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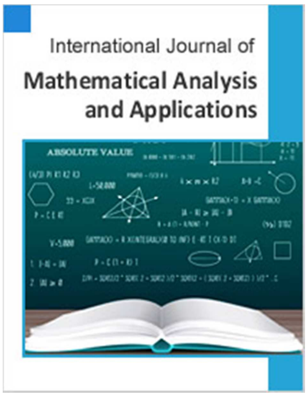

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Reconstruction of Functions Which Describe Motion of Rigid Body Under Measurements Using the Method of Trained Models

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Abstract
In this paper, we reconstruct the functions describing angular motion of rigid body (in particular, spacecraft) under measurements, and we carry out identification of attitude control system using discrete measurements. Main task is determination of the approximating functions in analytical form and constructing the mathematical model of motion. For finding the analytical functions which describe motion of rigid body about centre of mass, the method of trained models is used. It is shown that presented algorithms of data processing reconstruct continuous functions under discrete measurements with minimal error. Also, we have demonstrated the possibilities of designed method and its advantages for the solving the problems of identification of physical processes and technical systems. Our mathematical instruments can successfully be used for analyses of onboard systems of spacecraft. As an example, the problem of identification of attitude control system using result of measurements is studied in detail for multi-modular spacecraft. The problem of exact identification of a controlled motion (including a method of control) and determination of numerical values of main parameters of control algorithm at the regime of a programmed turn has been solved. High-precision reconstruction of actual controlled motion of a spacecraft allows us to make identification of attitude system with big reliability. Designed techniques demonstrate better characteristics. Mathematical technology, described in this article, was used in official reports on analysis of onboard control system of spacecraft motion (including multi-modular orbital station).

1. Introduction

During tests and analysis of technical systems we use measurements of different parameters under which we can conclude about correctness and quality of performance of system. Our field of researches is controlled motion of rigid body. In our application interest, we take spacecraft as rigid body, and we consider most complicated case when rigid body is multi-modular spacecraft.

We assume that subject of analysis is rotary motion of a spacecraft and its control system of motion. An investigation of attitude control system in different regimes is very important. In this article, we reconstruct the functions describing angular motion of rigid body under measurements and we carry out identification of attitude control system of multi-modular spacecraft by the method of trained models.

General task is identification of attitude control system of multi-modular spacecraft under discrete measurements of angular motion. In particular, it is the finding the analytical functions which describe motion of rigid body about centre of mass, and the finding the values of key parameters of motion control system with high-precision, and also specification of method by which maneuver was carried out. For this, it is necessary to solve main task which consist in determination of the approximating functions in the analytical form which are co-ordinated with discrete measurements in the best way. For achievement of the purpose, first of all it is necessary to select model depicting programmed (or nominal) behavior of parameters describing motion of spacecraft with respect to center of the mass. For solution of the problem, the method of trained models was used, as one of the most simple and at the same time most floppy and general methods of identification [1]. The projections of the vector of absolute angular velocity on an axes of spacecraft-bound coordinate system were adopted as the measurable parameters of an attitude.

The main results of mathematical processing and analysis of measurements about spacecraft motion at a regime of spatial turn are below briefly described. After that, the in-depth enough data of post-flight processing of measurements obtained from onboard of spacecraft are adduced, which has allowed not only to reconstruct actual motion of spacecraft in a turn regime, but also precisely enough to identify a method of turn control, which one the manoeuvres implemented.

The data processing of measurements is done using a technique based on identification with the pattern model. Within the framework of this technique, the measurements executed during some time interval are processed jointly by least squares process and integration of equations of spacecraft motion around a center of mass. The equations of angular motion is written according to the logic of a operating of control system which is carried out pursuant to a selected mode of turn. At processing, the parameters of a mathematical model are updated and the adjustment values of parameters of a control algorithm are determined.

The data, obtained by designed mathematical means in result of reconstruction of continuous functions under discrete measurements, were placed in official reports on analysis of onboard control system of spacecraft motion.

2. Data of Measurements and Methods of Their Processing

As example, we demonstrate reconstruction of functions which describe motion of multi-modular spacecraft under discrete measurements using the method of trained models. The spacecraft is considered absolutely solid body which centre of mass makes uncontrollable elliptic motion. For a study, analysis and identification, three-dimensional turn of spacecraft is very interesting. Indications of all orientation devices are interpreted in so-called building coordinate system *Oxyz* (or body-fixed coordinate system). It is a related rigidly with spacecraft a right-hand Cartesian system of the co-ordinates where the axes are directed as follows: the axis *Ox* is directed in parallel to longitudinal axis of the central (main) block; axis *Oz* is parallel to rotary axis of solar batteries of this block; and the axis *Oy* supplements system *Oxyz* to right-hand coordinate system.

For definition of nominal motion of bulky multi-modular spacecrafts, the vector of absolute velocity $\boldsymbol{\omega}$ were used only. The data about $\boldsymbol{\omega}$ is obtained by an onboard instrumentation system as its projections $\omega_x, \omega_y, \omega_z$ onto an axis of body-fixed coordinate system *Oxyz*, a center by which one coincides a center of the spacecraft mass. The measured values of different component of absolute angular velocity $\boldsymbol{\omega}$ are registered independently from each other and consequently can fall to the different moments of time.

At the turn, spacecraft executes a controlled motion, and it is provided with a control system by means of jet micro-engines. A variation of angular velocities $\omega_x, \omega_y, \omega_z$ during turn of multi-modular spacecraft is shown on Figure 1. They are the actually measured values of parameters of angular motion. As well as at any turn here there are temporary stages, at which one a size of all components of angular-velocity vector (so also a quantity of angular rate $|\boldsymbol{\omega}|$) simultaneously increases (decreases). The concern introduces time interval between gaining and damping of angular velocity. The character of variation of angular velocity ω_i ($i = x, y, z$) at this time interval also determines the type of a kinematical trajectory of turn and control method, applicable to it. Further, research of spacecraft rotation we shall conduct only at time period between an acceleration and braking of spacecraft (at a phase of nominal motion).

Set of the measurements included in a processing we will designate $t_i^{(k)}$, $\omega_i^{(k)}$ ($i = x, y, z$; $k = 1, 2,... N_i$, $t_i^{(k)} < t_i^{(k+1)}$). Here $\omega_i^{(k)}$ is the result of measurement of component ω_i of vector $\boldsymbol{\omega}$ at time moment $t_i^{(k)}$. The beginning and the finish of processing interval should not get at acceleration and braking phases. We believe $\min t_i^{(1)} \geq t_{\text{ac}}$, $\max t_i^{N_i} \leq t_{\text{br}}$ ($i = x, y, z$), where t_{ac} is the time of the ending of acceleration phase, t_{br} is the time of the beginning of braking phase.

Optimized functional which we should minimize has the form

$$G = \sum_{i=x}^{Z} \sum_{k=1}^{N_i} \left(\omega_i^{(k)} - \omega_i(t) \right)^2$$

We use a method of the least squares on the basis of the following assumption: regular errors in measurements of component ω_i are identical and equal Δ_i ($i = x, y, z$), and random errors in measurements of all components are independent and have identical normal distribution with zero average value and a standard deviation σ.

Identification problem is formulated as follows: it is necessary to find such calculated approximating functions ω_i

(t) (not only their mathematical form but also coefficients of the functions) which give a minimum for the functional:

$$F = \sum_{i=x}^{Z} \sum_{k=1}^{N_i} \left\{ \omega_i^{(k)} - \omega_i^0 \left[t_i^{(k)} \right] \right\}^2 \qquad (1)$$

where $\omega_i^0(t) = \omega_{i\,\mathrm{cal}}(t) + \Delta_i$; $\omega_{i\ \mathrm{cal}}$ is the calculated values of component ω_i of absolute angular velocity vector $\boldsymbol{\omega}$ according to functions $\omega_i(t)$; Δ_i is unknown constants (so-called «displacement of zero»).

For detection of features of spacecraft motion during turn and obtaining of its interesting characteristics, we shall take advantage of experience and consequences of analytical researches [2-4]. Now three ways of programmed turns of spacecraft can be used most frequently:

a) Series of the turns around spacecraft-fixed axises;

b) Turn of spacecraft around of the final turn vector (Euler turn);

c) Turn in a form of a precession of solid body (in particular, as regular precession).

At last case, some persistence of axial component angular velocity ω_1 on the one hand, and form of transverse angular velocities ω_2 and ω_3 are close to harmonious functions of time on the other hand, will give a basis to draw conclusion about presence of regular properties which have the characteristics as motion of spacecraft in relation to a center of the spacecraft mass as precessions of solid body around of some direction motionless in all-inertial space. Spacecraft is gyrated simultaneously around of some motionless axis $\boldsymbol{\eta}$ componented with longitudinal axis OX definite angle ϑ, and around of a centerline OX with angular velocities of precession $\dot{\psi}$ and of own rotation $\dot{\phi}$, accordingly. The indicated type of motion is described by following equations:

$$\omega_x = \dot{\phi} + \dot{\psi}\cos\vartheta\,,\ \omega_y = \dot{\psi}\sin\vartheta\sin\varphi(t),$$
$$\omega_z = \dot{\psi}\sin\vartheta\cos\varphi(t)$$

where φ is turn angle of own rotation.

By virtue of the universality last type of programmed turn of spacecraft introduces the greatest concern, though the turn of spacecraft along a trajectories of free motion is generally possible way [4, 5].

For determination of motion type and control method of spacecraft reorientation, it is necessary to select model depicting a dynamics of rotation and programmed (or nominal) variation of angular velocities describing motion of multi-modular spacecraft.

3. A choice of Pattern Mathematical Model

Let us consider one of turns of the orbital spacecraft representing a large heavy multi-modular construction. Data of onboard measurements of angular velocities $\omega_x, \omega_y, \omega_z$ are visually presented by Figure 1. The configuration of spacecraft is asymmetrical, therefore it is impossible to neglect discrepancy of the principal central axes of the ellipsoid of inertia with building axes of a spacecraft. At performance of the given regime of orientation, the orbital coordinate system (OCS) was basic (as reference basis), and a spacecraft was stabilized with respect to it before and after a turn. Therefore, before a turn and on its termination the magnitude of absolute angular rate is distinct from zero.

Feature is the unregularity of rotary motion of spacecraft which is expressed in dissimilarity of behaviour of angular velocities $\omega_x, \omega_y, \omega_z$ on their typical change at any of known ways of a turn of a solid body. Rotation occurs about all three axes Ox, Oy, Oz. However, rotary velocities in relation to axes of the body-fixed coordinate system $Oxyz$ are not constant, and proportions between them are not constants. Hence, the spacecraft turn is not planar [2], and it is not executed in the form of planar rotation around a motionless axis (Euler's axis) [3]. Thus all three angular velocities $\omega_x, \omega_y, \omega_z$ change dynamically enough. Any of components of absolute angular velocity vector $\boldsymbol{\omega}$ have not constant character (or at least close to it). Also, rotation of a spacecraft is impossible to consider as free motion [4, 5]. Nevertheless, at enough long interval of time, a quantity of angular velocity $\boldsymbol{\omega}$ changes slightly. Quite probably, that motion of vector $\boldsymbol{\omega}$ possesses some regular properties, but concerning any other rectangular coordinate system $Om_1m_2m_3$, motionless in relation to the body-fixed coordinate system $Oxyz$ (or in inertial space). If such coordinate system $Om_1m_2m_3$ exists, spacecraft motion around the centre of mass has regular character during an investigated time interval between acceleration and braking.

Let there will be such plane m_2Om_3, the vector projection $\boldsymbol{\omega}$ on which monotonously turns with some angular velocity distinct from zero. If even thus specified angular velocity has almost constant value, it is possible to speak about a precession of the vector $\boldsymbol{\omega}$ and, accordingly, of a spacecraft. Direction Om_1 which is orthogonal to plane m_2Om_3 and supplementing the system $Om_1m_2m_3$ to right coordinate system, is an axis of own rotation. In this case, the problem of determination of characteristics of spacecraft's true motion around the centre of mass is reduced to definition of mutual position of rectangular coordinate systems $Oxyz$ and $Om_1m_2m_3$ (for example, matrix of the directing cosines $B = \|b_{ij}\|_{3\times3}$ for axes Om_1, Om_2, Om_3 with respect to the axes Ox, Oy, Oz), to a transfer of vector $\boldsymbol{\omega}$ absolute angular velocity from the body-fixed coordinate system $Oxyz$ into coordinate system $Om_1m_2m_3$ related with a direction of an axis of own rotation Om_1 (to vector display $\boldsymbol{\omega}$ on axis Om_1, Om_2, Om_3), and to a construction of the valid motion $\boldsymbol{\omega}(t)$ and the main characteristics $\dot{\phi}(t)$, $\dot{\psi}(t)$, $\vartheta(t)$, $\omega(t) \equiv |\boldsymbol{\omega}(t)|$ of spacecraft's angular motion using known methods [1, 7, 8].

A finding of matrix of the directing cosines is a separate independent problem. Let the matrix of transition from the coordinate system $Oxyz$ to the coordinate system $Om_1m_2m_3$ is known: $B = \|b_{ij}\|_{3\times3}$, where b_{ij} is cosine of a angle between positive directions of axes Oi and Om_j ($i = x, y, z; j = 1, 2, 3$). Then, a display to axes Om_1, Om_2, Om_3 motion of absolute angular velocity vector **ω** in the course of spacecraft's turn can be received using the expressions:

$$\omega_j = \sum_{i=x}^{z} b_{ij}\omega_i \ (j = \overline{1,3}).$$

In our concrete case, the transition matrix B (the directing cosines) is

$$B = \begin{vmatrix} 0.6988475330 & -0.2198905795 & 0.6806322492 \\ 0.3087457564 & 0.9510947314 & 0.0000000000 \\ -0.6473457462 & 0.2101423186 & 0.7325604900 \end{vmatrix}$$

Results of the projections recalculation of the vector **ω** from the related coordinate system $Oxyz$ onto axis of system $Om_1m_2m_3$ are shown in Figure 2. The analysis of the received data allows to assume that in this case, at a phase between acceleration and a braking of angular velocity, rotation of spacecraft is in the form of a precession of solid body. In it specifies an approximate constancy of longitudinal angular velocity ω_1 and minor alteration of size of the transverse angular velocity $\omega_{tr} = \sqrt{\omega_2^2 + \omega_3^2}$. As pattern model, we can accept the following equations:

$$\omega_1 = \dot\phi + \dot\psi\cos\vartheta, \ \omega_2 = \dot\psi\sin\vartheta\sin\left(\dot\phi t + \phi_0\right),$$
$$\omega_3 = \dot\psi\sin\vartheta\cos\left(\dot\phi t + \phi_0\right) \quad (2)$$

where $\dot\psi$ is angular velocity of a precession (around some motionless axis **η** in inertial space), $\dot\phi$ is angular velocity of own rotation (about axis Om_1, motionless in the body-fixed coordinate system $Oxyz$), ϑ is the angle of deviation of axis Om_1 from precession axis **η**. The values $\dot\phi$, $\dot\psi$, and ϑ are the slowly-varying parameters (as functions of time).

Character of spacecraft's motion around the centre of a mass is defined by behaviour of elements $\dot\phi$, $\dot\psi$ and ϑ as time functions. Therefore, essentially important authentically and precisely to estimate a varying during time of parameters $\dot\phi$, $\dot\psi$, ϑ in the course of turn. Approximation of the measured values of angular velocities by expressions (2) is made by splitting of the given time-interval of processing into final number of subintervals and by a replacement of variable parameters $\dot\phi$, $\dot\psi$, ϑ inside everyone subinterval by the constants equal to any values of the same parameters inside or at boundary of considered subintervals. And, initial process with variable parameters $\dot\phi$, $\dot\psi$, ϑ at given final interval of time can be approximated by a process with piecewise-constant parameters, with any degree of accuracy [6].

The recovery problem of programmed motion of multi-modular spacecraft at the site [t_{ac}, t_{br}] consists in a numerical determination of parameters of control algorithm $\dot\phi$, $\dot\psi$, ϑ (and of constant parameters Δ_i also), and decision of the system (2), which give a minimum for the functional F (as the form (1)), when in the formula (1) $\omega_i^{(k)}$ is the measured values of the projections of a vector **ω** onto body-fixed axes at time moments $t^{(k)}$ ($t^{(k)} < t^{(k+1)}$); ω_j ($t^{(k)}$) is the values of component ω_j in the projections of axes of coordinate system $Om_1m_2m_3$ which are computed along the decision of model system (2); ω_i ($t^{(k)}$) is the calculation values of angular velocities $\omega_x, \omega_y, \omega_z$ received from the solution ω_j ($t^{(k)}$) of system (2).

Set of the measurements $\omega_i^{(k)}$, $t^{(k)}$ selected for processing and reconstruction of spacecraft motion correspond to for time moments which are not overstepping the bounds of time interval [t_{ac}, t_{br}]. The components ω_i ($i = x, y, z$) are computed using the matrix of the directing cosines "B" and the solution of the equations (2) ω_j (t) as time functions ($j = \overline{1,3}$). Obviously,

$$\omega_i(t) = \sum_{j=1}^{3} b_{ij}\omega_j(t)$$

where b_{ij} is the elements for matrix $B = \|b_{ij}\|_{3\times3}$ of transition from the body-fixed coordinate system $Oxyz$ to coordinate system $Om_1m_2m_3$.

Use the functional F means the hypothesis acceptance, that the measurement errors of component ω_i of all vectors **ω**($t^{(k)}$) in related coordinate system $Oxyz$ are independent random variables with identical standard deviation.

If the tensor of spacecraft inertia is known, the identification problem of angular motion can be facilitated. In this case, most likely, it is necessary to study motion with respect to the coordinate system related with the principal central axes of inertia of multi-modular spacecraft. For this, we shall enter the right Cartesian system of coordinates $On_1n_2n_3$ formed by the principal central axes of inertia. Thus, axis On_1 is a longitudinal axis of a spacecraft, and axis On_2 and On_3 are directed so that mismatch angles between them and axises of the related coordinate system were minimal. For a correct choice of reference model of motion, it is necessary to compute preliminary angular velocity vector **ω** in projections to the principal central axes of the ellipsoid of inertia On_1, On_2, On_3. For vector representation **ω** in projections to the principal central axes of inertia, we should have all three components ω_i ($i = x, y, z$) a vector **ω** for the same moments of time. The missing values ω_i of component of vector **ω** for time interval $t_i^{(k)}$, we receive by interpolation of a corresponding set of values. A matrix of transition from the body-fixed coordinate system $Oxyz$ to the system $On_1n_2n_3$ we will designate $A = \|a_{ij}\|_{3\times3}$, where a_{ij} is cosine of a angle between axises Oi and On_j ($i = x, y, z; j = 1, 2, 3$).

If results of recalculation of a vector **ω** from body-fixed coordinate system $Oxyz$ in a projection to axises of system

$On_1n_2n_3$ will show a picture (as variation form) similar to those that is studied from Figure 2, then character of change of angular rates ω_1, ω_2, ω_3 in projections to the principal central axes of the ellipsoid of inertia $On_1n_2n_3$ that motion of a multi-modular spacecraft around the centre of mass occurred as the precession of solid body; and we can apply the way considered above where instead of system $Om_1m_2m_3$ it is necessary to take $On_1n_2n_3$, and matrix B = A ($b_{ij} = a_{ij}$). Some constancy of longitudinal making angular velocity ω_1 on the one hand and variation of transverse angular velocities ω_2 and ω_3 on close by the form to harmonious functions of a time, on the other hand, specify in presence of regular properties, characteristic for motion in a form precession of solid body around some direction, motionless with respect to inertial space. Spacecraft rotates simultaneously around some motionless axis $\boldsymbol{\eta}$ is inclined to longitudinal axis On_1 by the rated angle ϑ, and around longitudinal axis On_1 with angular velocities of precession $\dot{\psi}$ and own rotation $\dot{\phi}$ accordingly. For such type of motion, equations (2) in which parameters $\dot{\phi}$, $\dot{\psi}$, and ϑ are slowly-varying functions of time are fair.

Rotation of multi-modular spacecraft can be described by mathematical model in the form of dependences (2), in which variables ω_j (j = 1, 2, 3) are projections of angular-velocity vector $\boldsymbol{\omega}$ onto axises of system $On_1n_2n_3$ ($On_1n_2n_3$ is coordinate system related with principal central axes of spacecraft inertia). The description of spacecraft motion is probably and in angular velocities ω_i ($i = x, y, z$), being projections of a vector $\boldsymbol{\omega}_{pr}$ of programmed absolute angular velocity onto axises of the body-fixed coordinate system $Oxyz$ (we have this possibility). For this purpose, it is necessary for function ω_1 (t), ω_2 (t), ω_3 (t) which are the solution of the equations (2), to transform in functions ω_x (t), ω_y (t), ω_z (t) using the matrix of the directing cosines between axises of coordinate systems $Oxyz$ and $On_1n_2n_3$.

In this case, determination of spacecraft's rotary motion for the segment [t_{ac}, t_{br}] consists in a finding of such parameters of motion model (the pattern model) and solutions of the equations (2), which allow to coordinate in the best way, in a sense of a method of the least squares, the measured (counted under onboard information) and calculated values of components ω_i ($i = x, y, z$); here t_{ac} is the time of a termination of acceleration stage; t_{br} is the time of the beginning of braking stage.

4. Results of a Solving the Problem of Reconstruction of Analytical Functions

Let us take for the analysis, processing and solution of the identification problem the turn presented at Figure 1. We accept, t_B is the moment of maneuver beginning, and t_E is the moment of maneuver ending. Interval from $t = t_B$ to $t = t_E$ represents a interest. We notice, that all time interval [t_B, t_E] can be broken into three basic phases: a increase of angular rate (an acceleration), a decrease of angular rate (a braking), and an interval between acceleration and braking. It is clearly visible from Figure 1. For the moments of time corresponding to stages of increase and decrease of angular rate, a linear interpolation is sufficient. At time interval between the specified stages the number of measurements is not great, owing to this a interpolation was carried out by the polynomials.

Feature of the turn chosen for research is the seeming at first sight unregularity of angular motion. However such behaviour of angular velocities easily to explain and to understand, if we will consider that axes of sensitivity of measuring instruments of angular velocity do not coincide with the principal central axes of the ellipsoid of spacecraft inertia.

For definition of true character of spacecraft motion, it is necessary to check up presence of regular properties of rotation during time interval between acceleration (increase of angular rate) and a braking (decrease of angular rate). In particular, we will ask a question: whether there is a direction, motionless in the related coordinate system with respect to which the vector of absolute angular velocity $\boldsymbol{\omega}$ is rotated with almost constant angular velocity? If yes, it makes sense to determinate and investigate more in detail motion of the vector $\boldsymbol{\omega}$ in new coordinate system $Om_1m_2m_3$ which axises are directed as follows: axis Om_1 coincides with the specified direction, around which a vector $\boldsymbol{\omega}$ does precession; axes Om_2 and Om_3 are in a plane which is perpendicular to axis Om_1; they supplement system $Om_1m_2m_3$ to right coordinate system and are located so that the angle of final turn between the coordinate systems $Oxyz$ and $Om_1m_2m_3$ was minimal.

In example shown by Figure 1, the motionless axis around which the vector of absolute angular velocity $\boldsymbol{\omega}$ does precession exists. We take the mathematical model (2) as the pattern model (for use the method of trained models [6]). Pattern model in the form of dependences (2), where variables ω_j are projections of a vector of angular velocity $\boldsymbol{\omega}$ onto axes of system $Om_1m_2m_3$ (j = 1, 2, 3), is closest to real process. After recalculation we have data about the vector $\boldsymbol{\omega}$ as projections onto axis of system $Om_1m_2m_3$ in the form demonstrated at Figure 2.

Programmed spacecraft motion is described by the variables ω_i ($i = x, y, z$), which are translated from the functions ω_1 (t), ω_2 (t), ω_3 (t), being the solutions of the equations (2), in functions ω_x (t), ω_y (t), ω_z (t) using the matrix of the directing cosines between axises of coordinate systems $Oxyz$ and $On_1n_2n_3$. Resulting functions for this concrete example of spacecraft turn are shown in the Figure 3. A construction of approximation of spacecraft's actual motion with respect to center of mass at the segment [t_{ac}, t_{br}] consists in a finding of parameters $\dot{\phi}$, $\dot{\psi}$, ϑ and the solution of the system (2), delivering a minimum of the functional

$$\Phi = \sum_{i=x}^{z} \sum_{k=1}^{N_i} \left\{ \omega_i^{(k)} - \omega_i \left(t^{(k)} \right) \right\}^2$$

The use of the functional Φ fairly because in our case (for our concrete sensors of angular velocity) the measurement errors of components of all vectors **ω** ($t^{(k)}$) in the system *Oxyz* are independent random variables with zero average value and an identical standard deviation. If systematic error of measurements of component ω_i is distinct from zero then we include it into the approximating function $\omega_i(t)$ as one more unknown constant parameter.

Earlier we has shown that is better to describe the motion of rigid body as simultaneous rotation about some motionless axis in the inertial space, making with a longitudinal axis of a spacecraft a calculated angle ϑ, and about longitudinal axis of a spacecraft with some angular velocities of precession $\dot{\psi}$ and of own rotation $\dot{\phi}$ (such type of motion is described by the analytical model (2)). For this case of realization of a programmed turn, we had following indicators of angular motion: $\dot{\phi} = -0.03933$ deg/s, $\dot{\psi} = 0.1769$ deg/s, $|\boldsymbol{\omega}| = 0.5518$ deg/s. Turn angle has made α = 158.5 degrees. Thus, received values $\dot{\phi}$, $\dot{\psi}$, ϑ are equal to nominal values of parameters of control law of a turn. Key problem is the constructing the functions ϑ (t) and φ (t) with minimal discrepancies between the reconstructed functions $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$ and the measurements $\omega_i^{(k)}$ in discrete moment of time $t_i^{(k)}$. Nominal value of nutation angle (its estimation): ϑ = 118.6 degree. If to accept the sought function φ (t) in the form of polynomial dependence, for example, $\varphi(t) = \varphi_0 + \varphi_1 t + \varphi_2 t^2 + \varphi_3 t^3$, required values of approximation coefficients will turn out the following:

$$\phi_0 = -38.56 \text{ degree}, \ \phi_1 = -0.03933 \text{ deg/s}, \ \phi_2 \approx -3\cdot 10^{-7} \text{ deg/s}^2, \ \phi_3 = 0.000 \text{ deg/s}^3$$

Actual values of the elements $\dot{\phi}$, $\dot{\psi}$, ω, ϑ during the turn of an orbital spacecraft are practically constant (ω is modulus of angular velocity). Presence of insignificant deviations from their nominal values points out that control system is not ideal. It is considered to be, a measure of inconstancy of the estimated parameter P is a relative deviation δP from its average value $\overline{P}$. Degree of a constancy of the parameters $\dot{\phi}$, $\dot{\psi}$, ω is defined by the standard technical norms: δP ≤ 0,05, where δP – the maximum relative deviation of parameter P from the set of a rating values. A constant value, satisfying to an inequality, is considered:

$$P_{\max} - P_{\min} \le 0{,}05\,|P_{\max} + P_{\min}|$$

where $P_{\min} = \min P(t)$, and $P_{\max} = \max P(t)$ are the minimum and maximum values of parameter P.

In the accepted designations, we have following indicators of realization quality of control method of turn (they is received by results of processing of the available measurements):

$$\delta\dot{\psi} = 0.030, \ \delta\dot{\phi} = 0.005, \ \delta\omega = 0.028$$

As criterion of reliability of identification of spacecraft's motion type in the form of the precession of solid body around some motionless direction in space **η** with constant angular velocities $\dot{\psi}$ and $\dot{\phi}$ we accept conditions: $\delta\dot{\phi} \le 0.05$, $\delta\dot{\psi} \le 0.05$, $\delta\omega \le 0.05$ inside the time interval with duration t_{nom}, under condition $t_{\text{nom}} \ge (T - \tau_{\text{ac}} - \tau_{\text{br}})/2$, where t_{nom} is supervision time; T is general time of a turn; τ_{ac} is the duration of acceleration phase; τ_{br} is the duration of braking phase.

Programmed motion $\boldsymbol{\omega}_{\text{pr}}(t)$ restored after the solution of the identification problem of angular motion has a form:

$$\begin{aligned} \omega_1(t) &= -0.03933 + 0.1769\cos\vartheta(t), \\ \omega_2(t) &= 0.1769\sin\vartheta(t)\sin(-0.03933t - 38.56^0), \\ \omega_3(t) &= 0.1769\sin\vartheta(t)\cos(-0.03933t - 38.56^0) \end{aligned} \quad (3)$$

Returnable recalculation of the vector $\boldsymbol{\omega}_{\text{pr}}$ from the coordinate system $Om_1m_2m_3$ into body-fixed coordinate system *Oxyz* will give the required functions $\omega_{i\,\text{pr}}(t)$ which reflect programmed motion of spacecraft during a turn. The Figure 3 gives visual representation of results of reconstruction of calculated motion of spacecraft at the phase of nominal rotation. Continuous lines correspond to spacecraft motion by the method [9] (they correspond to analytical model (2)); the markers correspond to direct measurements of angular velocities $\omega_x, \omega_y, \omega_z$. The given figure demonstrates that the consent of the measured values with a modeled variation of parameters of rotary motion is high enough. Root-mean-square deviations (as approximation errors) have turned out equal to:

$$\sigma_x = 0.0006 \text{ deg/s}, \ \sigma_y = 0.0018 \text{ deg/s}, \ \sigma_z = 0.0023 \text{ deg/s}.$$

The resulted estimates σ_x, σ_y, σ_z include measurement errors and errors of control (including the execution errors of control commands). Taking into account that an admissible error of attitude system Δω = 0.007 deg/s, we receive acknowledgement of the accepted law of motion:

$3\sigma < \Delta\omega$

As result, analytical form of the reconstructed functions $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$ is

$\omega_x(t) = 0.698847533\,\omega_1(t) - 0.2198905795\,\omega_2(t) + 0.6806322492\,\omega_3(t)$

$\omega_y(t) = 0.3087457564\,\omega_1(t) + 0.9510947314\,\omega_2(t)$

$\omega_z(t) = -0.6473457462\,\omega_1(t) + 0.2101423186\,\omega_2(t) + 0.73256049\,\omega_3(t)$

where $\omega_1(t)$, $\omega_2(t)$, $\omega_3(t)$ are computed by the equations (3), and ϑ (t) is constructed by formula

$$\vartheta(t) = \frac{\pi}{2} - \text{arctg}\frac{(\omega_1 - \dot{\varphi})}{\sqrt{\omega_2^2 + \omega_3^2}}$$

with use the interpolating functions for variables ω_1, ω_2, ω_3 which are calculated taking into account the transition matrix B and the interpolating polynomials for $\omega_x, \omega_y, \omega_z$ (for the set of the measurements $t_i^{(k)}$, $\omega_i^{(k)}$).

Oscillatory character of the constructed functions ω_i near their programmed values is caused by inconstancy, in particular, fluctuations of parameter ϑ which are caused by errors of execution of control commands and by nonlinearity of actuators of spacecraft's attitude system. The range of change of key parameters $\dot{\phi}$, $\dot{\psi}$, ω is insignificant, and we can conclude about their insignificant fluctuations which can be neglected.

We have considered the dynamics of turns of the multi-modular spacecraft which moments of inertia are not known precisely, and we have investigated a character of its motion. Data processing of onboard measurements was executed by a method of trained models [6]. As a result of processing of the information on angular motion of multi-modular spacecraft, actual motion around the centre of mass in the course of a three-dimensional turn is reconstructed. The presented processing results of data about angular velocities of spacecraft allow to conclude that its reorientation has been made in the form of solid-body precession at which the angular velocities $\dot{\phi}$, $\dot{\psi}$ are constant, and the angle of nutation ϑ varies slightly. Actually takes place nearly-regular precession of multi-modular spacecraft as solid body.

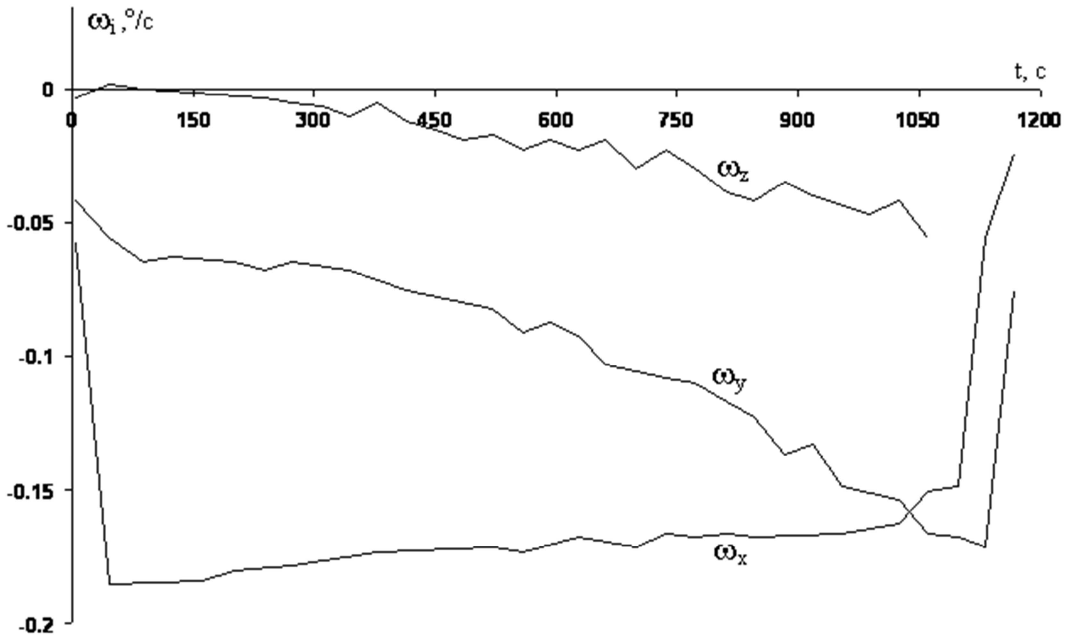

Figure 1. *Angular velocities of multi-modular spacecraft.*

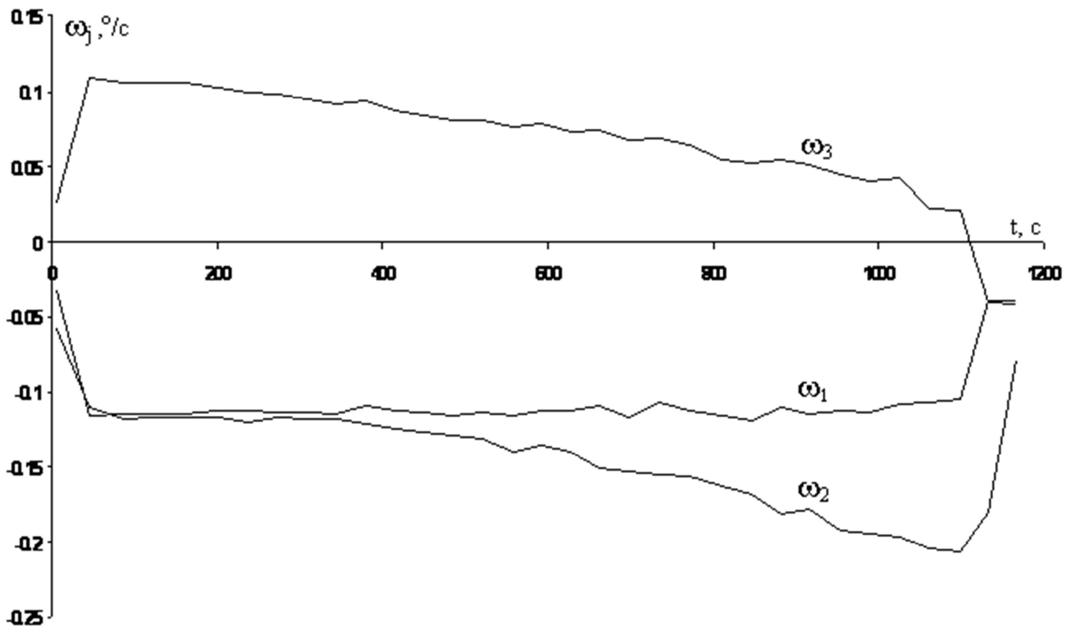

Figure 2. *Result of recalculation of angular velocities.*

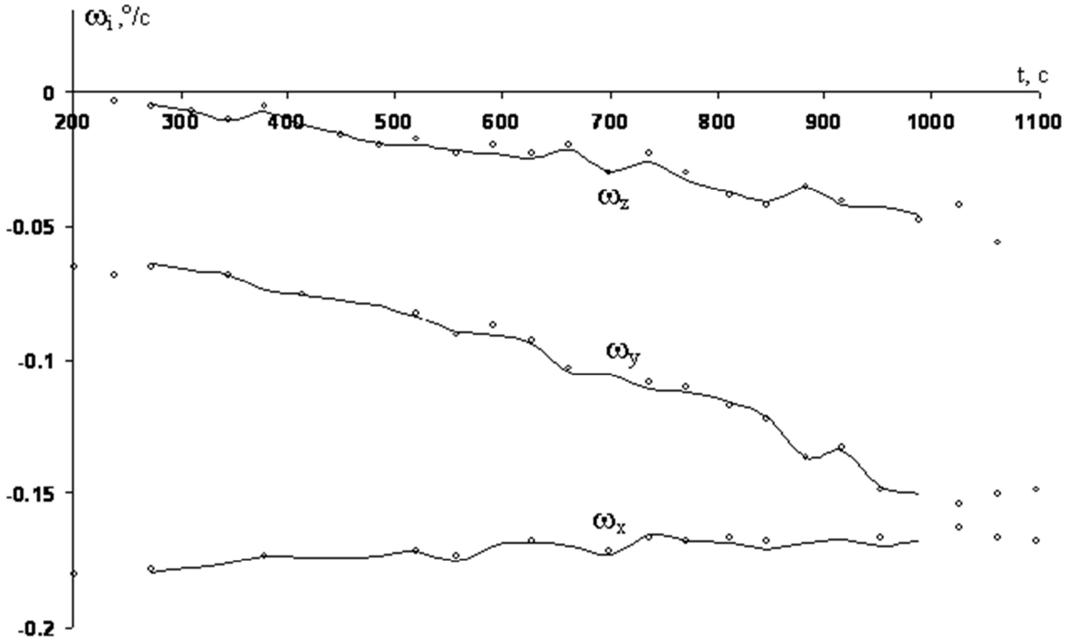

Figure 3. *Result of identification and approximation.*

5. Conclusions

In this article, the methods of mathematical analysis and its numerical methods are used for describing the controlled motion of rigid body (in particular, rotation of multi-modular spacecraft) in analytical form using results of measurements. The study of rotary motion of rigid body by methods of mathematical analysis has allowed to solve two primary tasks: a) restoration of nominal motion of angular-velocity vector formed according to a law realized by a control system; b) identification of control mode of spacecraft turn. Not less important problem consisting in a definition of numerical values of parameters of control algorithm of an orbital spacecraft in the regime of the spatial turn, defining a steered (a targeted) motion, has been in passing solved. For finding the analytical functions which describe motion of rigid body about centre of mass, the method of trained models is used.

Results of processing of the measuring information about parameters of angular motion on turns of multi-modular spacecraft are adduced. The development of a method of trained models has allowed to reconstruct programmed motion of multi-modular spacecraft in a regime of a spatial turn and authentically to identify a method by which the turn was carried out. Results of mathematical processing of onboard measurements convincingly show that the perspective method [9] has been used for turns. Is better to describe spacecraft motion inside main site of a turn (between a acceleration and a braking) as simultaneous rotation about some motionless axis in the inertial space, making with a longitudinal axis of a spacecraft a calculated angle ϑ, and about longitudinal axis of a spacecraft with quasi-constant angular rates of precession $\dot{\psi}$ and of own rotation $\dot{\phi}$. Control of a turn of orbital spacecraft was carried out by jet engines of orientation (as attitude control means) which operate in an impulse regime. Because of it occurrence of errors of working off by a control system of programmed angular velocities is inevitable. Estimations show that deviations of actual angular rates from their calculated values caused by nonlinearity of actuators can reach 0.005 – 0.01 deg/s for orbital station. For this reason, all deviations between the reconstructed functions of angular velocities and their measured values, not exceeding this size, can be fairly carried to errors of execution by attitude system of operating commands. Comparison of the analysis results of processes of orbital spacecraft turn leads to a conclusion that control of spacecraft reorientation was executed with necessary accuracy. High-precision reconstruction of controlled motion of a spacecraft allows to make identification of attitude system with the big reliability. Exact determination of numerical values of parameters of control algorithm of programmed turn of difficult spacecrafts is important for the practice.

The methods of attitude control which use rotation of solid body in the form of Euler-Poinsot motion are known [10]. Results of made research are important for investigation of this rotations, and the chosen model (2) is ideal for description of angular velocity functions when two moment of inertia have equal value (or they are close). Notice, optimal rotation obtained in the work [11] corresponds to the model (2) if $\dot{\phi}$, $\dot{\psi}$, and ϑ are constant. Also, the solution described in [12] satisfies the model (2) if two coefficients of the optimized functional are equal. This fact specifies significance of solved problem and topicality of it.

Note, the data which were obtained as result of reconstruction of continuous functions under discrete measurements by mathematical instruments, described in this article, was used in official reports of Korolev Rocket Space Corporation on analysis of onboard control system of spacecraft motion (including multi-modular orbital station).

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The developed mathematical techniques and software were used in real work and during analysis of onboard control system of spacecraft motion (including multi-modular spacecrafts and orbital stations). The author would like to express greatest thanks to colleagues who discussed (in constructive form) perspective applications of designed method for processing of measurements, and to specialists who gave recommendations and valuable suggestions about improvements of method for processing of telemetry data.

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