Abstract
In this work, we introduce generalized types of multiplicative metric spaces so-called left and right dislocated quasi multiplicative metric spaces (abbrev. l d q-multiplicative metric space and r d q-multiplicative metric space respectively). We state and prove two fixed point theorems in these spaces.

1. Introduction
In 1906, Frechet introduced the notion of metric space. We can see recent generalizations of metric space and concern topological structures in literature. (See, e.g., [1-5]).

In 2001, the concept of dislocated metric spaces was introduced by Hitzler [5]. He generalized metric spaces in the sense that self-distance of points need not to be zero necessarily. Also, he established Banach’s contraction principle in these spaces.

In 2005, Zeyada et al. [8] introduced the notion of dislocated quasi metric spaces and generalized the result of Hitzler’s result to dislocated quasi metric spaces. On the other hand, in 2013, Ahmed et al. [1] introduced two generalized types of dislocated metric spaces and proved some fixed point theorems in these spaces. Recently Ahmed et al. [2] introduce the concept of dislocated quasi multiplicative metric spaces. Also, Ahmed et al. [3] established the fixed point results for two rational contraction self-mappings on dislocated quasi multiplicative metric spaces.

Definition 1.1. [7] Let X be a nonempty set. A distance on X is a function d: X × X → [0, ∞). A pair (X, d) is called a distance space. If d satisfies the following conditions for all x, y, z ∈ X:
(1) If d(x, y) = 0, then x = y;
(2) d(x, y) = d(y, x);
(3) d(x, y) ≤ d(x, z) + d(z, y),
then it is called a dislocated metric on X (or simply d-metric). It is obvious that if d satisfies (1) – (3) and d(x, x) = 0 for all x ∈ X, then d is a metric on X. It’s clear that a metric is a d-metric and the converse is not true (see, e.g., [5]).

Definition 1.2. [1] Let X be a nonempty set and a d: X × X → [0, ∞) be a function satisfying the conditions for all x, y, z ∈ X:
(1) If d(x, y) = 0, then x = y;
(2) d(x, y) ≤ d(y, x);
(3) d(x, y) ≤ d(x, z) + d(z, y),
for all x, y, z ∈ X, then it is called a dislocated metric on X (or simply d-metric). It is obvious that if d satisfies (1) – (3) and d(x, x) = 0 for all x ∈ X, then d is a metric on X. It’s clear that a metric is a d-metric and the converse is not true (see, e.g., [5]).

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then it is called a dislocated quasi metric on X (d q-metric).
On the third hand, Bashirov et al. [4] defined the concept of multiplicative metric
Definition 1.3. Let $X$ be a nonempty set. A function $d: X \times X \rightarrow R$, $R := \{0, 1\}$, is called a multiplicative metric if $d$ satisfies the following conditions for all $x, y, z \in X$:

1. $d(x, y) \geq 1$;
2. $d(x, y) = 1$ if and only if $x = y$;
3. $d(x, y) = d(y, x)$;
4. $d(x, y) \leq d(x, z) \cdot d(z, y)$.

In this case, $(X, d)$ is called a multiplicative metric space.

Example 1.1. [7] Let $R^+$ be the collection of all $n$-tuples of positive real numbers.

Let $d: R^n \times R^n \rightarrow R$ be defined as follows

$$d((x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n)) = \sqrt{|a|},$$

where $x = (x_1, x_2, \ldots, x_n) \in R^n$, $y = (y_1, y_2, \ldots, y_n) \in R^n$ and $|\cdot|: R^n \rightarrow R$ is defined as follows

$$|a| = \begin{cases} a: if \ a \geq 1 \\ a: if \ a < 1 \end{cases}$$

Then it is obvious that the all conditions of multiplicative metric are satisfied.

After that Özavsar et al. [6] were the first researchers who discussed multiplicative metric mapping by giving some topological properties of the relevant multiplicative metric space. Furthermore, Özavsar [6] et al. mentioned the concept of multiplicative contraction.

Definition 1.4. [6] Let $(X, d)$ be a multiplicative metric space. A mapping $f: X \rightarrow X$ is called multiplicative contraction if there exists a real constant $\lambda \in (0, 1)$ such that

$$d(f(x), f(y)) \leq \lambda d(x, y) \quad \forall x, y \in X.$$
point. Secondly, assume \( x_n 0 \neq x_n 0 + 1 \), \( \forall n \). Then we have from (3.1)

\[
d(x_n, x_{n+1}) = d(F x_{n-1}, F x_n)
\]

\[
\leq \max \{d(x_{n-1}, x_n), d(x_{n-1}, F x_{n-1}), d(x_n, F x_n), d(x_{n-1}, F x_n), d(x_n, F x_{n-1})\}
\]

\[
\leq \max \{d(x_{n-1}, x_n), d(x_{n-1}, x_n), d(x_{n-1}, x_n), d(x_{n-1}, x_n), d(x_n, x_{n-1})\}
\]

\[
= d(x_n, x_{n+1})^\lambda, \quad d(x_n, x_{n+1})^\lambda
\]

Thus

\[
d(x_n, x_{n+1}) \leq d(x_n, x_{n}^\lambda) \cdot d(x_{n-1}, x_n)^\lambda, \quad \forall n, N.
\]

Similarly

\[
d(x_{n+1}, x_n + 2) \leq d(x_{n+1}, x_n + 2) \cdot d(x_n, x_{n+1})^\lambda
\]

Therefor

\[
d(x_n, x_{n+1}) \leq d(x_n, x_{n}^\lambda) \cdot d(x_{n-1}, x_n)^\lambda
\]

\[
\leq d(x_n, x_{n-2})^\lambda, \quad d(x_n, x_{n-3})^\lambda
\]

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\]

Now we claim that the sequence \( \{x_n\} \) satisfies the multiplicative Cauchy convergence, then let \( m, n, m \in N \) be such that \( m > n, n \) then by multiplicative triangular inequality, we have

\[
d(x_{n+m}, x_n) \leq d(x_{n+m-1}, x_{n+m}) \cdot d(x_{n+m-2}, x_{n+m-1}) \cdot \ldots \cdot d(x_n, x_{n+1})
\]

\[
\leq [d(x_1, x_n)^{i_{a-1}} \cdot d(x_{n+1}, x_n)^{i_{a-1}}] \cdot [d(x_1, x_n)^{i_{a-1}} \cdot d(x_{n+1}, x_n)^{i_{a-1}}]
\]

Letting \( n \to \infty \), in (2.6), we have \( d(x_n, x_{n+m}) \to 1 \). Hence the sequence \( \{x_n\} \) is satisfies multi- plicative Cauchy sequence.

By the completeness of \( X \), there exist \( x \in X \) such that \( x_n \to x \). From lemma (2.1), we obtain \( x_{n+1} = F x_n \). Since \( F \) is continuous, then \( x_{n+1} = F x_n \to F x \). Since the multiplicative limit is unique, thus \( x = F x \).

### 3. Right Dislocated Quasi-Multiplicative Metric Spaces

**Definition 3.1.** Let \( X \) be a nonempty set. A function \( d: X \times X \to R \) is called right dislocated quasi-multiplicative metric (abbrv \( r d q \)-multiplicative metric) if it satisfies the following conditions for all \( x, y, z \in X \)

1. \( d(x, y) \geq 1 \);
2. If \( d(x, y) = 1 \), then \( x = y \);
3. \( d(x, y) \leq d(x, z) \cdot d(y, z) \).

Here \( R := [0, \infty) \) denotes the set. Then the \( (X, d) \) is called left dislocated quasi-multiplicative metric space.

**Example 3.1.** Let \( X = [0, \infty) \). Define a mapping \( d: X \times X \to R \), by

\[
d(x, y) = a^{\frac{1}{2}} \cdot d(y, x) = a^{\frac{1}{2}} \cdot d(y, x),
\]

where \( a > 1 \) is any fixed number. Then for each \( a > 1 \), \( d \) is satisfy \( (X, d) \) is called right dislocated quasi-multiplicative metric space.

Now, we state the following definition.

**Definition 3.2.** A sequence \( \{x_n\} \) in \( r d q \)-multiplicative metric space.

\[
\lim_{n \to \infty} d(x_n, x) = \lim_{n \to \infty} d(x_n, x_n) = 1
\]

In this case \( x \) is called a r d q-limit of \( \{x_n\} \).

**Definition 3.3.** A sequence \( \{x_n\} \) in \( r d q \)-multiplicative metric space \( (X, d) \) is called Cauchy if for all \( \epsilon > 0 \), there exist \( n_0 \in N \) such that for all \( n, m \geq n_0 \), we have

\[
d(x_n, x_m) < \epsilon , \quad d(x_n, x_m) < \epsilon.
\]

**Definition 3.4.** A \( r d q \)-multiplicative metric space \( (X, d) \) is called complete if every Cauchy sequence in it is \( d q\)-convergent.

**Lemma 3.1.** Let \( (X, d) \) be a \( r d q \)-multiplicative metric space, every subsequence of any convergent sequence is convergent.

**Lemma 3.2.** Let \( (X, d) \) be a \( r d q \)-multiplicative metric space. If a sequence \( \{x_n\} \) in \( r d q \)-convergent, then the \( r d q \)-limit point is unique.

**Theorem 3.1.** Let \( (X, d) \) be complete \( r d q \)-multiplicative metric space and \( F: X \to X \) be a map such that

\[
d(F x, F y) \leq \max \{d(x, y), d(x, F x), d(y, F y), d(x, F y), d(y, F x)\},
\]

\[
\quad d(y, F x) \lambda.
\]

for all \( x, y \in X \), where \( \lambda \in (0, \frac{1}{2}) \), and \( F \) is continuous. Then \( F \) has unique fixed point.

**Corollary 3.1.** Let \( (X, d) \) be a \( r d q \)-multiplicative metric space and let \( F: X \to X \) be a multi- plicative contraction. If \( (X, d) \) is complete, then \( F \) has a unique fixed point.

**Remark 3.1.** Corollary (3.1) generalizes Theorem (1.1).

### References


