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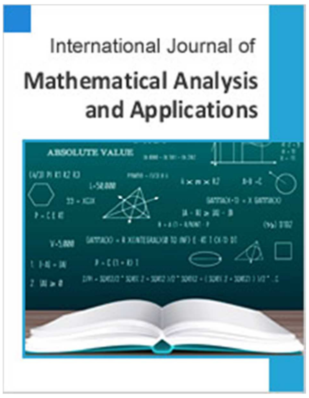

Fixed Point Theorems in Two Intuitionistic Generalized Fuzzy Metric Spaces

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Abstract

Fisher [4] proved a related fixed point theorem in two metric spaces. We proved fixed point theorems in two complete intuitionistic generalized fuzzy metric space for contractive type mappings and non-expansive mapping by generalizing the results of Veerapandi et al [12] on fuzzy metric spaces.

1. Introduction

As a generalization of fuzzy sets, Atanassov [1] introduced and studied the concept of intuitionistic fuzzy sets. Park [10] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalized of fuzzy metric spaces, George and Veeramani [5] showed that every metric induced an intuitionistic fuzzy metric, every fuzzy metric space in an intuitionistic fuzzy metric space. Fisher [4] proved a related fixed point theorem in two metric spaces.

In 2006, Sedghi and Shobe [11] defined $\mathcal{M}$- fuzzy metric spaces and proved a common fixed point theorem for four weakly compatible mappings in this space. In 2009, Mehra and Gugnani [9] defined the notion of an intuitionistic $\mathcal{M}$ – fuzzy metric space due to Sedghi and Shobe and proved a common fixed point theorem for six mappings for property (E) in this newly defined space. In this paper, fixed point theorems are proved in two complete intuitionistic generalized fuzzy metric space for contractive type mappings and non-expansive mapping by generalizing the result of Veerapandi et al [12] on fuzzy metric spaces.

Definition 1.1:

A binary operation $*$: $[0, 1] \times [0, 1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(i) is associative and commutative,
(ii) is continuous,
(iii) $a * 1 = a$, for all $a \in [0, 1]$,
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 1.2:

A binary operation $\lozenge$: $[0, 1] \times [0, 1] \to [0, 1]$ is said to be a continuous t- conorm if it satisfies the following conditions:

i) ◊ is associative and commutative.
ii) ◊ is continuous.
iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$.
iv) $a\lozenge b \leq c\lozenge d$ whenever $a \leq c$ and $b \leq d$, for each a, b, c, d ∈ [0, 1].

Definition 1.3:

A 5-tuple $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$ is said to be an intuitionistic generalized fuzzy metric space (shortly IGFM-Space), if X is an arbitrary set,* is a continuous t-norm, ◊ is a continuous t-conorm and $\mathcal{M}, \mathcal{N}$ are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions:

For all x, y, z, a ∈ X and s, t > 0

(IFM-1): $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) = 1$,

(IFM-2): $\mathcal{M}(x, y, z, t) > 0$,

(IFM-3): $\mathcal{M}(x, y, z, t) = 1$ if and only if $x = y = z$,

(IFM-4): $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function,

(IFM-5): $\mathcal{M}(x, y, z, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$,

(IFM-6): $\mathcal{M}(x, y, z,): (0, \infty) \to [0, 1]$ is left continuous,

(IFM-7): $\mathcal{N}(x, y, z, t) > 0$,

(IFM-8): $\mathcal{N}(x, y, z, t) = 0$ if and only if $x = y$,

(IFM-9): $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$, where p is a permutation function,

(IFM-10): $\mathcal{N}(x, y, z, a, t) \lozenge \mathcal{N}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$,

(IFM-11): $\mathcal{N}(x, y, z,): (0, \infty) \to [0, 1]$ is continuous,

Then, $(\mathcal{M}, \mathcal{N})$ is called an intuitionistic generalized fuzzy metric on X.

Definition 1.4:

Let $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$ be an intuitionistic generalized fuzzy metric space. Then

a) A sequence {Xian} in Xis said to be a Cauchy sequence if for all t > 0 and p > 0 $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_n, x_n, t)$ and $\lim_{n\to\infty} \mathcal{N}(x_{n+p}, x_n, x_n, t) = 0$.
b) A sequence $\{x_n\}$ in X is said to be convergent to a point x ∈X, if for all t > 0, $\lim_{n\to\infty} \mathcal{M}(x_n, x_n, x_n, t) = 1$ and for all t > 0 and $\lim_{n\to\infty} \mathcal{N}(x_n, x_n, x_n, t) = 0$.
c) An intuitionistic generalized fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$ is said to be complete if and only if every Cauchy sequence is convergent.

Lemma 1.5:

Let $\{x_n\}$ be a sequence in a generalized intuitionistic fuzzy Metri c space $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$, if there exists a constant k ∈ (0, 1) such that

$$\mathcal{M}(x_n, x_{n+1}, x_{n+1}, kt) \geq \mathcal{M}(x_{n-1}, x_n, x_n, t)$$

and $\mathcal{N}(x_n, x_{n+1}, x_{n+1}, kt) \leq \mathcal{N}(x_{n-1}, x_n, x_n, t)$ for all t > 0.

Then $\{x_n\}$ is Cauchy sequence in X.

2. Main Results

Theorem 2.1:

Let $(X, \mathcal{M}_1, \mathcal{N}_1, *, \lozenge)$ and $(Y, \mathcal{M}_2, \mathcal{N}_2, *, \lozenge)$ be two complete intuitionistic generalized fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying the following conditions.

$\mathcal{M}_2(Tx, TSy, TSy, t) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t),$
$\mathcal{M}_1(Sy, STx, STx, t), \mathcal{M}_2(y, Tx, Tx, t) * \mathcal{M}_2(y, TSy, TSy, t),$
$\mathcal{M}_1(x, STx, STx, t)\}$

$\mathcal{N}_2(Tx, TSy, TSy, t) \leq \max\{\mathcal{N}_1(x, Sy, Sy, t), \mathcal{N}_1(Sy, STx, STx, t), \mathcal{N}_2(y, Tx, Tx, t) \lozenge \mathcal{N}_2(y, TSy, TSy, t), \mathcal{N}_1(x, STx, STx, t)\}$

$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t) *$
$\mathcal{M}_1(x, STx, STx, t), \mathcal{M}_2(y, TSy, TSy, t), \mathcal{M}_2(y, Tx, Tx, t),$
$\mathcal{M}_2(Tx, TSy, TSy, t)\}$

$\mathcal{N}_1(Sy, STx, STx, qt) \leq \max\{\mathcal{N}_1(x, Sy, Sy, t) \lozenge \mathcal{N}_1(x, STx, STx, t),$
$\mathcal{N}_2(y, TSy, TSy, t), \mathcal{N}_2(y, Tx, Tx, t), \mathcal{N}_2(Tx, TSy, TSy, t)\}$

for all x in X and y in Y where q<1, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Proof:

Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X and $\{y_n\}$ in Y, as follows.

$x_n = (ST)^n x_0$, $y_n = T(x_n\text{-}1)$ for n=1, 2..., we have

$\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) = \mathcal{M}_1((ST)^n x_0, (ST)^{n+1} x_0, (ST)^{n+1} x_0, qt)$
$= \mathcal{M}_1(ST(ST)^{n-1} x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt)$
$= \mathcal{M}_1(STx_{n-1}, STx_n, STx_n, qt) = \mathcal{M}_1(Sy_n, STx_n, STx_n, qt)$
$\geq \min\{\mathcal{M}_1(x_n, Sy_n, Sy_n, t) * \mathcal{M}_1(x_n, STx_n, STx_n, t),$
$\mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_2(y_n, Tx_n, Tx_n, t), \mathcal{M}_2(Tx_n, TSy_n, TSy_n, t)\}$
$= \min\{\mathcal{M}_1(x_n, x_n, x_n, t) * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t),$
$\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_{n+1}, y_{n+1}, y_{n+1}, t)\}$
$= \min\{1 * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t),$
$\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), 1\}$
$\geq \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t)$ and

$\mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, qt) = \mathcal{N}_1((ST)^n x_0, (ST)^{n+1} x_0, (ST)^{n+1} x_0, qt)$
$= \mathcal{N}_1(ST(ST)^{n-1} x_0, ST(ST)^n x_0, ST(ST)^n x_0, qt)$
$= \mathcal{N}_1(STx_{n-1}, STx_n, STx_n, qt) = \mathcal{N}_1(Sy_n, STx_n, STx_n, qt)$
$\leq \max\{\mathcal{N}_1(x_n, Sy_n, Sy_n, t) \lozenge \mathcal{N}_1(x_n, STx_n, STx_n, t),$
$\mathcal{N}_2(y_n, TSy_n, TSy_n, t), \mathcal{N}_2(y_n, Tx_n, Tx_n, t), \mathcal{N}_2(Tx_n, TSy_n, TSy_n, t)\}$
$= \max\{\mathcal{N}_1(x_n, x_n, x_n, t) \lozenge \mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t),$
$\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{N}_2(y_{n+1}, y_{n+1}, y_{n+1}, t)\}$
$= \max\{0 \lozenge \mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, t), \mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t),$
$\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t), 0\}$
$\leq \mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t)$

Also we have, $\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, qt) = \mathcal{M}_2(Tx_{n-1}, Tx_n, Tx_n, qt)$
$= \mathcal{M}_2(Tx_{n-1}, TSy_n, TSy_n, qt)$
$\geq \min\{\mathcal{M}_1(x_{n-1}, Sy_n, Sy_n, t), \mathcal{M}_1(Sy_n, STx_{n-1}, ST x_{n-1}, t),$
$\mathcal{M}_2(y_n, Tx_{n-1}, Tx_{n-1}, t) * \mathcal{M}_2(y_n, TSy_n, TSy_n, t),$
$\mathcal{M}_1(x_{n-1}, STx_{n-1}, STx_{n-1}, t)\}$
$= \min\{\mathcal{M}_1(x_{n-1}, x_n, x_n, t), 1, 1 * \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(x_{n-1}, x_n, x_n, t)\}$
$\geq \mathcal{M}_1(x_{n-1}, x_n, x_n, t)$ and

$\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, qt) = \mathcal{N}_2(Tx_{n-1}, Tx_n, Tx_n, qt) = \mathcal{N}_2(Tx_{n-1}, TSy_n, TSy_n, qt)$
$\leq \max\{\mathcal{N}_1(x_{n-1}, Sy_n, Sy_n, t), \mathcal{N}_1(Sy_n, STx_{n-1}, STx_{n-1}, t),$
$\mathcal{N}_2(y_n, Tx_{n-1}, Tx_{n-1}, t) \lozenge \mathcal{N}_2(y_n, TSy_n, TSy_n, t),$
$\mathcal{N}_1(x_{n-1}, STx_{n-1}, STx_{n-1}, t)\}$

$= \max\{\mathcal{N}_1(x_{n-1}, x_n, x_n, t), 0, 0\Diamond\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{N}_1(x_{n-1}, x_n, x_n, t)\}$

$\leq \mathcal{N}_1 (x_{n-1}, x_n, x_n, t)$.

Now, $\mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, qt) \geq \mathcal{M}_2 (y_n, y_{n+1}, y_{n+1}, t)$

$\geq \mathcal{M}_1 (x_{n-1}, x_n, x_n, t/q)$...

$\geq \mathcal{M}_1 (x_0, x_1, x_1, t/q^{2n-1}) \to 1$ as $n\to\infty$.

$\mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, qt) \leq \mathcal{N}_2 (y_n, y_{n+1}, y_{n+1}, t)$

$\leq \mathcal{N}_1 (x_{n-1}, x_n, x_n, t/q)$...

$\leq \mathcal{N}_1 (x_0, x_1, x_1, t/q^{2n-1}) \to 0$ as $n\to\infty$.

Thus $\{x_n\}$ is a Cauchy sequence in $(X, \mathcal{M}_1, \mathcal{N}_1,*, \Diamond)$. Since $(X, \mathcal{M}_1, \mathcal{N}_1,*, \Diamond)$ is complete, it converges to a point z in X. Similarly, itcan prove that the sequence $\{y_n\}$ is also a Cauchy sequence in $(X, \mathcal{M}_1, \mathcal{N}_1,*, \Diamond)$. Since $(Y, \mathcal{M}_2, \mathcal{N}_2,*, \Diamond)$ and it converges to a point w in Y.

Now, we prove Tz = w, suppose Tz ≠ w. We have

$\mathcal{M}_2(Tz, w, w, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, qt)$

$= \lim_{n\to\infty} \mathcal{M}_2(Tz, TSy_n, TSy_n, qt)$

$\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(z, Sy_n, Sy_n, t), \mathcal{M}_1(Sy_n, STz, STz, t), \mathcal{M}_2(y_n, Tz, Tz, t) * \mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_1(z, STz, STz, t)\}$

$= \lim_{n\to\infty} \min\{\mathcal{M}_1(z, x_n, x_n, t), \mathcal{M}_1(x_n, STz, STz, t)$ $\mathcal{M}_2(y_n, Tz, Tz, t) * \mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_1(z, STz, STz, t)\}$

$\geq \mathcal{M}_1 (z, STz, STz, t)$ and

$\mathcal{N}_2(Tz, w, w, qt) = \lim_{n\to\infty} \mathcal{N}_2(Tz, y_{n+1}, y_{n+1}, qt)$

$= \lim_{n\to\infty} \mathcal{N}_2(Tz, TSy_n, TSy_n, qt)$

$\leq \lim_{n\to\infty} \text{ma } \{\mathcal{N}_1 (z, Sy_n, Sy_n, t), \mathcal{N}_1 (Sy_n, STz, STz, t), \mathcal{N}_2(y_n, Tz, Tz, t)\Diamond\mathcal{N}_2(y_n, TSy_n, TSy_n, t), \mathcal{N}_1(z, STz, STz, t)\}$

$= \lim_{n\to\infty} \max\{\mathcal{N}_1(z, x_n, x_n, t), \mathcal{N}_1(x_n, STz, STz, t),$ $\mathcal{N}_2(y_n, Tz, Tz, t)\Diamond\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{N}_1(z, STz, STz, t)\}$

$\leq \mathcal{N}_1 (z, STz, STz, t)$.

Now, $\mathcal{M}_1(z, STz, STz, qt)$

$= \lim_{n\to\infty} \mathcal{M}_1(x_n, STz, STz, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sy_n, STz, Tz, qt)$

$\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(z, Sy_n, Sy_n, t) * \mathcal{M}_1(z, STz, STz, t),$ $\mathcal{M}_2(y_n, TSy_n, TSy_n, t), \mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_2(Tz, TSy_n, TSy_n, t)\}$

$\lim_{n\to\infty} \min\{\mathcal{M}(z, x_n, x_n, t), * \mathcal{M}_1 (z, STz, STz, t),$ $\mathcal{M}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_n, Tz, Tz, t), \mathcal{M}_2(Tz, y_{n+1}, y_{n+1}, t)\}$

$= \min \{1 * \mathcal{M}_1(z, STz, STz, t), 1, \mathcal{M}_2(w, Tz, Tz, t), \mathcal{M}_2(Tz, w, w, t)\}$

$\geq \mathcal{M}_2 (Tz, w, w, t)$.

Hence, $\mathcal{M}_2(Tz, w, w, qt) \geq \mathcal{M}_1(z, STz, STz, t) \geq \mathcal{M}_2(Tz, w, w, t/q)$.

$\mathcal{N}_1(z, STz, STz, qt)$

$= \lim_{n\to\infty} \mathcal{N}_1(x_n, STz, STz, qt) = \lim_{n\to\infty} \mathcal{N}_1(Sy_n, STz, STz, qt)$

$\leq \lim_{n\to\infty} \max\{\mathcal{N}_1(z, Sy_n, Sy_n, t)\Diamond\mathcal{N}_1(z, STz, STz, t),$ $\mathcal{N}_2(y_n, TSy_n, TSy_n, t), \mathcal{N}_2(y_n, Tz, Tz, t), \mathcal{N}_2(Tz, TSy_n, TSy_n, t)\}$

$= \lim_{n\to\infty} \max \mathcal{N}(z, x_n, x_n, t)\Diamond\mathcal{N}_1 (z, STz, STz, t),$ $\mathcal{N}_2(y_n, y_{n+1}, y_{n+1}, t), \mathcal{N}_2(y_n, Tz, Tz, t), \mathcal{N}_2(Tz, y_{n+1}, y_{n+1}, t) \}$

$= \min \{0\Diamond\mathcal{N}_1(z, STz, STz, t), 0, \mathcal{N}_2(w, Tz, Tz, t), \mathcal{N}_2(Tz, w, w, t)\}$

$\leq \mathcal{N}_2 (Tz, w, w, t)$.

Hence $\mathcal{N}_2(Tz, w, w, qt) \leq \mathcal{N}_1(z, STz, STz, t) \leq \mathcal{N}_2(Tz, w, w, t/q)$

Thus Tz = w. Now we prove Sw =z. Suppose Sw ≠ z.

We have $\mathcal{M}_1(Sw, z, z, qt)$

$= \lim_{n\to\infty} \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{M}_1(Sw, STx_n, STx_n, qt)$

$\geq \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t) * \mathcal{M}_1(x_n, STx_n, STx_n, t),$ $\mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_2(w, Tx_n, Tx_n, t), \mathcal{M}_2(Tx_n, TSw, TSw, t)\}$

$= \lim_{n\to\infty} \min\{\mathcal{M}_1(x_n, Sw, Sw, t) * \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t),$ $\mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_2(w, y_{n+1}, y_{n+1}, t), \mathcal{M}_2(y_{n+1}, w, w, t)\}$

$\geq \mathcal{M}_2 (w, TSw, TSw, t)$ and

$\mathcal{N}_1(Sw, z, z, qt)$

$= \lim_{n\to\infty} \mathcal{N}_1(Sw, x_{n+1}, x_{n+1}, qt) = \lim_{n\to\infty} \mathcal{N}_1(Sw, STx_n, STx_n, qt)$

$\leq \lim_{n\to\infty} \max\{\mathcal{N}_1(x_n, Sw, Sw, t)\Diamond\mathcal{N}_1(x_n, STx_n, STx_n, t),$ $\mathcal{N}_2(w, TSw, TSw, t), \mathcal{N}_2(w, Tx_n, Tx_n, t), \mathcal{N}_2(Tx_n, TSw, TSw, t)\}$

$= \lim_{n\to\infty} \max\{\mathcal{N}_1(x_n, Sw, Sw, t)\Diamond\mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, t),$ $\mathcal{N}_2(w, TSw, TSw, t), \mathcal{N}_2(w, y_{n+1}, y_{n+1}, t), \mathcal{N}_2(y_{n+1}, w, w, t)\}$

$\leq \mathcal{N}_2 (w, TSw, TSw, t)$

Now, $\mathcal{M}_2(w, TSw, TSw, qt)$ $= \lim_{n\to\infty} \mathcal{M}_2(y_{n+1}, TSw, TSw, qt) = \lim_{n\to\infty} \mathcal{M}_2(Tx_n, TSw, TSw, qt)$

$\geq \lim_{n\to\infty} \min\{ \mathcal{M}_1(x_n, Sw, Sw, t), \mathcal{M}_1(Sw, STx_n, STx_n, t),$ $\mathcal{M}_2(w, Tx_n, Tx_n, t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, STx_n, STx_n, t)\}$

$= \lim_{n\to\infty} \min\{ \mathcal{M}_1(x_n, Sw, Sw, t), \mathcal{M}_1(Sw, x_{n+1}, x_{n+1}, t),$ $\mathcal{M}_2(w, y_{n+1}, y_{n+1}, t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, x_{n+1}, x_{n+1}, t)\}$

$\geq \mathcal{M}_1(z, Sw, Sw, t)$ and

$\mathcal{N}_2(w, TSw, TSw, qt)$ $= \lim_{n\to\infty} \mathcal{N}_2(y_{n+1}, TSw, TSw, qt) = \lim_{n\to\infty} \mathcal{N}_2(Tx_n, TSw, TSw, qt)$

$\leq \lim_{n\to\infty} \max\{ \mathcal{N}_1(x_n, Sw, Sw, t), \mathcal{N}_1(Sw, STx_n, STx_n, t),$ $\mathcal{N}_2(w, Tx_n, Tx_n, t)\Diamond\mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(x_n, STx_n, STx_n, t)\}$

$= \lim_{n\to\infty} \max\{ \mathcal{N}_1(x_n, Sw, Sw, t), \mathcal{N}_1(Sw, x_{n+1}, x_{n+1}, t),$ $\mathcal{N}_2(w, y_{n+1}, y_{n+1}, t)\Diamond\mathcal{N}_2(w, TSw, TSw, t), \mathcal{N}_1(x_n, x_{n+1}, x_{n+1}, t)\}$

$\leq \mathcal{N}_1(z, Sw, Sw, t)$.

Hence $\mathcal{M}_1(Sw, z, z, qt) \geq \mathcal{M}_2(w, TSw, TSw, t) \geq \mathcal{M}_1(z, Sw, Sw, t/q)$ and

$\mathcal{N}_1(Sw, z, z, qt) \leq \mathcal{N}_2(w, TSw, TSw, t) \leq \mathcal{N}_1(z, Sw, Sw, t/q)$.

Which is a contradiction.

Thus Sw = z. we have STz = Sw = z and TSw = Tz = w.

Thus the pointz is a fixed point of ST in X and the point w is a fixed point of TS.

Uniqueness: Let $z' = z$ be the another fixed point of ST in X. We have

$\mathcal{M}_1(z, z', z', qt) = \mathcal{M}(Sw, STz', STz', qt)$

$\geq \min \{\mathcal{M}_1(z', Sw, Sw, t) * \mathcal{M}_1(z', STz', STz', t),$ $\mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_2(w, Tz', Tz', t), \mathcal{M}_2(Tz', TSw, TSw, t)\}$

$= \min \{\mathcal{M}_1(z', z, z, t) * 1, 1, \mathcal{M}_2(w, Tz', Tz', t),$ $\mathcal{M}_2(Tz', w, w, t)\} \geq \mathcal{M}_2(Tz', w, w, t)$ and

$\mathcal{N}_1(z, z', z', qt) = \mathcal{N}(Sw, STz', STz', qt)$

$\leq \max \{\mathcal{N}_1(z', Sw, Sw, t) \Diamond\mathcal{N}_1(z', STz', STz', t),$ $\mathcal{N}_2(w, TSw, TSw, t), \mathcal{N}_2(w, Tz', Tz', t), \mathcal{N}_2(Tz', TSw, TSw, t)\}$

$= \max\{\mathcal{N}_1(z', z, z, t) \Diamond 1, 1, \mathcal{N}_2(w, Tz', Tz', t), \mathcal{N}_2(Tz', w, w, t)\}$

$\geq \mathcal{N}_2(Tz', w, w, t)$.

Now, $\mathcal{M}_2(Tz', w, w, qt) = \mathcal{M}_2(Tz', TSw, TSw, qt)$

$\geq \min\{\mathcal{M}_1(z', Sw, Sw, t), \mathcal{M}_2(Sw, STz', STz', t),$
$\mathcal{M}_2(w, Tz', Tz', t) * \mathcal{M}_2(w, TSw, TSw, t), \mathcal{M}_1(z', STz', STz', t)\}$
$= \min\{\mathcal{M}_1(z', z, z, t), \mathcal{M}_1(z, z', z', t), \mathcal{M}_2(w, Tz', Tz', t) * 1, 1\}$
$\geq \mathcal{M}_1(z, z', z', t)$ and
$\mathcal{N}_2(Tz', w, w, qt) = \mathcal{N}_2(Tz', TSw, TSw, qt)$
$\leq \max\{\mathcal{N}_1(z', Sw, Sw, t), \mathcal{N}_2(Sw, STz', STz', t),$
$\mathcal{N}_2(w, Tz', Tz', t) \lozenge \mathcal{N}_2(w, TSw, TSw, t), \mathcal{N}_1(z', STz', STz', t)\}$
$= \max\{\mathcal{N}_1(z', z, z, t), \mathcal{N}_1(z, z', z', t), \mathcal{N}_2(w, Tz', Tz', t) \lozenge 0, 0\}$
$\leq \mathcal{N}_1(z, z', z', t)$

Hence, $\mathcal{M}_1(z, z', z', qt) \geq \mathcal{M}_2(Tz', w, w, t) \geq \mathcal{M}_1(z, z', z', t/q)$ and $\mathcal{N}_1(z, z', z', qt) \leq \mathcal{N}_2(Tz', w, w, t) \leq \mathcal{N}_1(z, z', z', t/q)$.

Which is a contradiction. Thus z = z'. So the point z is a unique fixed point of ST. Similarly, we prove the point w is also a unique point of TS.

Corollary 2.2:

Let $(X, \mathcal{M}_1, \mathcal{N}_1, *, \lozenge)$ and $(Y, \mathcal{M}_2, \mathcal{N}_2, *, \lozenge)$ be two complete intuitionistic generalized fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying following conditions.

$\mathcal{M}_2(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t),$
$\mathcal{M}_2(y, Tx, Tx, t) * \mathcal{M}_2(y, TSy, TSy, t)\}$
$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t) *$
$\mathcal{M}_1(x, STx, STx, t), \mathcal{M}_2(y, Tx, Tx, t)\}$ and
$\mathcal{N}_2(Tx, TSy, TSy, qt) \leq \max\{\mathcal{N}_1(x, Sy, Sy, t),$
$\mathcal{N}_2(y, Tx, Tx, t) \lozenge \mathcal{N}_2(y, TSy, TSy, t)\}$
$\mathcal{N}_1(Sy, STx, STx, qt) \leq \max\{\mathcal{N}_1(x, Sy, Sy, t) \lozenge$
$\mathcal{N}_1(x, STx, STx, t), \mathcal{N}_2(y, Tx, Tx, t)\}$, for all x in X and y in Y where $q<1$, then ST has a unique fixed point z in X and TS has a unique fixed point win Y. Further Tz = w and Sw = z.

Corollary 2.3:

Let $(X, \mathcal{M}_1, \mathcal{N}_1, *, \lozenge)$ and $(Y, \mathcal{M}_2, \mathcal{N}_2, *, \lozenge)$ be two complete intuitionistic generalized fuzzy metric spaces. If T is a mapping from X into Y and S is a mapping from Y into X, satisfying the following conditions.

$\mathcal{M}_2(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_2(y, Tx, Tx, t),$
$\mathcal{M}_2(y, TSy, TSy, t), \mathcal{M}_1(x, STx, STx, t), \mathcal{M}_1(Sy, STx, STx, t),\}$
$\mathcal{M}_1(Sy, STx, STx, qt) \geq \min\{\mathcal{M}_2(y, Tx, Tx, t),$
$\mathcal{M}_1(x, Sy, Sy, t), \mathcal{M}_1(x, STx, STx, t), \mathcal{M}_2(Tx, TSy, TSy, t),$
$\mathcal{M}_2(y, TSy, TSy, t)\}$
$\mathcal{N}_2(Tx, TSy, TSy, qt) \leq \max\{\mathcal{N}_1(x, Sy, Sy, t), \mathcal{N}_2(y, Tx, Tx, t),$
$\mathcal{N}_2(y, TSy, TSy, t), \mathcal{N}_1(x, STx, STx, t), \mathcal{N}_1(Sy, STx, STx, t),\}$
$\mathcal{N}_1(Sy, STx, STx, qt) \leq \max\{\mathcal{N}_2(y, Tx, Tx, t),$
$\mathcal{N}_1(x, Sy, Sy, t), \mathcal{N}_1(x, STx, STx, t), \mathcal{N}_2(Tx, TSy, TSy, t),$
$\mathcal{N}_2(y, TSy, TSy, t)\}$, for all x in X and y in Y where $q<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Corollary 2.4:

Let $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$ be a complete intuitionistic generalized fuzzy metric space, If S and T are mapping from X into itself satisfying the following conditions.

$\mathcal{M}(Tx, TSy, TSy, qt) \geq \min\{\mathcal{M}(x, Sy, Sy, t), \mathcal{M}(y, Tx, Tx, t), \mathcal{M}(y, TSy, TSy, t), \mathcal{M}(x, STx, STx,, t), \mathcal{M}(Sy, STx, STx, t)\}$

$\mathcal{N}(Tx, TSy, TSy, t) \leq \max\{\mathcal{N}(x, Sy, Sy, t),$
$\mathcal{N}(y, Tx, Tx, t), \mathcal{N}(y, TSy, TSy, t), \mathcal{N}(x, STx, STx, t,) \mathcal{N}(Sy, STx, STx, t)\}$

$\mathcal{M}(Sy, STx, STx, t) \geq \min\{\mathcal{M}(y, Tx, Tx, t), \mathcal{M}(x, Sy, Sy, t),$
$\mathcal{M}(x, STx, Stx, t), \mathcal{M}(Tx, TSy, TSy, t), \mathcal{M}(y, TSy, TSy, t)\}$
$\mathcal{N}(Sy, STx, STx, t) \leq \max\{\mathcal{N}(y, Tx, Tx, t)$
$\mathcal{N}(x, Sy, Sy, t) \lozenge \mathcal{N}(x, STx, Stx, t), \mathcal{N}(Tx, TSy, TSy, t),$
$\mathcal{N}(y, TSy, TSy, t)\}$, for all x, y in X where $q<1$, then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further Tz = w and Sw = z.

Corollary 2.5:

Let $(X, \mathcal{M}, \mathcal{N}, *, \lozenge)$ be a complete intuitionistic generalized fuzzy metric space, If S and T are mapping from X into itself satisfying the following conditions.

$\mathcal{M}(Tx, TSy, TSy, t) \geq \min\{\mathcal{M}(x, Sy, Sy, t),$
$\mathcal{M}(Sy, STx, STx, t), \mathcal{M}(y, Tx, Tx, t), \mathcal{M}(y, TSy, TSy, t),$
$\mathcal{M}(x, STx, STx, t)\}$
$\mathcal{M}(Sy, STx, STx, t) \geq \min\{\mathcal{M}(x, Sy, Sy, t) *$
$\mathcal{M}(x, STx, Stx, t), \mathcal{M}(y, TSy, TSy, t),$
$\mathcal{M}(y, Tx, Tx, t), \mathcal{M}(Tx, TSy, TSy, t)\}$ and
$\mathcal{N}(Tx, TSy, TSy, t) \leq \max\{\mathcal{N}(x, Sy, Sy, t),$
$\mathcal{N}(Sy, STx, STx, t), \mathcal{N}(y, Tx, Tx, t), \mathcal{N}(y, TSy, TSy, t),$
$\mathcal{N}(x, STx, STx, t)\}$
$\mathcal{N}(Sy, STx, STx, t) \leq \max\{\mathcal{N}(x, Sy, Sy, t) \lozenge \mathcal{N}(x, STx, Stx, t),$
$\mathcal{N}(y, TSy, TSy, t), \mathcal{N}(y, Tx, Tx, t), \mathcal{N}(Tx, TSy, TSy, t)$

for all x, y in X where $q < 1$, then ST has a unique fixed point z in X and TS has a unique fixed point win Y. Further Tz = w and Sw = z.

3. Conclusion

Efferts have been taken in generalizing the concepts of fixed point theorems for contractive mappings in fuzzy metric spaces and the existence and uniqueness of common fixed points in abstract spaces like two intuitionistic generalized fuzzy metric space and three intuitionistic generalized fuzzy metric space are proved. This work can be easily extended by increasing the number of self mappings and establishing the fixed point theorems in more generalized settings.

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