} +$$

$$+\sum_{\substack{p=1\\ p<q}}^{n}\sum_{q=1}^{n} \frac{\varkappa_p(z_p)\varkappa_q(z_q)}{(2\pi i)^{n-2}} \int_{\Delta_{[pq]}} \frac{\Delta f\left(\tau_{t_{pq}};t\right)}{(\tau - t)_{[pq]}} d\tau_{[pq]} + \cdots +$$

$$+\sum_{\substack{p=1\\ p<q}}^{n}\sum_{q=1}^{n} \frac{\varkappa_{[pq]}}{(2\pi i)^2} \int_{\gamma^p}\int_{\gamma^q} \frac{\Delta f\left(\tau_{t_{pq}};t\right)}{(\tau_p - t_p)(\tau_q - t_q)} d\tau_p d\tau_q + \cdots$$

$$+\sum_{p=1}^{n} \frac{\varkappa(z)_{[p]}}{2\pi i} \int_{\gamma^p} \frac{\Delta f(\tau_{t_p};t)}{\tau_p - t_p} d\tau_p + f(t)z(t))\Bigg| \le$$

$$\le c\Bigg[Z(\omega_f^N; |z - t|, Q) + \sum_{p=1}^{n} Z(\omega_f^N; |z - t|_{[p]}, d_p, \theta) +$$

$$+\sum_{\substack{p=1\\ p<q}}^{n}\sum_{q=1}^{n} Z(\omega_f^N; |z - t|_{[pq]}, d_p, d_q, \theta) + \cdots +$$

$$+\sum_{\substack{p=1\\ p<q}}^{n}\sum_{q=1}^{n} Z(\omega_f^N; d_{[pq]}, |z_p - t_p|, ||z_q - t_q|\theta) +$$

$$+\sum_{p=1}^{n} Z(\omega_f^N; d_{[p]}, |z_p - t_p|, Q)\Bigg) + \left(Z\left(\omega_f^{N_p}; |z - t|_{[p]} Q_{[p]}\right) +\right.$$

$$+\sum_{q=1}^{n} Z(\omega_f^{N_p}; |z-t|_{[pq]}, d_q, \theta_{[p]}\Big) +$$

$$+\sum_{\substack{q=1 \\ q<r; \\ q,r=\overline{1,n}}}^{n} \sum_{r=1}^{n} Z(\omega_f^{N_p}; |z-t|_{[pqr]}, d_q, d_r\theta_{[p]}) + \cdots$$

$$+\sum_{\substack{q=1 \\ q<r;q,r=\overline{1,n}}}^{n} \sum_{r=1}^{n} Z(\omega_f^{N_p}; d_{[pqr]}, |z_q-t_q|, |z_r-t_r|, \theta_{[p]}) +$$

$$+\sum_{\substack{q=1 \\ q\neq p}}^{n} Z(\omega_f^{pq}; d_{[pq]}, |z_q-t_q|, \theta_{[p]})\Big) + \cdots +$$

$$+(Z(\omega_f^{pq}; |z_p-t_p|, |z_q-t_q|, d_p, \theta_p, \theta_q) + Z(\omega_f^{pq}; |z_p-t_p|, d_p, \theta_p, \theta_q) +$$

$$+Z(\omega_f^{pq}; |z_q-t_q|, d_p, \theta_p, \theta_q)) + Z(\omega_f^{p}; |z_q-t_q|, \theta_p)]$$

holds.

The proof follows from Theorem 3.1.

Theorem 3.3. Let γ^k *(k=$\overline{1,n}$)* - c.j.r.c., $f \in J_0(\Delta, \theta)$. Then function $\Phi(z)$ continuously extendable on a core Δ from each of 2^n semicylindrical domains for which the core is common.

By Theorems 3.1 and 3.2 and taking into account (2), we get that the function $\Phi(z)$ is continuously extended to the cores Δ and for the limiting values of the function $\Phi(z)$ equitable Sokhotskii's formulas:

$$\Phi^+(t) = f(t) + \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\Delta f(\tau;t)}{\tau - t} d\tau$$

$$+\sum_{p=1}^{n} \frac{1}{(2\pi i)^{n-1}} \int_{\Delta_{[p]}} \frac{\Delta f(\tau_{t_p};t)}{(\tau-t)_{[p]}} d\tau_{[p]} + \sum_{\substack{p=1 \\ p<q}}^{n} \sum_{q=1}^{n} \frac{1}{(2\pi i)^{n-2}} \int_{\Delta_{[pq]}} \frac{\Delta f\left(\tau_{t_{pq}};t\right)}{(\tau-t)_{[pq]}} d\tau_{[pq]}$$

$$+\cdots+\sum_{\substack{p=1 \\ p<q}}^{n} \sum_{q=1}^{n} \frac{1}{(2\pi i)^2} \int_{\gamma^p} \int_{\gamma^q} \frac{\Delta f(t_{\tau_{pq}};t)}{(\tau_p-t_p)(\tau_q-t_q)} d\tau_p d\tau_q + \sum_{p=1}^{n} \frac{1}{2\pi i} \int_{\gamma^p} \frac{\Delta f\left(\tau_{t_p};t\right)}{\tau_p - t_p} d\tau_p,$$

$$\Phi_{-p}^{+}(t) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\Delta f(\tau;t)}{\tau - t} d\tau +$$

$$+\sum_{\substack{k=1 \\ k\neq p}}^{n} \frac{1}{(2\pi i)^{n-1}} \int_{\Delta_{[k]}} \frac{\Delta f(\tau_k;t)}{(\tau-t)_{[k]}} d\tau_{[k]} + \sum_{\substack{q=1 \\ q<r; \\ q,r\neq p}}^{n} \sum_{r=1}^{n} \frac{1}{(2\pi i)^{n-2}} \int_{\Delta[kr]} \frac{\Delta f(\tau_{t_{kr}};t)}{(\tau-t)_{[kr]}} d\tau_{[kr]} +$$

$$+\cdots+\sum_{\substack{k=1 \\ k<r\,;\,k,r\neq p}}^{n} \sum_{r=1}^{n} \frac{1}{(2\pi i)^{n-3}} \int_{\gamma^p} \int_{\gamma^k} \int_{\gamma r} \frac{\Delta f(t_{\tau_{pkr}};t)}{(\tau_p-t_p)(\tau_k-t_k)(\tau_r-t_r)} d\tau_p d\tau_k d\tau_r +$$

$$+\sum_{\substack{k=1 \\ k\neq p}}^{n} \frac{1}{(2\pi i)^2} \int_{\gamma^p} \int_{\gamma^k} \frac{\Delta f(t_{\tau_{pk}};t)}{(\tau_p-t_p)(\tau_k-t_k)} d\tau_p d\tau_k + \frac{1}{2\pi i} \int_{\gamma^p} \frac{\Delta f(t_{\tau_p};t)}{\tau_p - t_p} d\tau_p,$$

$$\Phi_{-pq}^{+} = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\Delta f(\tau;t)}{\tau - t} d\tau + \sum_{\substack{k=1 \\ k\neq p,q}}^{n} \frac{1}{(2\pi i)^{n-1}} \int_{\Delta_{[k]}} \frac{\Delta f(\tau_{t_k};t)}{(\tau-t)_{[k]}} d\tau_{[k]} +$$

$$+\sum_{\substack{k=1\\k<r;\\k,r\neq p,q}}^{n}\sum_{r=1}^{n}\frac{1}{(2\pi i)^{n-2}}\int\limits_{\Delta_{[kr]}}\frac{\Delta f(\tau_{t_{kr}};t)}{(\tau-t)_{[kr]}}d\tau_{[kr]}+\cdots+$$

$$+\sum_{\substack{k=1\\k<r;\\k,r\neq p,q}}^{n}\sum_{r=1}^{n}\frac{1}{(2\pi i)^{4}}\int\limits_{\gamma^p}\int\limits_{\gamma^q}\int\limits_{\gamma^k}\int\limits_{\gamma^r}\frac{f(t_{\tau_{pqkr}};t)}{(\tau_p-t_p)(\tau_q-t_q)(\tau_k-t_k)(\tau_r-t_r)}d\tau_p d\tau_q d\tau_k d\tau_r+$$

$$+\sum_{\substack{r=1\\k,r\neq p,q}}^{n}\frac{1}{(2\pi i)^{3}}\int\limits_{\gamma^p}\int\limits_{\gamma^q}\int\limits_{\gamma^k}\frac{\Delta f(t_{\tau_{pqk}};t)}{(\tau_p-t_p)(\tau_q-t_q)(\tau_k-t_k)}d\tau_p d\tau_q d\tau_k+$$

$$+\frac{1}{(2\pi i)^{2}}\int\limits_{\gamma^p}\int\limits_{\gamma^q}\frac{\Delta f(t_{\tau_{pq}};t)}{(\tau_p-t_p)(\tau_q-t_q)}d\tau_p d\tau_q,$$

$$\Phi^{+}{}_{\substack{-pql\\p<q<l\\p,q,l=\overline{1,n}}}(t)=\frac{1}{(2\pi i)^{n}}\int\limits_{\Delta}\frac{\Delta f(\tau;t)}{\tau-t}d\tau\sum_{\substack{k=1\\k\neq p,q,l}}^{n}\frac{1}{(2\pi i)^{n-1}}\int\limits_{\Delta[k]}\frac{\Delta f(\tau_{t_k};t)}{(\tau-t)_{[k]}}d\tau_{[k]}+$$

$$+\sum_{\substack{k=1\\k<r;\\k,r\neq p,q,l}}^{n}\sum_{k,r\neq p}^{n}\frac{1}{(2\pi i)^{n-2}}\int\limits_{\Delta_{[kr]}}\frac{\Delta f(\tau_{t_{kr}};t)}{(\tau-t)_{[kr]}}d\tau_{[kr]}+$$

$$+\sum_{\substack{k=1\\k<r<s}}^{n}\sum_{\substack{r=1\\k,r,s\neq p,q,l}}^{n}\sum_{s=1}^{n}\frac{1}{(2\pi i)^{n-3}}\int\limits_{\Delta_{[krs]}}\frac{\Delta f(\tau_{t_{krs}};t)}{(\tau-t)_{[krs]}}d\tau_{[krs]}+\cdots+$$

$$+\sum_{\substack{k=1\\k<r;\\k,r\neq p,q,l}}^{n}\sum_{r=1}^{n}\frac{1}{(2\pi i)^{5}}\int\limits_{\gamma^p}\int\limits_{\gamma^q}\int\limits_{\gamma^l}\int\limits_{\gamma^k}\int\limits_{\gamma^r}\frac{\Delta f\left(t_{\tau_{pqlkr}};t\right)d\tau_p d\tau_q d\tau_l d\tau_k}{(\tau_p-t_p)(\tau_q-t_q)(\tau_l-t_l)(\tau_k-t_k)(\tau_r-t_r)}+$$

$$+\sum_{\substack{k=1\\k\neq p,q,l}}^{n}\frac{1}{(2\pi i)^{4}}\int\limits_{\gamma^p}\int\limits_{\gamma^q}\int\limits_{\gamma^l}\int\limits_{\gamma^k}\frac{\Delta f(t_{\tau_{pqlk}};t)d\tau_p d\tau_q d\tau_l d\tau_k}{(\tau_p-t_p)(\tau_q-t_q)(\tau_l-t_l)(\tau_k-t_k)}+$$

$$+\frac{1}{(2\pi i)^{3}}\int_{\gamma^p}\int_{\gamma^q}\int_{\gamma^k}\frac{\Delta f(t_{\tau_{pqk}};t)}{(\tau_p-t_p)(\tau_q-t_q)(\tau_k-t_k)}d\tau_p d\tau_q d\tau_k\ ,$$

$$\Phi^{-}{}_{\substack{+pql\\p<q<l\\p,q,l=\overline{1,n}}}(t)=\frac{1}{(2\pi i)^{n}}\int\limits_{\Delta}\frac{\Delta f(\tau;t)}{\tau-t}d\tau+\sum\nolimits_{k=p,q}^{n}\frac{1}{(2\pi i)^{n-1}}\int\limits_{\Delta[k]}\frac{\Delta f(\tau_{t_k};t)}{(\tau-t)_{[k]}}d\tau_{[k]}+$$

$$+\sum\nolimits_{\substack{k=p,q,l\\k<r}}^{n}\sum\nolimits_{r=p,q}^{n}\frac{1}{(2\pi i)^{n-2}}\int_{\Delta[kr]}\frac{\Delta f(\tau_{tkr};t)}{(\tau-t)_{[kr]}}d\tau_{[kr]}+\frac{1}{(2\pi i)^{3}}\int_{\Delta_{[pql]}}\frac{\Delta f(\tau_{t_{ql}};t)}{(\tau-t)_{[pql]}}d\tau_{[pql]},$$

$$\Phi^{-}{}_{\substack{+pq\\p<q\\p,q=\overline{1,n}}}(t)=\frac{1}{(2\pi i)^{n}}\int\limits_{\Delta}\frac{\Delta f(\tau;t)}{(\tau-t)}d\tau+\sum\nolimits_{k=p,q}^{n}\frac{1}{(2\pi i)^{n-1}}\int_{\Delta_{[k]}}\frac{\Delta f(\tau_{tk};t)}{(\tau-t)_{[k]}}d\tau_{[k]}+$$

$$+\frac{1}{(2\pi i)^{n-2}}\int_{\Delta_{[pq]}}\frac{\Delta f(\tau_{t_{pq}};t)}{(\tau-t)_{[pq]}}d\tau_{[pq]},\tag{24}$$

$$\Phi^{-}{}_{\substack{+p\\p=\overline{1,n}}}(t)=\frac{1}{(2\pi i)^{n}}\int_{\Delta}\frac{\Delta f(\tau;t)}{\tau-t}d\tau+\frac{1}{(2\pi i)^{n-1}}\int_{\Delta[p]}\frac{\Delta f(\tau_{tp};t)}{(\tau-t)_{[p]}}d\tau_{[p]}\ ,$$

$$\Phi^{-}(t) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\Delta f(\tau;t)}{\tau - t} d\tau.$$

In particular, for the case $n = 1$, the Sokhotskii's formulas (24) take the forms

$$\begin{cases} \Phi^{+}(t) = \frac{f(t)}{2} + \frac{1}{2\pi i} \int_{\gamma^1} \frac{f(\tau)}{\tau - t} d\tau, \\ \Phi^{-}(t) = -\frac{f(t)}{2} + \frac{1}{2\pi i} \int_{\gamma^1} \frac{f(\tau)}{\tau - t} d\tau, \end{cases} \quad n = 1; \tag{25}$$

and for the case $n = 2$:

$$\Phi^{++}(t_1, t_2) = \frac{1}{(2\pi i)^2} \int_{\gamma^1} \int_{\gamma^2} \frac{\Delta f(\tau_1, t_1, \tau_2, t_2)}{(\tau_1 - t_1)(\tau_2 - t_2)} d\tau_1 d\tau_2 +$$

$$+ \frac{1}{2\pi i} \int_{\gamma^2} \frac{f(t_1, \tau_2) - f(t_1, t_2)}{\tau_2 - t_2} d\tau_2 + \frac{1}{2\pi i} \int_{\gamma^1} \frac{f(\tau_1, t_2) - f(t_1, t_2)}{\tau_1 - t_1} d\tau_1 + f(t_1, t_2), \tag{26}$$

$$\Phi^{+-}(t_1, t_2) = \frac{1}{(2\pi i)^2} \int_{\gamma^1} \int_{\gamma^2} \frac{\Delta f(\tau_1, t_1, \tau_2, t_2)}{(\tau_1 - t_1)(\tau_2 - t_2)} d\tau_1 d\tau_2 + \frac{1}{2\pi i} \int_{\gamma^2} \frac{f(t_1, \tau_2) - f(t_1, t_2)}{(\tau_2 - t_2)} d\tau_2, \tag{27}$$

$$\Phi^{-+}(t_1, t_2) = \frac{1}{(2\pi i)^2} \int_{\gamma^1} \int_{\gamma^2} \frac{\Delta f(\tau_1, t_1, \tau_2, t_2)}{(\tau_1 - t_1)(\tau_2 - t_2)} d\tau_1 d\tau_2 + \frac{1}{2\pi i} \int_{\gamma^1} \frac{f(\tau_1, t_1) - f(t_1, t_2)}{(\tau_1 - t_1)} d\tau_{2,} \tag{28}$$

$$\Phi^{--}(t_1, t_2) = \frac{1}{(2\pi i)^2} \int_{\gamma^1} \int_{\gamma^2} \frac{\Delta f(\tau_1, t_1, \tau_2, t_2)}{(\tau_1 - t_1)(\tau_2 - t_2)} d\tau_1 d\tau_2. \tag{29}$$

Above we have shown the behavior of Cauchy-type integrals on the core Δ of the border. Now we explain the behavior of the integrals on the whole boundary of the semicylindrical domain**.** To reduce the entries in detail, consider the case n=3 for

$$D_1^+ \times D_2^- \times D_3^+.$$

The boundary of this domain consists of the sets: $\gamma^1 \times D_2^- \times D_3^+$; $D_1^+ \times \gamma^2 \times D_3^+$; $D_1^+ \times D_2^- \times \gamma^3$; $\gamma^1 \times \gamma^2 \times D_3^+$;$\gamma^1 \times D_2^- \times \gamma^3$;$D_1^+ \times \gamma^2 \times \gamma^3$ and $\Delta = \gamma^1 \times \gamma^2 \times \gamma^3$.

The integral (1) for *n=3* we write as follows

$$\Phi(z) = \frac{1}{2\pi i} \int_{\gamma^p} \left[\frac{1}{(2\pi i)^2} \int_{\Delta_{[p]}} \frac{f(\tau) d\tau_{[p]}}{\prod_{\substack{k=1 \\ k \neq p}}^{3} (\tau_k - z_k)} \right] \frac{d\tau_p}{\tau_p - z_p}, p = 1,2,3, \tag{30}$$

$$\Phi(z) = \frac{1}{(2\pi i)^2} \int_{\Delta_{[p]}} \left[\frac{1}{2\pi i} \int_{\gamma^p} \frac{f(\tau) d\tau_{[p]}}{(\tau_p - z_p)} \right] \frac{d\tau_{[p]}}{\prod_{\substack{k=1 \\ k \neq p}}^{3} \tau_k - z_k}, p = 1,2,3. \tag{31}$$

Let us consider integral (30) as integral of Cauchy-type of a complex variable with a core

$$\varphi_p\left(z_{\tau_p}\right) = \frac{1}{(2\pi i)^2} \int_{\Delta_{[p]}} \frac{f(\tau) d\tau_{[p]}}{\prod_{\substack{k=1 \\ k \neq p}}^{3} (\tau_k - z_k)},$$

depending on the parameters $z_k, k = 1,2,3, k \neq p$ and applying to them Sokhotskii's formulas (26) of the variable z_p and we obtain

$$\Phi^{+-+}\left(z_{t_p}\right) = \frac{(-1)^{p-1}}{(2\pi i)^2} \int_{\Delta_{[p]}} \frac{f\left(\tau_{t_p}\right) d\tau_{[p]}}{\prod_{\substack{k=1 \\ k \neq p}}^{3} (\tau_k - z_k)} + \frac{1}{2\pi i} \int_{\Delta} \frac{f(\tau) d\tau}{(\tau_p - z_p) \prod_{\substack{k=1 \\ k \neq p}}^{3} (\tau_k - z_k)}. \tag{32}$$

At p=1,2,3 the formula (32) gives boundary values of integral (1) in points of boundary sets: $\gamma^1 \times D_2^- \times D_3^+$; $D_1^+ \times \gamma^2 \times D_3^+$; $D_1^+ \times D_2^- \times \gamma^3$.

Considering integrals (31) as integral of Cauchy-type of two complex variables of $z_k, k = 1,2,3, k \neq p$ with the core

$$\Psi_p\left(\tau_{z_p}\right) = \int_{\gamma^p} \frac{f(\tau) d\tau_p}{(\tau_p - z_p)},$$

depending on the parameter z_p and applying to them Sokhotskii's formulas (26), (27) and (28) on the corresponding variables and obtain

$$\Phi^{+-+}(z_{t_3}) = -\frac{1}{4}\frac{1}{2\pi i}\int_{D_3}\frac{f(\tau_{t_3})d\tau_3}{(\tau_3 - z_3)} - \frac{1}{2}\frac{1}{(2\pi i)^2}\int_{\Delta_{[2]}}\frac{f(\tau_{t_2})d\tau_2}{(\tau_1 - t_1)(\tau_3 - z_3)} +$$

$$+\frac{1}{2}\cdot\frac{1}{(2\pi i)^2}\int_{\Delta_{[1]}}\frac{f(\tau_{t_1})d\tau_{[1]}}{(\tau_2 - t_2)(\tau_3 - z_3)} + \frac{1}{(2\pi i)^3}\int_{\Delta}\frac{f(\tau)d\tau}{(\tau_3 - z_3)\prod_{k=1}^{2}(\tau_k - z_k)}, \quad (33)$$

$$\Phi^{+-+}(z_{t_2}) = \frac{1}{4}\frac{1}{2\pi i}\int_{D_3}\frac{f(\tau_{t_2})d\tau_2}{(\tau_2 - z_2)} + \frac{1}{2}\frac{1}{(2\pi i)^2}\int_{\Delta_{[3]}}\frac{f(\tau_{t_3})d\tau_{[3]}}{(\tau_1 - t_1)(\tau_2 - z_2)} +$$

$$+\frac{1}{2}\cdot\frac{1}{(2\pi i)^2}\int_{\Delta_{[1]}}\frac{f(\tau_{t_1})d\tau_{[1]}}{(\tau_2 - t_2)(\tau_3 - z_3)} + \frac{1}{(2\pi i)^3}\int_{\Delta}\frac{f(\tau)d\tau}{(\tau_2 - z_2)\prod_{k=1}^{3}(\tau_1 - z_1)(\tau_2 - z_2)}, \quad (34)$$

$$\Phi^{+-+}(z_1) = -\frac{1}{4}\frac{1}{2\pi i}\int_{D_1}\frac{f(\tau_{t_1})d\tau_1}{(\tau_1 - z_1)} + \frac{1}{2}\frac{1}{(2\pi i)^2}\int_{\Delta_{[3]}}\frac{f(\tau_{t_3})d\tau_{[3]}}{(\tau_1 - t_1)(\tau_2 - z_2)} +$$

$$-\frac{1}{2}\cdot\frac{1}{(2\pi i)^2}\int_{\Delta_{[2]}}\frac{f(\tau_{t_2})d\tau_{[2]}}{(\tau_1 - t_1)(\tau_3 - z_3)} + \frac{1}{(2\pi i)^3}\int_{\Delta}\frac{f(\tau)d\tau}{(\tau_1 - z_1)\prod_{k=2}^{3}(\tau_k - z_k)}. \quad (35)$$

Formulas (33), (34) and (35) give values of integral (1) at points of sets $\gamma^1 \times \gamma^2 \times D_3^+$; $\gamma^1 \times D_2^- \times \gamma^3$; $D_1^+ \times \gamma^2 \times \gamma^3$. All shifts integrals at a conclusion of formulas (32)-(35) are admissible as only shifts of special integrals with routine were applied. On a core Δ boundary values $\Phi^{+-+}(t)$ are defined by Sokhotskii's formula (24).

Thus the following theorem is proven.

Theorem 3.4. Let $\gamma^k\ (k = \overline{1,n}) -$ r.j.c.c., $f \in J_0(\Delta, \theta)$. Then the function $\Phi(z)$ defined as (1) is continuously extended to the entire of border of the semicylindrical domain and the limiting values of the function $\Phi(z)$ are formulas such as Sokhotskii's formulas for the case of the core (24).

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