International Journal of Mathematical Analysis and Applications
2018; 5(2): 44-48
http://www.aascit.org/journal/ijmaa
ISSN: 2375-3927

American Association for Science and Technology

Alternative Attri-Var Quality Control Sampling Inspection Methods

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Citation

Malik Beshir Malik. Alternative Attri-Var Quality Control Sampling Inspection Methods. *International Journal of Mathematical Analysis and Applications*. Vol. 5, No. 2, 2018, pp. 44-48.

Received: May 6, 2017; **Accepted**: August 6, 2017; **Published**: May 31, 2018

Abstract: Sampling inspection methods used in industrial quality control normally take the form of inspection-by-attributes or inspection-by-variables methods. Inspection-by-attributes sampling plans are noted for their robustness with respect to any distributional form of the characteristic of an assumed continuous distribution (usually a normal distribution) and therefore are not necessarily robust as departures from this assumed distribution are encountered in practice but do permit relatively smaller sample sizes than would be required under an equivalent attributes sampling plan. In this paper we provide a new method for sampling inspection. The sample size levels and robustness of the new method lies in between the two classical inspection-by-variables and inspection-by-attributes sampling plans. The *new* method will be designed and explained, and its equivalence to the classical methods will be established. The sample size performance is thoroughly investigated and compared for the traditional and equivalent new methods. Their robustness will be discussed at a preliminary level.

Keywords: Equivalence of Plans, Robustness, Sample Size Performance, Attri-var Sampling Plans, Batch Inspection

1. Introduction

Sampling inspection methods are commonly used in quality control to assess the acceptability of units of products formed into lots. The assessment involves selecting a random sample from the lot and following the procedure specified by a sampling plan established for that purpose. The two most popular approaches are designated as sampling by attributes and sampling by variables. The attribute-types plans are based on counts of an attribute of the quality of the items in the batch for a given sample size. The variables-type plans assume that the observed measurements are continuous observations drawn from a normally distributed population with mean μ and standard deviation σ. For convenience and without any loss of generality, we will assume $\sigma = 1$.

For different sampling plans to be considered equivalent, they should have very similar operating characteristic (OC) curves. Due to the nature of OC curves, two sampling plans are considered to be equivalent if two points on their respective OC curves match. In this paper, we use this criterion to produce equivalent sampling plans within our *new* approach and show its equivalence to the traditional attributes and variables sampling plans. In the next sections, we give a description of our *new* approach and show its equivalence to the traditional attributes and variables sampling plans. Based on its equivalence, we compared all of these plans, assessing their performance for other good properties. We found that our *new* methods are actually a bridging gap between the equivalent attributes and variables plans in terms of sample size and robustness.

2. Description and Parameters of the New Method

By and large, inspection-by-variables methods are dependent on the mean, $\bar{x}$, of the sample, $x_1, x_2, \ldots, x_n$, from a normal distribution. In the presence of extreme sample values, the sample mean is known to be highly unstable as an estimator of the lot mean. Consequently, this approach lacks robustness when the normality assumption is violated. To lessen the impact of these setbacks, we have introduced a transformation similar to the Winsorization approach in the *new* method. This transformation designated $Q(x)$ and is a quality function that measures the quality of each sample observation, x_i, such that $Q(x)$ is an increasing function of x, defined as follows:

$$Q(\text{x}) = \begin{cases} 0 \, for \; x \leq A \\ (x-A)(A-B) \, for \; A < x < B \\ 1 \, for \; x \geq B \end{cases} \quad (1)$$

where A and B are normal distribution cut-off points such that $A < B$, giving for each given μ a proportion of totally nonconforming quality of P_0 in the lower tail and a proportion of totally conforming quality of P_1 in the upper tail. In the actual tabulations, and without any real loss of generality, the parameter A is set to 0. The displacement of μ will then determine P_0 and P_1. Though the context is different the ideas here are in a way similar to the three-parameter (n, t_1, t_2) plan of Kumar and Ramyamol [7] whereby for sample size n a lot is rejected of the time between successive failures (Y_t) is < t_1, and accepted if $Y_t \geq t_2$. There is also the idea of a three-step solution procedure endorsed by Qin, Cudney and Hamzic [10] which effectively reduces the solution time for large size problems.

This simple but powerful definition of quality can be used to develop a statistic,$\overline{Q_n}(x)$, i.e., the mean of quality function Q(x), which is then used in developing the *new* methods. It is obvious that Q(x) and hence $\overline{Q_n}(x)$ have a mixture of discrete and continuous parts as is evident by the U-shaped nature of Q(x). As $\overline{Q_n}(x)$ is an n-convolution of Q(x), we expect the cumulative sampling distribution for $\overline{Q_n}(x)$ to be a mixture of continuous parts and discrete jumps.

Based on the works of Cleroux and McConalogue [3] and McConalogue [8, 9], a computational numerical distribution of $\overline{Q_n}(x)$ was evaluated using the concept of a numerical n-convolution, continuous spline-fits and discrete binomial jumps. Such distributions were tabulated as percentage points at given percentiles, *T,* for given sample sizes and P_0 and P_1 values. Table 1 below gives an example of such tables. We used these cumulative probability distributions and their given parameters and test criteria to find the decision rules of our new method based on equivalence. One restriction imposed is that at the discrete jumps, test criteria may not exist, a fact very familiar in all discrete distributions. In these cases, we will not have a plan as experienced in inspection-by-attributes methods.

These distributions and test criteria are used to set up and establish the decision rules for our *new* method for each set of the parameters (*n, T,* P_0, P_1) which can be translated as (*n, μ, A, B*).

With the decision rules established for the *new* method based on given OC curves, we establish equivalences to the traditional variables and attributes plans, and examples of the new decision rules and plans will be shown as we build equivalence among the three approaches.

Table 1 Distribution of the r. v. $\overline{Q_n}(x)$ for given sample size n, P1 and normal mean μ (i.e., P0)

For Sample size n = 8 μ = 1.30 {i.e., P0 = 0.09680}

Table 1. *Distribution of the r. v.* $\overline{Q_n}(x)$ *for given sample size n, P_1 and normal mean μ (i.e. P_0).*

***n* = 8**	**μ=1.30 {i.e. P_0 = 0.09680}**							
QMIN:	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SCALE:	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
P_1:	0.1151	0.1775	0.2478	0.3239	0.4032	0.4825	0.5586	0.6289

P[$\overline{Q_n}(x)$ < T]	**TEST CRITERION, T**							
0.04	0.5241	*NE*	0.4702	0.4371	0.4055	0.4000	0.3456	0.3143
0.05	0.5769	0.5413	0.5101	0.4791	0.4456	0.4135	0.3826	0.3535
0.06	0.6126	0.5706	0.5369	0.5042	0.4712	0.4381	0.4062	0.3761
0.04	*NE*	0.5931	0.5573	0.5237	0.4905	0.4566	0.4240	0.3931
0.05	0.7500	0.6117	0.5740	0.5396	0.5055	0.4717	0.4386	0.4070
0.06	0.6419	*NE*	0.5882	0.5531	0.5187	0.4845	0.4509	0.4189
0.49	*NE*	0.8379	0.8020	0.7671	0.7329	0.6953	0.6574	0.6196
0.50	*NE*	0.8415	0.8054	0.7705	0.7362	0.6986	0.6607	0.6228
0.51	*NE*	0.8451	0.8088	0.7739	0.7395	0.7019	0.6639	0.6260
0.89	*NE*	0.9802	0.9465	0.9141	0.2802	0.8481	0.8113	0.7731
0.90	*NE*	0.9860	0.9525	0.9202	0.8866	0.8542	0.8175	0.7795
0.91	*NE*	0.9919	0.9586	0.9266	0.8933	0.8606	0.8242	0.7862
0.94	*NE*	*NE*	0.9789	0.9484	0.9164	0.8820	0.8476	0.8102
0.95	*NE*	*NE*	0.9865	0.9568	0.9255	0.8917	0.8571	0.8201
0.96	*NE*	*NE*	0.9947	0.9661	0.9357	0.9027	0.8681	0.8315

Notes about the table:

1) This table shows the percentage probabilities of $\overline{Q_n}(x)$ on the first column while the entries in the other columns give the corresponding i^{th} percentile, *T*, for each *B* value (for *B* = 0.5, 0.7, 0.9,…, 1.9 which reflect the tail spike probability P_1).
2) Here, we need to compute $\overline{Q_n}(x)$ value for the i^{th} cell using *i*, *Qmin*, and *SCALE*. For *i* = 49, $\overline{Q_n}(x)$=Qmin + *i*(scale) = 0.0 + 49(0.01) = 0.49.
3) For example, if *n*=8, *μ*=1.3, (i.e. P_0 = 0.0968), *B*=1.7 (i.e. P_1 = 0.5586), the 3rd cell (*i* = 3), $\overline{Q_n}(x) = Qmin + 3(0.01) = 0.03$ has a cumulative probability of 0.4062. This means that *T*=0.4062 is the 3rd percentile of $\overline{Q_n}(x)$ for *n* = 8.
4) The entry "*NE*" in the table indicates that the value of *T* is non-existent as it corresponds to a jump in the probability distributions.

3. Equivalence Procedures and the New Method's Decision Rules

The concept of equivalence based on OC curves is extensively covered in the literature of quality control and acceptance sampling. Basically there are two definitions of equivalence. One is attributable to Hamaker and Von Strick [6] and the other is due to Bravo and Wetherill [2]. Hamaker and Von Strick stated that OC curves of different plans are equivalent when they share some indifference point of 50% acceptance, say p_{50}, and have the same relative slope h at that point; where

$$h = -\left(\delta \ln P_a / \delta \ln p\right) |p_{50} = -2\partial P_a / \partial p \, |p_{50} \quad (2)$$

Clearly, this is not very practical as it at least assumes an analytically defined continuous function for (p, P_a) or that the relative slope exists, two facts that are not defined for plans that are based on discrete distributions.

Bravo and Wetherill [2] suggested matching at the two points corresponding to the acceptance quality limit (AQL) point and the point of indifference at P_a = 50%. Since OC curves are known to be nicely behaving, these two definitions are basically the same but the latter is more practical for us to adopt.

In the *new* method, the values (n, T, P_0, B) are used to fix the two points on the OC curve, thus determining the decision rule and criterion for these plans. Then, we look up the equivalent decision rule and criterion for each of the traditional plans matching these two OC points. In other words, for given $n = n^*$ and $B = B^*$, the cut-off point $t = t^*$ of the distribution of $\overline{Q_n}(x)$, i.e., the average quality, is determined such that an inspected lot with $p_0(1)$ proportion nonconforming (i.e., p associated with AQL) will have a probability of acceptance P_a = 1-α (for a pre-specified α) while a lot with $p_0(2)$ proportion nonconforming is accepted with probability $P_a = \beta$ where $\beta <$ 1-α and $p_0(1) < p_0(2)$. The OC curve so discussed now passes through the two points: (AQL, 1-α) and ($p_0(2)$, β). Noting how our method's distribution is constructed, AQL can be translated as μ(1-α) and $p_0(2)$ as $\mu(\beta)$, and we have now fixed (n^*, t^*, μ(1-α), $\mu(\beta)$, B^*) as our *new* plan corresponding to a quality protection of a specific OC level. Likewise, we can fix any and many OC levels of quality protection. In the next paragraph, we explain how our plans work.

The decision parameters above are used as in the following typical basic rules:

i. Decide the specific values of n^* and B^*.
ii. Determine t^* value by choice of the protection OC level of (AQL, 1-α) and ($p_0(2)$, β) for given α and β.
iii. Take a random sample x_1, $x2,\ldots,$ xn drawn independently from an inspected lot, believed to be normally distributed.
iv. Evaluate $Q(x_i)$ for each x_i, and compute $\overline{Q_n}(x)$, the sample mean.
v. If $\overline{Q_n}(x) > t^*$, accept the lot. Otherwise reject the lot as having poor quality.

Like this plan, Al-Omari [1] suggested the idea of truncation at a predetermined level, yet his was limited to the Inverse Rayleigh Distribution.

4. Determination of the Traditional Equivalents to the New Plans

The OC two-points principles discussed earlier and used for building the *new* plans are also applied to seek traditional plans that match ours. In each case, we simply pick the given OC points of (AQL, 1-α) and ($p_0(2)$, β) and seek the binomial distribution and normal distribution sampling plan parameters that give that some OC levels.

For the case of attributes, we need the sample size n' and the acceptance number c. The relevant distribution model to use here is the binomial distribution with parameters n' and p such that if $P_a(p)$ is the probability of acceptance for any given proportion p of nonconforming quality and acceptable number of nonconforming unites c, i.e.:

$$P_a(\mathrm{p}) = P_a(\mathrm{p}) \sum_{i=1}^{c} \binom{n}{i} p^i (1-p)^{n-i} \quad (3)$$

We need to solve c for pre-specified (AQL, 1-α) and ($p_0(2)$, β). But sometimes, because of the discrete nature of the model in (1), we can conveniently use the approximating system of the following inequalities:

$$P_a(\mathrm{AQL}) \geq 1-\alpha \quad (4)$$

$$P_a(p_0(2)) \leq \beta \quad (5)$$

Noting that the Poisson distribution Po(c, $n'p$) is a good approximation of (3) while the Poisson distribution in turn is best approximated by the Chi-square (χ^2) distribution with 2(c+1) degrees of freedom, with cut-off point at $2n'p$ and variance 2(c+1), the above system of inequalities above can be replaced with:

$$\Pr(\chi^2 > 2n'p | 2(c+1) \text{ degrees of freedom}) \geq 1-\alpha \quad (6)$$

$$\Pr(\chi^2 > 2n'p | 2(c+1) \text{ degrees of freedom}) \leq \beta \quad (7)$$

A method by Hald [4, 5] was invaluable in solving for n' and c is used, with the solutions defined, thus determining the equivalent traditional two-class attribute plan.

For the case of traditional sampling-by-variables equivalents, there are the two common approaches, depending on whether or not the lot (or process) standard deviation σ is known. The σ-method leads to use of the standard normal distribution, and the s-method (where σ is unknown and s is the sample standard deviation is used to estimate σ) leads to use of a non-central Student t-distribution. We know for a fact that sample sizes for the σ-method are always smaller than for the s-method, so it made sense to create equivalent s-method plans only for comparison.

The σ-method plan requires the parameters n_σ (the sample size) and k_σ (the critical cut-off value for acceptance or rejection) such that:

$$\left[k_\sigma - \mu_{(1-\alpha)}\right] * \sqrt{n_\sigma} = \sigma \Phi^{-1}(1-\alpha) \quad (8)$$

and

$$\left[k_\sigma - \mu_{(\beta)}\right] * \sqrt{n_\sigma} = \sigma \Phi^{-1}(\beta) \quad (9)$$

will give P_a(AQL) = 1-α and $P_a(p_0(2)) = \beta$ corresponding to the two points on OC curve for equivalence purposes discussed earlier. The above equations solve for n_σ and k_σ for given α and β giving:

$$n_\sigma = \frac{(Z_{1-\alpha} - Z_\beta)^2 \sigma^2}{(\mu_{1-\alpha} - \mu_\beta)^2} \quad (10)$$

and

$$k_\sigma = \frac{(Z_{1-\alpha} * \mu_\beta - Z_\beta * \mu_{1-\alpha})^2}{(Z_{1-\alpha} - Z_\beta)} \quad (11)$$

where $Z_{1-\alpha}$ and Z_β are the standard normal z-values or percentiles. These values of n_σ and k_σ are the required parameters for establishing the equivalent σ-method plan. Integral-valued n_σ could be a bit problematic; however for approximations the general rounding principles will be sufficient for our comparison purposes.

With the OC equivalence principle, Hald [4] suggested an iterative approximating procedure for finding the s-method decision rules and the parameters of sample size n_s and the test criterion k_s. Hald's method was advanced by Wetherill and Kollerstrom [11] and was checked to be adequate by Bravo and Wetherill [2]. Using the same method, our results compare well with Bravo and Wetherill's published results. We have found that the iterative process converges relatively quickly for the solution values of n_s and k_s, giving solutions for all the cases of equivalent variables s-method plans.

Given the σ-method plan parameters, n_σ and k_σ, the plan accepts the batch if the sample mean, $\bar{x}_\sigma \le A - \sigma k_\sigma$. Now for s-method plans, σ is unknown and we need to match the two random variables, $\bar{x}_\sigma + \sigma k_\sigma$ and $\bar{x}_s + s k_s$, such that they have the same mean and variance. As a result, we get the following approximate solutions:

$$k_\sigma = k_s\,(4n_s - 5)/(4n_\sigma - 4) \quad (12)$$

$$n_\sigma = n_s\,(1 + k_s^2/2) \quad (13)$$

Solving for n_s and k_s is done via numerical iteration, noting from (2) above, that n_s would be greater than n_σ by an incremental factor of $1 + k_s^2/2$ and an initial value of $k_s = k_\sigma$, since the factor $(4n_s - 5)/(4n_\sigma - 4)$ is almost 1. So with $k_\sigma = k_s$ in (1), we can get an approximation of n_s which is applied to (2), giving a final approximation for n_s. Again, this is the procedure for obtaining an equivalent s-method plan.

With all the equivalent plans established, we are able to assess the sample size comparisons and the performance of the *new* method's plans.

5. Sample Size Performance

The sample sizes for all the equivalent plans for different AQL levels were computed and compared using the ratio of sample size of any of the traditional plans to the sample size of their equivalent *new* plan. The results are reflected in Figure 1 where the horizontal broken line indicates the case of the *new* plan. The two bottom curves are the cases of the traditional variables sampling plans. The lowest one refers to the σ-method plan while the other close one refers to the s-method plan. Obviously, the top curve is the attribute plan as it has the highest sample sizes for all equivalent plans. For each B, there is a separate figure for different sample sizes of the *new* method.

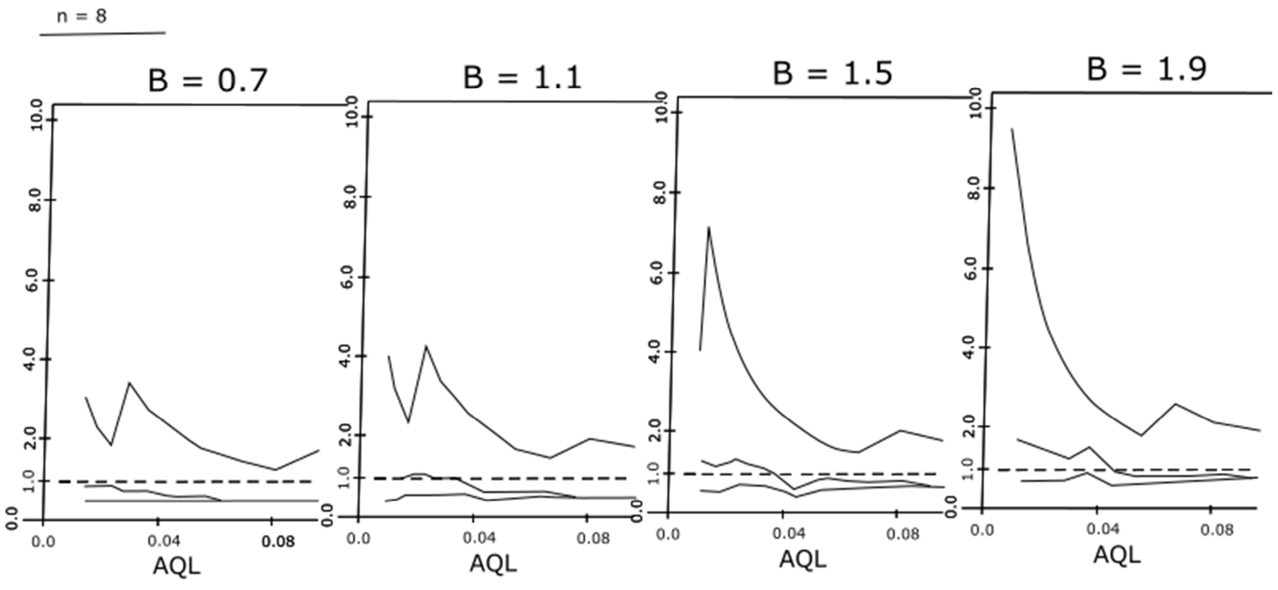

Figure 1. *Ratio of sample size of the traditional plans to that of their equivalent new plans for given n = 8 and AQL levels, at different B values of 0.7, 1.1, 1.5, and 1.9.*

Notes on the figure:

1. The y-axis represents the ratio of traditional plan sample size to the Ramp (new) sample size
2. The dotted (broken) line indicates the sample size ratio n_j/n of the traditional plans' n_j to the ramp n is unity.
3. The upper-most curve gives the sample size ratio of the attributes case to that of the *new* equivalent plan. The second is for the s-method plan while the lowest curve is that of the σ-method plan sample size ratio.
4. AQL is the Acceptance Quality Limit reflecting the quality level that is the worst tolerable process average when a continuing series of lots is submitted for acceptance sampling.
5. The new Ramp plans gets better sample size as B gets larger.

As a function of AQL, each figure shows quite clearly that the lower the AQL is, the larger the gap is between the attributes curve and any other plan's curve. It is beyond any doubt that there is a large savings in sample size under the *new* method as compared to the attributes sampling case. All of the curves indicate the *new* method can act as a transition between the attributes and variables sampling methods whenever sample size is an issue.

Figure 1 shows how the *new* plans fair on sample size performance for any *B*. For low *B* values, the *new* plan is closer to the attributes plan. While for higher *B* values, the attributes sample size required is quite high. In all cases, the *new* plans compare very well with all traditional plans, and interestingly enough, in some cases, they are better than the *s*-method plans as indicated by the graphs for $B = 1.5$ and 1.7 when the AQL is less than 0.04. However, as we expected, the *new* plans can never be better than the σ-method variables plans (whose *n* ratio to that of the *new* plans is shown by the lowest graph in each figure).

6. The Bridging Role of Parameter *B*

The parameter *B* is a very important one in really bridging the gap between the variables and attributes methods. The smaller the value of *B* is, the closer our plan resembles the attributes plan. Since *B* approaches *A*, *Q(x)* will approach a Bernoulli variable. In that case, we would expect a larger sample size. On the other hand, as *B* is increased, a greater portion of *Q(x)* becomes continuous. Thus, the plan becomes closer to a variables plan and the sample size will generally tend to be smaller.

7. Discussion

Having found that the *new* method faired well with encouraging results, with some forthcoming strands of research. First, some statistical simulations using various non-normal distributions could be conducted to see how the four different methods will compare. In other words, further robustness studies should be conducted to fully ascertain the degree of robustness of the *new* attri-var method. Secondly, more research is needed to establish more diversified development and design of *new* attri-var plans, based on the principles laid out by this paper to introduce more equivalents to the current traditional ones. Another strand of research is to consider non-normality robustness along the lines suggested by Zimmer and Burr [12].

8. Conclusion

This paper recommended, developed and also set up some new attri-var plans that are equivalent to some of the existing attribute and variables plans that compared well with existing schemes. Not only that, they bridged the gap between variables and attributes plans. Thus they removed the variable versus attribute dichotomy that exists in the current schemes through use of the parameter B. The smaller B (as it approaches 0) the new Ramp becomes an attribute plan, while larger values of B lead to variables plans. Sample size savings especially for the attribute plans was prominent in the new attri-var plans. Moreover, as B increases the new Ramp plan becomes more prominently better in sample size savings. Both the sample size levels and robustness of the new bridge Ramp method lie in between the two classical by-variables and by-attributes sampling plans

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