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# Improved Mathematical Modeling of the Hourly Solar Diffuse Fraction (HSDF) - Adrar, Algeria Case Study

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### Abstract

Solar energy is among the excellent alternative energy resources; however, it suffers from significant problems. These problems are mainly due to the inherent variability, and intermittency of the solar resource; however, proper predictability of the resource can reduce the consequent impacts of the mentioned problems. Enhancing the predictability of the solar resource provides an essential tool for the design, performance analysis, and economic evaluation of various solar energy projects. In this paper, highly accurate mathematical models for estimating the hourly diffuse solar fraction are presented for enhancing the predictability of the solar resource over Adrar, Algeria big south desert. The presented modeling is based on clearance index measurements. The best found model for the considered site is found to be the sigmoid logistic empirical model. This model shows the highest accuracy in comparison with other models where its correlation coefficient (R), and the Nash-Sutcliffe NSE are found to be 93.7% and 84.2% respectively. In addition, the sigmoid logistic model shows very low values of the mean bias error (MBE), and root mean square error (RMSE).

### 1. Introduction

The solar energy that is received on earth is, in round numbers, about 10,000 times the total amount of energy consumed by the entire humanity. In other words, capturing 0.01% of this energy would allow us to dispense with oil, gas, coal and uranium. Information on solar flux have become an important issue for renewable energy issues stemming from oil crises and other ecological issues, hence the need for reliable measurements of surface solar radiation. The quantity of solar radiation that strikes a surface relies on various affecting factors, as a location of the sun in the sky and the

clearness of the atmosphere as well as on the nature and the orientation of the striking surface.

Foreseeing the amount of hourly diffuse fraction ( $F_d$ ), is the highest importance in building energy systems as well as an accurate evaluation of thermal environment within buildings and several engineering applications on earth, with about 20% average ground reflection of the global irradiance [1]. Unfortunately, data measurements almost non-existent in rural areas. Therefore, it is necessary to find a model of diffuse fraction, correlating the diffuse radiation to the global radiation, which is usually available in the reports from the meteorological stations. The hourly global horizontal irradiance ( $I_g$ ) comprises of two sections [2]: hourly diffuse horizontal irradiance ( $I_d$ ) and hourly beam horizontal irradiance ( $I_b$ ). We can define the hourly clearness index  $k_t$  as the ratio of the hourly observed horizontal global solar radiation to the calculated hourly horizontal extraterrestrial

global solar radiation[2]:

$$k_t = \frac{I_g}{I_0} \quad (1)$$

The hourly diffuse fraction ( $F_d$ ) takes the form,

$$F_d = \frac{I_d}{I_g} \quad (2)$$

The atmosphere top limit of hourly solar radiation in unit ( $\text{W/m}^2$ ) is defined by the following equation [3]:

$$I_0 = (12/\pi) \cdot I_{sc} \cdot E_0 \int_{\omega_2}^{\omega_1} (\cos \delta \cos \varphi \sin \omega + \sin \delta \sin \varphi) d\omega \quad (3)$$

where,  $I_{sc}$ : 1367  $\text{W/m}^2$  solar constant,  $\varphi$ : place latitude,  $E_0$  is the eccentricity correction factor  $\delta$  is the solar declination in degrees; and  $\omega_2$  and  $\omega_1$  is time other than an hour in degrees.

**Table 1.** Summarizes the nominated models from the two categories.

Models	Years	Ref	Diffuse fraction ( $F_d$ )		
			Interval (I)	Interval (II)	Interval (III)
M1	1977	[4]	0.94 for $k_t \leq 0.4$	$1.29 - 1.19k_t/1.00 - 0.33k_t$ for $0.4 < k_t \leq 1$	-
M2	2003	[5]	$0.9995 - 0.05k_t - 2.4156k_t^2$ $+ 1.4926k_t^3$ for $k_t \leq 0.78$	0.2 for $0.78 < k_t \leq 1$	-
M3	1977	[6]	$1 - 0.249k_t$ for $k_t \leq 0.35$	$1.557 - 1.84k_t$ for $k_t \leq 0.75$	0.177 for $k_t > 0.75$
M4	1984	[7]	0.915 for $k_t \leq 0.225$	$1.135 - 0.9422k_t - 0.3878k_t^2$ for $k_t \leq 0.775$	0.215 for $k_t > 0.775$
M5				2001	[8]
M6	1994	[9]	$1.0086 - 0.78k_t$ for $k_t \leq 0.21$	$0.9995 - 0.05k_t - 2.4156k_t^2 + 1.49263k_t^3$ for $k_t \leq 0.78$	0.215 for $k_t > 0.775$
M7	2006	[10]	0.987 for $k_t \leq 0.1$	$0.94 + 0.937k_t - 5.01k_t^2 + 3.20k_t^3$ for $k_t \leq 0.8$	0.177 for $k_t > 0.8$
M8	2001	[11]	$\frac{1}{1 + \exp(-4.90 + 8.78k_t)}$		
M9	2008	[12]	$\frac{1}{1 + \exp(-5.003 + 8.602k_t)}$		
M10			2016	[13]	$0.13 + 0.86 \frac{1}{1 + \exp(-6.29 + 8.78k_t)}$

The first correlation for predicting models the hourly solar diffuse fraction constructed by Liu and Jordan [14]. The focus of many distributions of deciding on the hourly diffuse irradiance by the method for connections that relate the diffuse part ( $F_d$ ) with the clearness list ( $k_t$ ) considering polynomials of different requests. Among the models existing in the literature, including, but not limited to, the establishment of Reindl, Beckman, and Duffie(1990) [15], of Khalil and Shaffie(2013)[16].

The main purpose of this study is to develop a novel model to identify the most suitable radiometric diffuse fraction model to characterize Adrar at Algerian Big South Desert. The 16 made radiometric estimation models, are tested and validated on the two years for all sky. Several statistical indicators are used for assessing the accuracy of various models.

## 2. Method for Estimation

Numerous models are available to estimate the hourly diffuse

fraction from irradiance on a horizontal one. But, they require information at the same time on the global radiation on a horizontal surface. After a brief summary, several studies were identified, which offer computational models of the hourly diffuse fraction to any site for an indicated period. Among the models existing in the literature, we have selected some models for our study (see Table 1). Generally, the empirical models of the hourly diffuse fraction are classified into two categories; the multi-zone models, most of the existing models divide the hourly clearness index ( $k_t$ ), domain into at most 3 sub-domains (between two to three different zones), it has been used for computing the hourly diffuse fraction as a function of the hourly clearness index, ( $k_t$ ), and several climatic variables, including the ambient temperature and relative humidity (RH),  $F_d = f(k_t)$  and the one-zone models, these models are relatively new. One-zone models for estimating the hourly diffuse fraction is related to the hourly clearness index and,  $F_d = f(k_t)$ . The research to date has tended to focus on One-zone

models rather than Couplet zone models, we have selected some models for our study include in Table 1.

### 3. Data Source

The data used in this study are those of the whole two years of 2014 and 2015, the studied site is the one from the Research Unit in Renewable energies in Saharan Medium (URER. MS) Adrar located in southern Algeria with the following geographical data 27 ° 53 'North and 0 ° 11' West. Our work means to compare diffuse irradiance on a horizontal plan calculated by these models along with those measured at the site of Adrar [17]. The diffuse irradiance measurements are performed by the K&Z CMP21 pyranometer (Figure 1). Information quality control techniques are important for an application or analysis, which seeks to help mitigate the errors. The treatment conditions of Quality control measures during data collection include the following [18, 19]:

- a) Zenith angle condition ( $Z < 85$ )
  1.  $I_g < 1.50E_0 \cos^{1.2} Z + 100$
  2.  $I_d < 0.95E_0 \cos^{1.2} Z + 50$
  3.  $I_b < 1100 + 0.03E_{lev}$  and  $I_b < E_0$
  4.  $Abs(Closr) < 5\%$

where Elev is the elevation (m) and Closr =  $100 [(I_b \cos Z + I_d - I_g) / I_g]$ .

- b) Diffuse Fraction ( $F_d$ )

1. Case I:  $Z < 75$   $F_d < 1.05$  for  $I_g > 50$
2. Case II:  $Z > 1.10$   $F_d < 1.05$  for  $I_g > 50$



Figure 1. Pyranometer for measuring the diffuse radiation data in site of the test cell.

### 4. Model Selection and Validation

In order to verify of each model is statistically assessed through six statistical indicators; mean bias error (MBE), root mean square error (RMSE), root mean square difference (RMSD), correlation coefficient (R), the Nash-Sutcliffe (NSE), and the t-static errors (TS). The best model for the considered site is selected based on the superiority of the values of the statistical indices in comparison with other models. The equations of these statistical indicators are [20-25]:

$$MBE = \frac{1}{n} \sum_n (F_{d\ i,e} - F_{d\ i,o}) \quad (4)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_n (F_{d\ i,e} - F_{d\ i,o})^2} \quad (5)$$

$$RMSD = \frac{100}{\bar{F}_{i,o}} \sqrt{\frac{1}{n} \sum_n (F_{d\ i,e} - F_{d\ i,o})^2} \quad (6)$$

$$R = \frac{\sum_n (F_{d\ i,o} - \bar{F}_{d,o})(F_{d\ i,e} - \bar{F}_{d,e})}{\sqrt{\sum_n (F_{d\ i,o} - \bar{F}_{d,o})^2 \sum_n (F_{d\ i,e} - \bar{F}_{d,e})^2}} \quad (7)$$

$$NSE = 1 - \frac{\sum_n (F_{d\ i,e} - F_{d\ i,o})^2}{\sum_n (F_{d\ i,o} - \bar{F}_{d,o})^2} \quad (8)$$

$$TS = \sqrt{\frac{(n-1)MBE^2}{RMSE^2 - MBE^2}} \quad (9)$$

### 5. Results and Discussion

In this section, we present an analysis of reviews statistical data of various models using the data set of ground measurements of global and diffuse horizontal irradiance over a period of two years at Adrar. In a first step, a quality control testing is applied to our datasets based on the Gueymard methodology [19]. Applying these criteria to the data in Adrar, the number of valid points was reduced by 74.6 percent from the original data, taking into consideration night hours too. It adopted an additional correction to limiting minimum and maximum values of the estimated diffuse fraction to the values of 0 and 1 respectively in order to prevent the non-natural results. And lastly, the maximum allowable value of clearness index ( $k_t$ ) has been forced to 1. As can be seen from Figure 2 after applying a filtering process described above, the data set of diffuse fraction versus ( $k_t$ ).

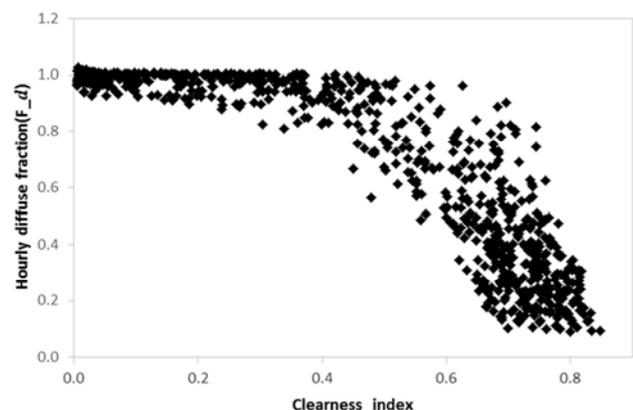


Figure 2. The experimental hourly diffuse fraction data as a function of clearness index.

Figure 3 gives an example of one statistical indicators, MBE(%). The first set of analyses examined the impact of the mean bias error MBE(%) where the values of MBE(%) are high in the first group (Multi-zone models) ranging between 6% to 67% while it changed between 1.2% to 9% in the second group (One-zone models).

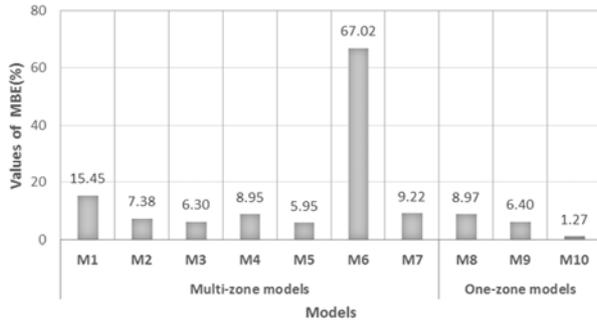


Figure 3. The values of mean bias error MBE(%) of the nominated models.

According to the filtered results, we will adopt a single-parameter function, i.e., a function in terms of the clearness index ( $k_t$ ) only, in the modeling work. In the two categories use the linear, a piecewise 2nd-order and 3rd-order and a sigmoid logistic formula to determine the diffuse fraction model in the region of Adrar. The regression coefficients results are given by the following expressions:

$$A\# 1. F_d = \begin{cases} 0.955 - 0.099k_t & \text{for } k_t \leq 0.35 \\ 11.87k_t^3 - 22.116k_t^2 + 11.485k_t - 0.866 & \text{for } k_t > 0.35 \end{cases} \quad (10)$$

$$A\# 2. F_d = \begin{cases} 0.996 - 0.130k_t & \text{for } k_t \leq 0.40 \\ 1.800 - 2.212k_t + 0.194k_t^2 & \text{for } 0.40 < k_t \leq 0.80 \\ 0.140 & \text{for } k_t > 0.80 \end{cases} \quad (11)$$

$$A\# 3. F_d = \frac{1}{1 + \exp(-5.979 + 9.101k_t)} \quad (12)$$

$$A\# 4. F_d = 0.142 + 0.847 / (1 + \exp(-7.121 + 11.428k_t)) \quad (13)$$

Table 2 gives errors in the different estimation models. Illuminations estimated by empirical models and those measured on Adrar site are virtually identical. Thus, the average committed errors are small for the eight models tested below MBE (%) <2.1% and (R%), the Nash-Sutcliffe (NSE%) exceed 92%, 80% respectively. Based upon assessment on the proposed model functions, A3 and A4 models gives more accuracy whatever sky state.

When fitting a variety of models to the same data, some objective criterion is required for determining the best fit. Using the Akaike Information Criterion (AIC) method, defined as

$$AIC = n \log(RSS) + 2p \quad (14)$$

with  $RSS$ ,  $p$  is residual sum of squares and the number of the model parameters.

Table 2. Error model results for the out-of-sample data in 2014, Adrar.

Models	MBE	RMSE	RMSD%	R	NSE
M10	0.013	0.125	19.963	0.931	0.825
M5	0.059	0.145	23.188	0.924	0.826
M3	0.063	0.145	23.107	0.926	0.825
M9	0.064	0.143	22.808	0.931	0.836
A1	0.006	0.125	19.920	0.930	0.834
A2	0.016	0.121	19.321	0.935	0.863
A3	-0.021	0.120	19.184	0.937	0.845
A4	-0.021	0.120	19.202	0.937	0.842

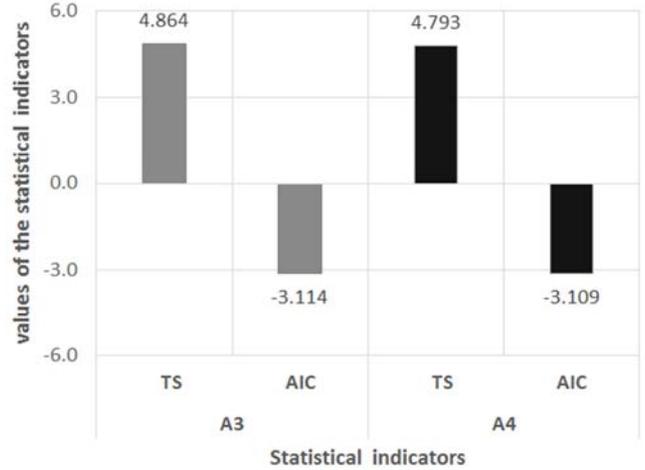


Figure 3. Comparison the t-static errors (TS) and Akaike information criterion (AIC  $\times 10^3$ ) of models A3 and A4.

As shown in Figure 1, comparison of the t-static errors (TS) index and Akaike information criterion (AIC) for the better models A3 and A4. The best regression is chosen via the AIC test and lowest value of TS. Accordingly, A4 model has a better performance than the A3 model. As Figure 4 shows, the estimated results using the best models (A4) to estimate the hourly diffuse fraction in Adrar.

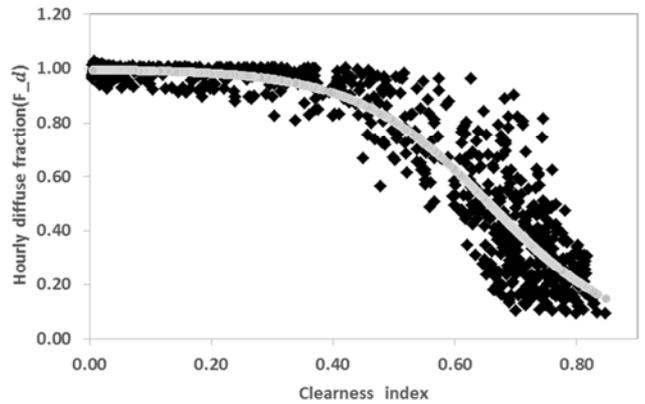


Figure 4. The estimated by the best model and experimental hourly diffuse fraction data over an Adrar region (2015).

## 6. Conclusion

The main goal of the current study was to develop a new correlation model to estimate the diffuse solar fraction for Adrar region for a better predictability of the solar resource. There are ten diffuse fraction correlations selected from the literature and nine models established, then classified in two categories: Multi-zone and One-zone models. Furthermore, based upon the assessments of the six statistical indicators, sigmoid logistic model (i.e.  $F_d = 1 / (1 + \exp(-5.979 + 9.101k_t))$ ) has the best performance over the all tested models. The results of this study will serve as an input to advance the solar energy project activities in Adrar region, Algerian Big South desert.

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