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A Mathematical Model for Recruitment and Developmental Sustainability of Fish Population in the Pond

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Abstract: Recruitment and sustainability for fish population are renewable natural resources, if correctly managed. The basic purpose of fish recruitment and sustainability is to provide advice on the optimum exploitation level of aquatic living resources such as fish. We formulate a mathematical model for recruitment and developmental sustainability of fish population in the pond by modifying growth model of Verhuls where we incorporate catch equation of Baranov as a function of time in the model. Runge-Kutta scheme of fourth order was used to solve the modified model. Furthermore, we collected data from Federal University Wukari fish pond to validate our modified model. We coded the Runge-Kutta scheme for our modified model by using Octave programming language, results are shown on Table 2 and figure 1, 2, 3, 4 and 5. It was observed that at P=1, P=20, P=100 and P=300 the fish recruited started increasing from 1st month to 5th month and at 6th month the fish population decrease equally because at 6th month fishes are expected to be harvested and top up. The result show that fish population recruited started increasing from first month to fifth month of recruitment and started decreasing equally at sixth month. We conclude that fish reach its maturity age at fifth month and our modified model can be use to predict expected fish population recruitment and sustainability from its initial recruitments.

Keywords: Fish Population, Pond, Recruitment and Sustainability

1. Introduction

Ecological communities are made up of a vast profusion of living things from trees to micro organism. Each species differs from every other. Furthermore, individuals within any one species are unique. Several factors that are known to affect the behaviour of each individual include genetic constitution, age, unpredictable changes of life up to that moment, and the prevailing conditions in the locality at the moment. There are many other influences that affect the individual' physiological processes. A community as a whole is being depleted by deaths and replenished by births. Often, immigration and emigration are other causes of continual turnover of individuals. As a result, the components of a community are never the same on two successive occasions. Past experience has shown that in the absence of outside disturbance most communities either remain in a steady state for long periods or pass through an orderly progression of successive stages that culminates in a steady state.

Fish is an important resources both as food (source of protein) and as a source of income to the family units and foreign exchange to the country. Fish stocks are renewable resources if correctly handled. If all mature fish are caught before spawning, then there will be no recruitment. Therefore, there should be sufficient numbers of mature fish in the stock at each time. The problem is to find the appropriate middle ground where it is possible to obtain good catches for a long time. It is necessary to estimate the size and productivity of fish, it is necessary to have measurement which relate to the stock size in some way. It is rarely possible to count the abundance of a marine species, yet it is known that measurements are made of quantities which are only indirectly related to the stock size.

Due to the profound differences in ecological systems, mathematical modelling has been found to play a key role. In an effort to determine the processes that permits maintenance of a steady state or the gradual orderly succession of states and the cause and consequences of sudden departures from steadiness, ecologists have found the need to employ mathematical models. Population modelling is a relatively old field just as the study of ecology is ([2, 3]). Some of the earlier works on population modelling include [4] who developed a mathematical model of the USA population. Later [5] independently applied mathematical models in problems of variations and fluctuation in the number of individuals and species of fish in Adriatic Sea. The results of these two works form the backbone of the modern deterministic models. In the 1930s Alexander Nicholson and victor Bailey developed a model to describe the population dynamics of a coupled predator-prey system. The model assumes that predators search for prey at random, and that both predators and prey are assumed to be distributed in a non-contiguous ("clumped") fashion in the environment.

In the late 1980s, a credible, simple alternative to the Lotka-Volterra predator-prey model emerged and the ratio dependent or Arditi-Ginzburg model. The two are the extremes of the spectrum of predator interference models. According to the authors of the alternative view, the data show that true interactions in nature are so far from the Lotka-Volterra extreme on the interference spectrum that the model can simply be discounted as wrong. They are much closer to the ratio dependent extreme, so if a simple model is needed one can use the Arditi-Ginzburg model as the first approximation.

The first principle of population dynamics is widely regarded as the exponential law of Malthus, as modelled by the Malthusian growth model. The early period was dominated by demographic studies such as the work of Benjamin Gompertz and Pierre François Verhulst in the early 19th century, who refined and adjusted the Malthusian demographic model. A more general model formulation was proposed by F. J. Richards in 1959, by which the models of Gompertz, Verhulst and also Ludwig von Bertalanffy are covered as special cases of the general formulation.

Most of the earlier models were deterministic non-linear representation of the population processes. [6]pointed out that the logistic equation of limited growth, the second most fundamental aspect of population dynamics, was first derived by [7]introduced the concept of limited population growth which have population been referred to as the logistic law [8]. Kolmogorov demonstrated the use of ordinary differential equation on single species interaction, but was also improved later by [9, 10], and [11] to include numerous species and interaction factors.

Runge-Kutta method population models was developed by two German men Carl Runge (1856 – 1927), and Martin kutta (1867 – 1944) in 1901. Carl Runge developed numerical method for solving differential equation that arose in his study of atomic spectra. In numerical analysis, the runge-Kutta methods are a family of implicit and explicit iteration methods, which includes the well-known routine called the Eulermethod, use in temporal discretization for the approximate solution of ordinary differential equation. Matrix population have had application in population modelling as in [12] who introduced stages of development of a species in the model. Runge-Kutta models have also been applied in harvesting problems, and also are applied on formulation of optimal harvesting policies. [13] found out that the stage structured models are more precise than the age based in capturing the important demographic characteristic and providing insights into a certain dynamical properties of a population that cannot be reveal. Stochastic models have also been used by various modellers to describe complex dynamics in ecological systems. [14] used stochastic model to compare patterns of density dependence and relative contribution of intrinsic versus extrinsic sources of variability to population dynamics. [15] and [16] showed that if two population are started at initial densities the smallest fraction apart, and if the sequence of stochastic events affecting the two populations is the same, then the two population will diverge through time if the dynamics are chaotic. Fisheries modelling have tended to develop independently from general population modelling ([17, 18]). Fish have a highly plastic growth and span a wide range of sizes over a single age and or stage. [19] have shown that the age estimation in fish is relatively easier than other aspects of fish. This is the reason why age structured models of fish are sometimes preferred to stage structured ones. But, [20] show that size dependent interaction provides a unifying framework for understanding mechanisms governing survival and recruitment in fishes.

The [21] age structured model only proved useful to species with distinct ages like mammals and birds, but such models have been shown to demonstrate less robustness when applied to species like fish and insect that produce large number of offspring and experience low survival rates during their early life history. [22] noted that it is extremely difficult to precisely fish mortality in the early life history stages while [23] developed elaborate models of fish habitat choice involving trade– offs of predation risk and growth in various habitat. A multi structured fishery model comprising of age and phase (or size) of population species would, then, be more appropriate. [24] applied the Leslie age structured population matrix on fish population and derived the parameters of the model for application in optimal fish harvesting.

Most of the cited works above are mainly deterministic in nature and therefore does not incorporate variability in key population parameters such as carrying capacity or natural mortality. The approach used in these methods for parameter has used classical likelihood-based mathematical theory. The problem of computing likelihood function requires high dimensional integration because it is necessary to integrate over the unobserved process errors [25]. [26] have considered both autonomous and non autonomous population models and establish that steady harvesting is for all time superior to reckless harvesting even though reckless harvesting can sometimes do as good as steady harvesting. Base on [29], steady harvesting is where a fixed number of fish were removed each year, while periodic harvesting is usually thought of as a sequence of intermittent closure and openings of different fishing grounds. Advocates of inhabitants harvesting have pointed out that constant populations of deer, fish, and other game animals, harvesting can be used to decrease the quantity of animals that needlessly die from hunger or other normal causes. On the other hand, free harvesting can lead inhabitants to the point of extinction, as is evidenced by well-known examples such as the North American Bison (Bison bison) and some populations of whales. Harvesting policy has been used to calminhabitants in surroundings with restricted resources or carrying capacity. According to [30], harvesting tactic that optimizes the entire acceptable harvest as maintaining the steady population of tilapia fish is logistic periodic (seasonal) harvesting tactic. A harvesting tactic using logistic periodic (seasonal) harvesting tactic can be used to get better productivity, cut down investment return time and decrease risk from changes in sale price of tilapia fish and expenses of productions of tilapia fish, mainly when relatively short return periods are used. The maturity of fish harvesting tactic probably can supply the market demand all through the year. It also can get better the commercial return to farmers before harvesting. The study help in raising the fish such as tilapia fishin freshwater ponds for the farmer just like any other agricultural activity. Base on [1], model results, maintenance of fish health and feed adaptation will need that the withholding time for each batch be 10 days since for the first 10 days, temperature and DO which are the mainly significant factors are still within the necessary ranges for fish survival.

But linearity and normality greatly restrict the realism and general applicability of the state-space model methodology. One way out of this problem would be to use penalized likelihood where process errors are treated as fixed parameters to be known. But this approach also has a problem in that using penalized likelihood to fit generalized linear models makes the estimates of fixed effects inconsistent when there are limited data per random effect [27]. The mathematical model approach uses the Runge-Kutta techniques for sampling from joint posterior which are very general in their application and do not rely on such assumption as linearity or normality. Mathematical model allows for the inclusion of information from diverse sources through use of prior probabilities. The thrust of this study will therefore be to attempt to apply this approach in order to devise an appropriate mathematical model for assessing stock in an aquatic population with application to a specific fish population. According to [19] the definitive study was made in 1999 by Ransom Myers. Myers solved the problem "by assembling a large base of stock data and developing a complex mathematical model to sort it out. Out of that came the conclusion that a female in general produced three to five recruits per year for most fish. Fish managers are interested in knowing how a sustainability of fish can be utilised in the long run or what will be the likely consequences of a certain actions as regarded catches and development of fish stock. A good mechanism for monitoring fish stock and putting into consideration of fish characteristics such as sex, age, species, morphology and even predation effect can help one devise technique that will optimize the fish breeding and harvesting. Fish production and consumption have attracted some significant interest in this 21st century of its low cholesterol. Fish producers are interested on the best time to harvest and replenish in the pond with minimal or no expenses. In view of meeting the public needs, we modified growth model by [7] incorporative catch equation of [28] in the model. Where the modified model is for recruitment and development of fish sustainable at various stages of life and empirical data are used to verify the model variables and parameters.

2. Method

2.1. Modified Model Equation

We modified growth model by verhulsts (1838) equation (1) by incorporating catch equation (3), here is the modified equation (2)

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{N}\right) \tag{1}$$

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{N}C\right) \tag{2}$$

Where
P = population size of fish in tones
r= growth rate
N = carrying capacity
C =catch of fish
t= time period
H = harvesting of fish
M=Natural mortality
Where the catch model is given below;

$$C = \frac{H}{H+M}(1 - e^{-}(H+M)t)p(t) \tag{3}$$

2.2. Assumption of the Modified Model

In general, the following are the assumptions of our modified model.

(i) If the initial population is less than the environmental carrying capacity, population will monotonically increase towards the carrying capacity.
(ii) The supply of resources such as food and space are limited.
(iii) Growth rate decreases as the population is sufficiently large.
(iv) Growth rate increases as the population is sufficiently small.
(v) An increase in growth and size results in a decrease in

mortality because mortality is a function of size.

(vi) Fishes reproduce during the period of consideration

2.3. Numerical Method

We use Runge-kutta method to solve the modified aquatic population model. Below is the description of the Runge-kutta method which consists of determine appropriate constants so that a formula such as:

$$p_{n+1} = p_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = hf\left(x_n, y_n\right)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, yn + \frac{k_2}{2}\right)$$

$$k_4 = hf\left(x_n + h,, y_n + k_3\right)$$

3. Results

Implementation of the Modified Fish Population Model

In this section, we use data collected from the Federal University Wukari fish pond to implement the modified model as shown in Table 1. The period of maturity for the fish is 5 months and about 80% will survive to maturity.

Table 1. *Data Collected from Federal University Wukari Fish Pond.*

NATURAL MORTALITY (M)	0.1
CARRYING CAPACITY (N)	2000Kg
STEP SIZE (h)	0.1
PERIOD OF TIME (t)	1 to 5 MONTHS
RATE OF GROWTH (r)	0.5
HARVESTING OF FISH (H)	0.85

From the data collected from the fish pond in the Federal University Wukari, weusingRunge-Kutta scheme to solve equation (2).

For n=0

$$P_{n+1} = P_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$C_1 = \frac{H}{H+M}\left(1 - \ell^{-(H+M)t}\right)P(t)$$

$$C_1 = \frac{0.85}{0.95}\left(1 - \ell^{-1.8}\right)$$

$$C_1 = 0.895(0.835)$$

$$C_1 = 0.7473$$

$$k_1 = h\left(rP_0\left(1 - \frac{P_0}{N}C\right)\right)$$

$$k_1 = 0.04998$$

$$k_2 = h\left(r\left(P_0 + \frac{k_1}{2}\right)\left(1 - \frac{P_0 + \frac{k_1}{2}}{2000}C\right)\right)$$

$$k_2 = 0.05122$$

$$k_3 = h\left(r\left(P_0 + \frac{k_2}{2}\right)\left(1 - \frac{P_0 + \frac{k_2}{2}}{2000}C\right)\right)$$

$$k_3 = 0.0513$$

$$k_4 = h\left(r\left(P_0 + k_3\right)\left(1 - \frac{P_0 + k_3}{2000}C\right)\right)$$

$$k_4 = 0.05254$$

Substitute k_1, k_2, k_3 and k_4,

$$P_{n+1} = P_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$P_1 = 1 + \frac{1}{6}\left(0.04998 + 2(0.05122) + 2(0.0513) + 0.05254\right)$$

$$P_1 = 1.05126$$

For n=1

$$P_2 = P_1 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$P_2 = 1.05126 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$C_2 = \frac{H}{H+M}\left(1-\ell^{-(H+M)t}\right)P_1(t)$$

$$C_2 = \frac{0.85}{0.85+0.1}\left(1-\ell^{-(0.95)2}\right)1.05126$$

$$C_2 = 0.800$$

$$k_1 = hf\left(rP_1\left(1-\frac{P_1}{N}C\right)\right)$$

$$k_1 = 0.1\left(0.5(1.05126)\left(1-\frac{1.05126(0.800)}{2000}\right)\right)$$

$$k_1 = 0.05254$$

$$k_2 = hf\left(r\left(P_1+\frac{k_1}{2}\right)\left(1-\frac{P_1+\frac{k_1}{2}}{2000}C\right)\right)$$

$$k_2 = 0.1\left(0.5\left(1.05126+\frac{k_1}{2}\right)\left(1-\frac{1.05126+\frac{0.05126(0.800)}{2}}{2000}\right)\right)$$

$$k_2 = 0.0538$$

$$k_3 = hf\left(r\left(P_1+\frac{k_2}{2}\right)\left(1-\frac{P_1+\frac{k_2}{2}}{2000}C\right)\right)$$

$$k_3 = 0.1\left(0.5\left(1.05126+\frac{0.0538}{2}\right)\left(1-\frac{1.05126+\frac{0.0538(0.800)}{2}}{2000}\right)\right)$$

$$k_3 = 0.0539$$

$$k_4 = hf\left(r(P_1+k_3)\left(1-\frac{P_1+k_3}{2000}C\right)\right)$$

$$k_4 = 0.1\left(0.5(1.05126+0.0539)\left(1-\frac{1.05126+0.0539(0.800)}{2000}\right)\right)$$

$$k_4 = 0.05523$$

Substitute k_1, k_2, k_3 and k_4

$$P_2 = 1.05126+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$P_2 = 1.05126+\frac{1}{6}(0.05254+2(0.0538)+2(0.0539)+0.05523)$$

$$P_2 = 1.10512$$

For n=2

$$C_3 = \frac{H}{H+M}\left(1-\ell^{-(H+M)t}\right)P_2(t)$$

$$C_3 = \frac{0.85}{0.85+0.1}\left(1-\ell^{-(0.95)3}\right)1.10512$$

$$C = 0.895\left(1-\ell^{-2.685}\right)1.10512$$

$$C = 0.895(0.9318)1.10512$$

$$C_3 = 0.922$$

$$k_1 = hf\left(r\left(P_2\left(1-\frac{P_2}{n}C\right)\right)\right)$$

$$k_1 = 0.1\left(0.5(1.10512)\left(1-\frac{1.10512(0.922)}{2000}\right)\right)$$

$$k_1 = 0.05523$$

$$k_2 = hf\left(r\left(P_2+\frac{k_1}{2}\right)\left(1-\frac{P_2+\frac{k_1}{2}}{2000}C\right)\right)$$

$$k_2 = 0.1\left(0.5\left(1.10512+\frac{0.05523}{2}\right)\left(1-\frac{1.10512+\frac{0.05523(0.922)}{2}}{2000}\right)\right)$$

$$k_2 = 0.05660$$

$$k_3 = h\left(r\left(P_2+\frac{k_2}{2}\right)\left(1-\frac{P_2+\frac{k_2}{2}}{2000}C\right)\right)$$

$$k_3 = 0.1\left(0.5\left(1.10512+\frac{0.05660}{2}\right)\left(1-\frac{1.10512+\frac{0.05660(0.922)}{2}}{2000}\right)\right)$$

$$k_3 = 0.05664$$

$$k_4 = hf\left(r(P_2+k_3)\left(1-\frac{P_2+k_3}{2000}C\right)\right)$$

$$k_4 = 0.0581$$

Substitute k_1, k_2, k_3 and k_4

$$p_3 = 1.10512 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$p_3 = 1.10512 + \frac{1}{6}\left(0.05523 + 2\left(0.05660\right) + 2\left(0.05664\right) + 0.0581\right)$$

$$p_3 = 1.1618$$

For n=3

$$C_4 = \frac{H}{H+M}\left(1 - \ell^{-(H+M)t}\right)P_3\left(t\right)$$

$$C_4 = \frac{0.85}{0.85+0.1}\left(1 - \ell^{-(0.95)4}\right)1.1618$$

$$C_4 = 1.017$$

$$P_4 = P_3 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$P_4 = 1.1618 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = hf\left(r\left(P_3\left(1 - \frac{P_3}{n}C_4\right)\right)\right)$$

$$k_1 = 0.1\left(0.5\left(1.1618\right)\left(1 - \frac{1.1618\left(1.017\right)}{2000}\right)\right)$$

$$k_1 = 0.0581$$

$$k_2 = hf\left(r\left(P_3 + \frac{k_1}{2}\right)\left(1 - \frac{P_3 + \frac{k_1}{2}}{2000}C_4\right)\right)$$

$$k_2 = 0.1\left(0.5\left(1.1618 + \frac{0.0581}{2}\right)\left(1 - \frac{1.1618 + \frac{0.0581\left(1.017\right)}{2}}{2000}\right)\right)$$

$$k_2 = 0.5951$$

$$k_3 = h\left(r\left(P_3 + \frac{k_2}{2}\right)\left(1 - \frac{P_3 + \frac{k_2}{2}}{2000}C_4\right)\right)$$

$$k_3 = 0.1\left(0.5\left(1.1618 + \frac{0.5951}{2}\right)\left(1 - \frac{1.1618 + \frac{0.5951\left(1.071\right)}{2}}{2000}\right)\right)$$

$$k_3 = 0.0732$$

$$k_4 = hf\left(r\left(P_3 + k_3\right)\left(1 - \frac{P_3 + k_3}{2000}C_4\right)\right)$$

$$k_4 = 0.1\left(0.5\left(1.1618 + 0.0732\right)\left(1 - \frac{1.1618 + 0.0732\left(1.017\right)}{2000}\right)\right)$$

$$k_4 = 0.0617$$

Substitute k_1, k_2, k_3 and k_4

$$P_4 = 1.1618 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$P_4 = 1.1618 + \frac{1}{6}\left(0.0581 + 2\left(0.5951\right) + 2\left(0.0732\right) + 0.0617\right)$$

$$P_4 = 1.4045$$

Table 2. *Show solve data from the Modified Model at various Populations (P).*

TIME OF PERIOD IN MONTH (n or t)	P=1	P=20	P=100	P=200	P=300
0	1.0000	20.0000	100.0000	200.0000	300.0000
1	1.0526	21.0624	106.8290	224.9250	390.6505
2	1.1079	22.1733	113.1424	243.7697	457.7666
3	1.1662	23.3392	119.3692	259.7484	505.2920
4	1.2275	24.5647	125.7258	274.6557	542.4638
5	1.2920	25.8537	132.3218	289.4248	575.0297
6	1.3600	1.3600	1.3600	1.3600	1.3600
7	1.4315	1.4315	1.4315	1.4315	1.4315
8	1.5067	1.5067	1.5067	1.5067	1.5067
9	1.5860	1.5860	1.5860	1.5860	1.5860
10	1.6693	1.6693	1.6693	1.6693	1.6693
11	1.7571	1.7571	1.7571	1.7571	1.7571
12	1.8495	1.8495	1.8495	1.8495	1.8495

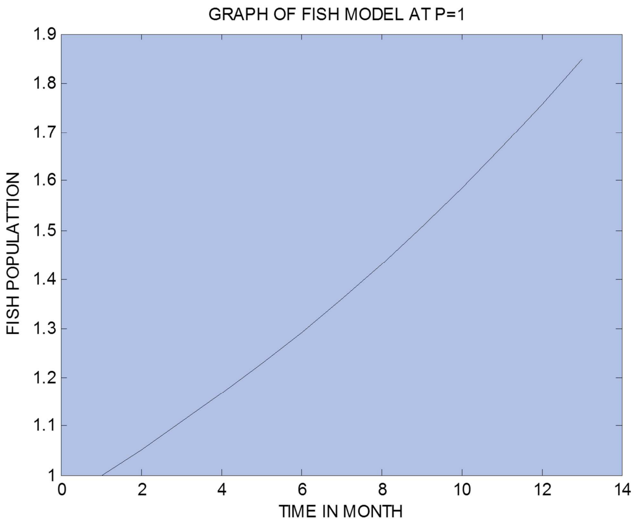

Figure 1. *Graphof Fish Model at P=1.*

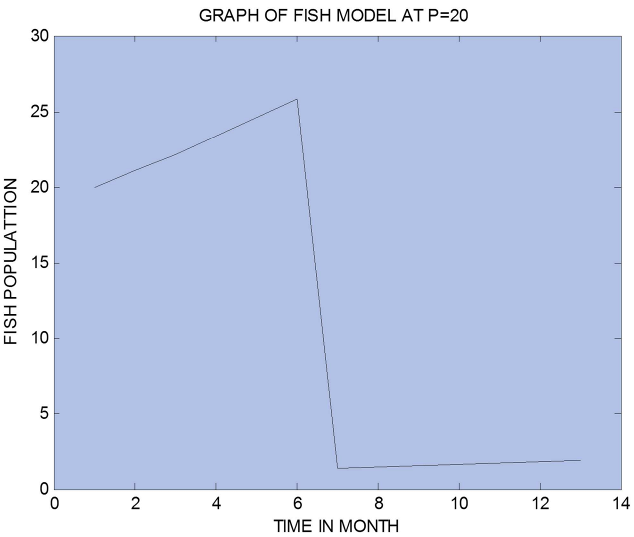

Figure 2. *Graphof Fish Model at P=20.*

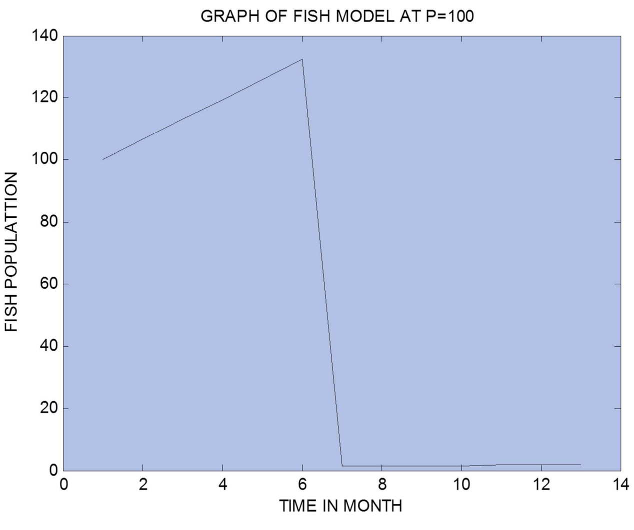

Figure 3. *Graphof Fish Model at P=100.*

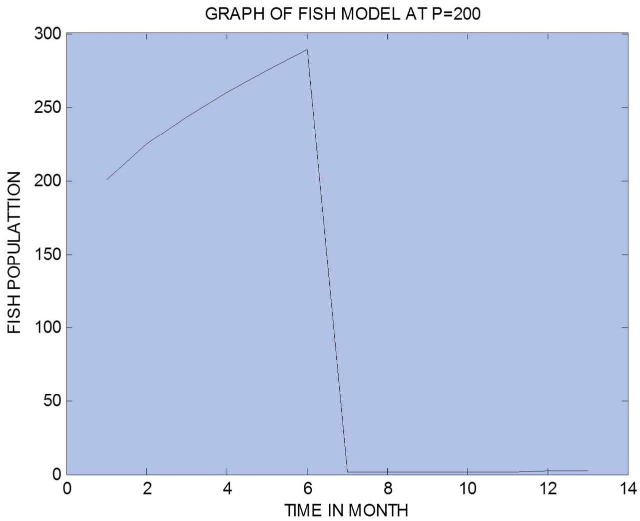

Figure 4. *Graphof Fish Model at P=200.*

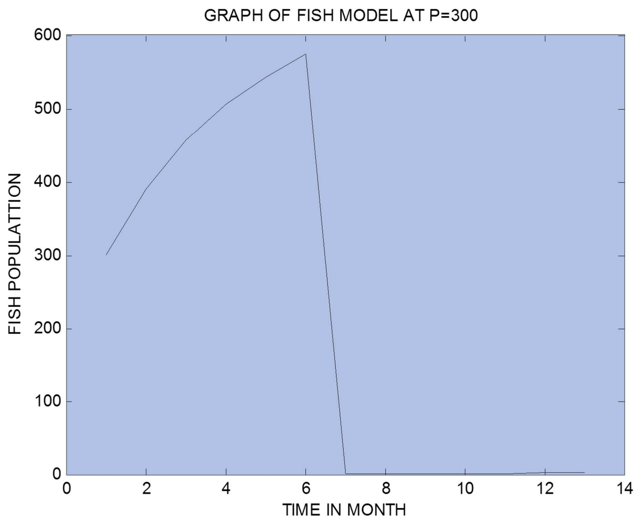

Figure 5. *Graphof Fish Model at P=300.*

4. Discussions

We coded the Runge-Kutta scheme for our modified model by using Octave programming language, results are shown on Table 2 and figure 1, 2, 3, 4 and 5. It was observed that at $P=1$, $P=20$, $P=100$, $P=200$ and $P=300$ the fish recruited started increasing from 1st month to 5th month and at 6th month the fish population decrease equally because at 6th month fishes are expected to be harvested and top up. Furthermore, at $P=1$, there is increase of one, $P=20$, there is increase of six, $P=100$, there is increase of thirty-two, $P=200$, there is increase of eighty nine and $P=300$, here is increase of two hundred and seventy five. These increments will inform fish producer the amount to add into the stock to meet up with initial recruitment.

5. Conclusion

In view of the discussion, we conclude that fish reach it maturity stage from 5th month of recruitment and harvesting should start at that period to enable sustainability of fish cycle in the pond. Our model is only viable for fish in the pond were all surviving factors are in place. Also, the model can be improved on by considering other aquatic organisms, so that the modified model can be generalised for aquatic organisms. In conclusion, our modified model can be use to predict expected fish sustainability from its initial recruitments.

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