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On a Problem Arising in a Two-Fluid Medium

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Abstract: A nonlinear system of equations describing the dynamics of the motion of two-phase fluids is considered. An initial-boundary value problem for the systems of viscous two-fluid media with phase equilibrium with respect to pressure whose solution or some integral characteristic of this solution becomes infinite in a finite time (blow-up) is investigated. Using the method of test functions proposed by Pokhozhaev and Mitidieri, the influence of boundary and initial conditions on the appearance, time and rate of destruction of solutions to these problems is examined. The theorem on the blow-up of the solution to the initial-boundary value problem is proved. The lifetime of the solution is estimated.

Keywords: Two-Speed Hydrodynamics, Mineralization, Porous Medium, Initial-Boundary Value Problem, Destruction in a Finite Time, Blow-Up

1. Introduction

The study of the influence of mineralization on the dynamic properties of heterophase media, on the character of filtration and on the propagation of acoustic waves in porous media is relevant for a wide class of applied problems. The salinity of the saturating fluid can lead to an increase in the effective viscosity [1], affecting the attenuation pattern of the bulk and surface waves [2]. Consideration of the salinity of the saturating porous fluid medium is necessary when designing models of modern technological systems. In this paper, using the method of conservation laws [3], the matched equations of filtration of mineralized liquids in viscoelastic porous media are obtained. The method based on matching the principles of thermodynamics, conservation laws, and group invariance of equations has been successfully used to study saturated porous media, including the presence of an impurity [4-7].

In describing processes in physics, geophysics, biology, chemistry, and a number of other sciences, nonlinear mathematical models are used which are based on nonlinear systems of partial differential equations. A detailed study of nonlinear mathematical models, as a rule, is carried out using computational experiments, but the search for possible analytical solutions is also an important stage of research. This is explained by the fact that in the numerical simulation of problems, errors may appear that can be eliminated by comparing the results with analytical solutions. Analytical studies of nonlinear models are often used in analyzing the stability of difference schemes for which calculations are made. In addition, the found analytical solutions of mathematical models are in their own way extremely useful, since in some cases their knowledge simplifies the understanding of the studied physical processes and allows us to estimate the role of certain parameters of the problem to be solved.

To study the destruction of a solution to a system of equations of a two-fluid medium with phase equilibrium with respect to pressure, we use the method developed by S. I. Pokhozhaev, E. Mitidieri, and V. A. Galaktionov, which was called the method of test functions, or the method of nonlinear capacity. For a greater detail about the capabilities of this method, see [8-13]. In particular, papers [14-, 17] deal with the application of test functions of a special type to hydrodynamics problems.

2. Dynamic Equations of a Mineralized Fluid in a Porous Medium

The model presented in [18] to describe the deformations in the porous core uses an effective deformation tensor and does not assume the constancy of the discharge density [19]. It is assumed that the porous core combined with the mineralized saturating fluid constitute a two-velocity continuum, where unit volume is characterized by the strain tensor, the density of the solid phase and the liquid phase containing the admixture, the corresponding velocities u_1, u_2, the entropy density S. The impurity content is described by the density $\rho_c = \rho c$, where $\rho = \rho_1 + \rho_2$, c is the impurity concentration. In this case, the internal energy of a unit volume of the two-phase medium E_0 is determined by the relation [6]:

$$dE_0 = TdS + \mu_r d\rho + qd\rho_1 + \mu_c d\rho_c + \frac{1}{2}\eta_{ik} d\varepsilon_{ik} + (\mathrm{u}_1 - \mathrm{u}_2, d\mathrm{j}_0). \quad (1)$$

Here μ_r, q, μ_c are the chemical potentials; T is the temperature; η_{ik} is the traceless stress tensor and ε_{ik} is the deformation tensor of the porous core; $\mathrm{j}_0 = \mathrm{j} - \rho \mathrm{u}_2$ is the relative momentum, $\mathrm{j} = \rho_1 \mathrm{u}_1 + \rho_2 \mathrm{u}_2$ is the momentum of the medium. Energy is determined by the formula $E = E_0 + (\mathrm{u}_2, \mathrm{j}_0) + \frac{1}{2}\rho \mathrm{u}_2^2$.

The evolution of the density of a two-phase medium, the partial density of the solid phase and impurities are determined by the mass conservation laws

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\mathbf{j} = 0,$$
$$\frac{\partial \rho_1}{\partial t} + \mathrm{div}(\rho_1 \mathbf{u}_1) = 0, \quad (2)$$
$$\frac{\partial \rho_c}{\partial t} + \mathrm{div}\left(\frac{\rho_c}{\rho}\mathbf{j}\right) = 0.$$

As a consequence, the law of conservation of mass of the liquid phase is valid. The laws of conservation of momentum, energy, and entropy are also fulfilled (without dissipation)

$$\frac{\partial j_i}{\partial t} + \partial_i \Pi_{ik} = 0,$$
$$\frac{\partial E}{\partial t} + \mathrm{div}\mathrm{Q} = 0, \quad (3)$$
$$\frac{\partial S}{\partial t} + \mathrm{div}\left(\frac{S}{\rho}\mathrm{j}\right) = 0,$$

where Π_{ik} is the momentum density tensor, E is the total energy per unit volume, Q is the energy flow. The form of the equation of motion of the liquid phase is determined by the conditions of thermodynamic equilibrium:

$$\nabla \mu_c = 0, \nabla \mu_r = 0,$$
$$\nabla T = 0, \mathrm{u}_1 = \mathrm{u}_2, \quad (4)$$
$$\frac{\partial \mathrm{u}_2}{\partial t} + (\mathrm{u}_2, \nabla)\mathrm{u}_2 = -\nabla \mu_r - \frac{\rho_c}{\rho}\nabla \mu_c - \frac{S}{\rho}\nabla T.$$

The evolution of deformations is described according to [19]:

$$\frac{\partial \varepsilon_{ik}}{\partial t} + u_{1,j}\,\partial_j \varepsilon_{ik} - \frac{1}{2}\left(\delta_{kj} - \varepsilon_{kj}\right)\partial_i u_{1,j} - \frac{1}{2}\left(\delta_{ij} - \varepsilon_{ij}\right)\partial_k u_{1,j} = 0. \quad (5)$$

The flows and the pressures in the context of the method of conservation laws are uniquely determined [4, 6]:

$$Q_i = \left(\mu + TS/\rho + \mathrm{u}_2^2/2 - (\mathrm{u}_1, \mathrm{u}_2)\right) j_i + \left(p_1 \delta_{ik} + (\mathrm{u}_1, \mathrm{j} - \rho \mathrm{u}_2)\delta_{ik} - 2h_{ik}\varepsilon_{kj}\right) u_{1,j}, \quad (6)$$

$$\Pi_{ik} = \rho_1 u_{1,i} u_{1,k} + \rho_2 u_{2,i} u_{2,k} + p\,\delta_{ik} + p_1\,\delta_{ik} + \eta_{ik}\varepsilon_{ik} \quad (7)$$

$$p = -E_0 + TS + \mu\,\rho + \mu_c\,\rho_c + (\mathrm{u}_1 - \mathrm{u}_2)(\mathrm{j} - \rho \mathrm{u}_2), \quad (8)$$

where $p_1 = \rho_1 q$. The stresses arising in the porous core are described by the tensor

$$\sigma_{ik} = -p_1\,\delta_{ik} - \eta_{ik}\varepsilon_{ik}.$$

Taking into account the dissipative processes leads to the appearance of additional flows in equations (2)-(5)

$$\frac{\partial j_i}{\partial t} + \partial_k\left(\Pi_{ik} + \pi_{u,ik}\right) = 0,$$
$$\frac{\partial E}{\partial t} + \mathrm{div}(\mathrm{Q} + \mathrm{f}_e) = 0,$$
$$\frac{\partial S}{\partial t} + \mathrm{div}\left(\frac{S}{\rho}\mathrm{j} + \mathrm{f}_s\right) = \frac{R}{T}, \quad (9)$$
$$\frac{\partial \rho_c}{\partial t} + \mathrm{div}\left(\frac{\rho_c}{\rho}\mathrm{j} + \mathrm{f}_c\right) = 0,$$

$$\frac{\partial \varepsilon_{ik}}{\partial t} + u_j\, \partial_j \varepsilon_{ik} - \frac{1}{2}\left(\delta_{kj} - \varepsilon_{kj}\right) \partial_i u_{1,j} - \frac{1}{2}\left(\delta_{ij} - \varepsilon_{ij}\right) \partial_k u_{1,j} = -\pi_{e,ik}.$$

Here f_e, f_s are the irreversible flows of energy and entropy; f_u is the interphase friction force; $\pi_{u,ik}$ is the irreversible flow of momentum; $\pi_{e,ik}$ is the flow describing the relaxation of tangential stresses [5]; R is the entropy production.

The procedure of matching equations (2) and (9) with the first law of thermodynamics (1), taking into account the definition of energy, leads to finding the irreversible fluxes and the dissipative function [20]

$$R = -\left(f_{u,i} + \tfrac{1}{\rho_2}\partial_k \pi_{ik}\right)\left(\boldsymbol{j}_i - \rho u_{1,i}\right) - f_{s,i}\tfrac{1}{T}\partial_i T - f_{c,i} T\ \partial_i\left(\tfrac{\mu_a}{T}\right) - \pi_{u,ik} u_{2,ik} - \pi_{e,ik}\eta_{ik}. \quad (10)$$

Here $u_{2,ik} = \frac{1}{2}\left(\partial_k u_{2,i} + \partial_i u_{2,k} - \frac{2}{3}\delta_{ik}\ \mathrm{div}\ \mathrm{u}_2\right)$.

The effects of volume viscosity and compactification of a porous core are not discussed in this paper; the introduction of the corresponding scalar flows into the model within the framework of this approach is obvious. The structure of the dissipative function allows one to introduce the linear dissipative relations for the flows of vectorial and tensor nature [20]:

$$-f_{u,i} = \lambda_{11}(j_i - \rho u_{1i}) + \lambda_{12}\frac{\partial_i T}{T} + \lambda_{13} T\, \partial_i\left(\frac{\mu_a}{T}\right) + \frac{1}{\rho_2}\partial_k \pi_{2ik}, \quad (11)$$

$$f_{s,i} = -\lambda_{21}(\boldsymbol{j}_i - \rho u_{1i}) - \lambda_{22}\frac{\partial_i T}{T} - \lambda_{23} T\, \partial_i\left(\frac{\mu_a}{T}\right), \quad (12)$$

$$f_{c,i} = -\lambda_{31}(\boldsymbol{j}_i - \rho u_{1i}) - \lambda_{32}\frac{\partial_i T}{T} - \lambda_{33} T\, \partial_i\left(\frac{\mu_a}{T}\right), \quad (13)$$

$$-\pi_{u,ik} = \zeta_{11}\, u_{2,ik} + \zeta_{12}\, \eta_{ik}, -\pi_{e,ik} = \zeta_{21}\, u_{2,ik} + \zeta_{22}\, \eta_{ik}. \quad (14)$$

The obtained equations of the dynamics of saturated porous media with the admixture in the case of small deformations and neglecting some cross effects can be represented as

$$\begin{gathered}
\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho_1 \mathrm{u}_1 + \rho_2 \mathrm{u}_2) = 0, \\
\frac{\partial \rho_2}{\partial t} + \mathrm{div}(\rho_2 \mathrm{u}_2) = 0, \\
\frac{\partial \rho_c}{\partial t} + \mathrm{div}(c\mathrm{j}\ - \lambda\ (\mathrm{u}_1 - \mathrm{u}_2)) = \mathrm{div}(\rho D \nabla c), \\
\frac{\partial \varepsilon_{ik}}{\partial t} - \frac{1}{2}\partial_i u_{1,k} - \frac{1}{2}\partial_k u_{1,i} = \frac{1}{\tau}\ \varepsilon_{ik}, \\
\frac{\partial j_i}{\partial t} + \partial_k\left(\rho_1 u_{1,i} u_{1,k} + \rho_2 u_{2,i} u_{2,k} + p\ \delta_{ik} + p_1\ \delta_{ik} + \eta_{ik}\varepsilon_{ik}\right) = \partial_k\left(\mu\ u_{2,ik}\right), \\
\frac{\partial \mathrm{u}_2}{\partial t} + (\mathrm{u}_2, \nabla)\mathrm{u}_2 = -\frac{1}{\rho}\nabla p + \frac{\rho_1}{\rho}\nabla\left(\frac{p_1}{\rho_1}\right) + \frac{\rho_1}{2\rho}\nabla(\mathrm{u}_1 - \mathrm{u}_2)^2 + \frac{\eta_{ik}}{\rho}\nabla \varepsilon_{ik} + \\
+b(\mathrm{u}_1 - \mathrm{u}_2) + \frac{1}{\rho_2}\nu\ \nabla T + \frac{1}{\rho_2}\partial_k\left(\eta\ u_{2,ik}\right), \\
\frac{\partial s}{\partial t} + \frac{1}{\rho}(\mathrm{j}, \nabla)s = -\frac{1}{\rho}\mathrm{div}\left(\kappa \frac{1}{T}\nabla T + \nu(\mathrm{u}_1 - \mathrm{u}_2)\right) + \frac{R}{\rho T}.
\end{gathered} \quad (15)$$

Here μ is the dynamic viscosity of the saturating fluid, D is the diffusion coefficient, τ is the characteristic relaxation time of tangential stresses, b is the coefficient of interphase friction, κ is the thermal conductivity of the two-phase medium, $\nu = \lambda_{12}$, $\lambda = \rho_1 \lambda_{33}$. The thermodynamics of the two-phase medium is given by the functional dependence $e_0 = e_0(\rho_1, \rho_2, \rho_c, \varepsilon_{ik}, \mathrm{u}_1 - \mathrm{u}_2, s)$,), whose specific choice determines the pressures p, p_1, the temperature T and the stress tensor σ_{ik}. The equation of state of the two-phase medium closes the full dynamic equations (15). Thus, the obtained system of the full dynamic equations determines a thermodynamically consistent model of heat and mass transfer in the porous media saturated with a mineralized liquid.

3. The Blow-Up Solution of the Initial-Boundary Value Problem Arising in the Incompressible Two-Velocity Medium Equation

In this section, we consider the initial-boundary value problem for the two-phase equation (15). We will neglect the salt concentration effect on the motion of fluids. Both phases are considered to be viscous and incompressible without regard for tangential stresses. Thus, let us consider the following system of differential equations:

$$\mathrm{div}\,(\rho_1 \mathrm{u}_1)=0, \mathrm{div}\,(\rho_2 \mathrm{u}_2)=0, \quad (16)$$

$$\frac{\partial \mathrm{u}_1}{\partial t}+(\mathrm{u}_1,\nabla)\mathrm{u}_1=-\frac{\nabla p}{\bar\rho}+\nu_1\,\Delta \mathrm{u}_1+\frac{\rho_2}{2\bar\rho}\nabla(\mathrm{u}_2-\mathrm{u}_1)^2, \quad (17)$$

$$\frac{\partial \mathrm{u}_2}{\partial t}+(\mathrm{u}_2,\nabla)\mathrm{u}_2=-\frac{\nabla p}{\bar\rho}+\nu_2\,\Delta \mathrm{u}_2-\frac{\rho_1}{2\bar\rho}\nabla(\mathrm{u}_2-\mathrm{u}_1)^2, \quad (18)$$

where u_1 and u_2 are the velocity vectors of the subsystems of the components of the two-velocity continuum with the respective partial densities ρ_1, ρ_2; ν_1, ν_2 are the respective kinematic viscosities, $\bar\rho=\rho_1+\rho_2$ is the total density of the two-velocity continuum, and $p=p(\bar\rho,(\mathrm{u}_2-\mathrm{u}_1)^2)$ is the equation of state of the two-velocity continuum.

Let us itroduce the cylindrical coordinates (r,φ,z). For the above system of equations, in the cylindrical domain $\Omega=[0,R]\times[0,2\pi]\times[0,L]$ let us investigate the following initial-boundary value problem with the initial conditions

$$\mathrm{u}_1|_{t=0}=\mathrm{u}_{1,0}(r,\varphi,z), \mathrm{u}_2|_{t=0}=\mathrm{u}_{2,0}(r,\varphi,z), \quad (19)$$

and the boundary conditions

$$u_{2,r}(R,\varphi,z)=u_{2,r}(0,\varphi,z)=0,\ u_{2,z}(r,\varphi,0)=0,$$
$$\int_0^{2\pi}\int_0^L R\,(z-L)\frac{\partial u_{2,z}}{\partial r}(R,\varphi,z)d\varphi dz-$$
$$-\int_0^{2\pi}\int_0^R\left(u_{2,z}(r,\varphi,L)+L\,\frac{\partial u_{2,z}}{\partial z}(r,\varphi,0)\right)d\varphi dr=h_2(t), \quad (20)$$

$$u_{1,z}(r,\varphi,0)=0, u_{1,r}(R,\varphi,z)=u_{1,r}(0,\varphi,z)=0,$$
$$\int_0^{2\pi}\int_0^L R\,(z-L)\,\frac{\partial u_{1,z}}{\partial r}(R,\varphi,z)d\varphi dz- \quad (21)$$
$$-\int_0^{2\pi}\int_0^R\left(u_{1,z}(r,\varphi,L)+L\,\frac{\partial u_{1,z}}{\partial z}(r,\varphi,0)\right)d\varphi dr=h_1(t).$$

In formulas (20) and (21), the functions h_1,h_2 are continuous functions on the interval $[0,\infty)$.

Let us choose the test function

$$\mathrm{u}=(0,0,z-L), z\in[0,L]. \quad (22)$$

By virtue of boundary conditions (20), (21) in the class $\mathrm{u}_1(\mathrm{x},t),\mathrm{u}_2(\mathrm{x},t)\in C^1((0,T];\ \mathrm{C}^{(2)}(\bar\Omega))$ $(\mathrm{C}^{(2)}(\bar\Omega)=C^{(2)}(\bar\Omega)\times C^{(2)}(\bar\Omega)\times C^{(2)}(\bar\Omega))$, we have the following relations:

$$\int_\Omega \Delta\,\mathrm{u}_1\mathrm{u}d\mathrm{x}=\int_\Omega\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial u_{1,z}}{\partial r}\right)+\frac{1}{r^2}\,\frac{\partial^2 u_{1,z}}{\partial \varphi^2}+\frac{\partial^2 u_{1,z}}{\partial z^2}\right)(z-L)\,r\,dr\,d\varphi\,dz$$
$$=\int_0^{2\pi}\int_0^L r\,\frac{\partial u_{1,z}}{\partial r}|_0^R(z-L)d\varphi dz+\int_0^R\int_0^L\frac{1}{r}\,\frac{\partial u_{1,z}}{\partial\varphi}|_0^{2\pi}(z-L)drdz \quad (23)$$
$$+\int_0^{2\pi}\int_0^R\left((z-L)\,\frac{\partial u_{1,z}}{\partial z}-u_{1,z}\right)|_0^L d\varphi dr=\int_0^{2\pi}\int_0^L R\,\frac{\partial u_{1,z}}{\partial r}(R,\varphi,z)(z-L)d\varphi dz$$
$$-\int_0^{2\pi}\int_0^R\left(u_{1,z}(r,\varphi,L)+L\,\frac{\partial u_{1,z}}{\partial z}(r,\varphi,0)\right)d\varphi dr=h(t),$$

$$\int_\Omega(\mathrm{u}_1,\nabla)\mathrm{u}_1\mathrm{u}dx=\int_\Omega\left(u_{1,r}\,\frac{\partial u_{1,z}}{\partial r}+\frac{1}{r}\,u_{1,\varphi}\,\frac{\partial u_{1,z}}{\partial\varphi}+u_{1,z}\,\frac{\partial u_{1,z}}{\partial z}\right)\,(z-L)\,r\,dr\,d\varphi\,dz$$
$$=\int_\Omega\left(\frac{\partial}{\partial r}(r\,u_{1,r}\,u_{1,z})+\frac{\partial}{\partial\varphi}(u_{1,\varphi}u_{1,z})+r\,u_{1,z}\,\frac{\partial u_{1,z}}{\partial z}\right)\,(z-L)\,dr\,d\varphi\,dz$$
$$-\int_\Omega\left(\frac{\partial(r\,u_{1,r})}{\partial r}+\frac{\partial u_{1,\varphi}}{\partial\varphi}\right)\,u_{1,z}\,(z-L)dr\,d\varphi\,dz- \quad (24)$$
$$=\int_0^{2\pi}\int_0^L r\,u_{1,r}\,u_{1,z}|_0^R(L-z)d\varphi dz+\int_0^L\int_0^R u_{1,\varphi}u_{1,z}|_0^{2\pi}(z-L)drdz$$

$$+\int_{\Omega}\left(\frac{\partial u_{1,z}}{\partial z}-\frac{1}{r}\left(\frac{\partial (r\, u_{1,r})}{\partial r}+\frac{\partial u_{1,\varphi}}{\partial \varphi}\right)\right) u_{1,r}\,(z-L)\, r\, dr\, d\varphi\, dz =$$

$$=\int_0^{2\pi}\int_0^R r\, u_{1,z}^2 (z-L)\,|_0^L dr d\varphi + \int_{\Omega}\left(\frac{\partial}{\partial z}\left(r\, u_{1,z}^2\,(z-L)\right)-r\, u_{1,z}^2\right) dr\, d\varphi\, dz$$

$$=-\int_{\Omega} u_{1,z}^2\, r\, dr\, d\varphi\, dz$$

Here we use the continuity equation for the vector field u_1 and

$$-\int_{\Omega}\nabla\, p u dx=\int_{\Omega}\frac{\partial p}{\partial z}\,(L-z)\, dx=\int_{\Omega} p dx+\int_0^R\int_0^{2\pi}(L-z)\, p|_0^L r\, dr d\varphi$$

$$\geq -\int_0^R\int_0^{2\pi} r\, L\, p(r,\varphi,0) dr d\varphi \equiv g(t), \tag{25}$$

where $\int_{\Omega} p d\mathbf{x}\geq 0,\ g(t)\in C[0,\infty)$.

We multiply both sides of equation (17) by ρ_1, equation (18) – by ρ_2 and, as a result, we obtain the law of conservation of momentum:

$$\frac{\partial(\rho_1\, u_1+\rho_2\, u_2)}{\partial t}+\rho\,(u_1,\nabla)u_1+\rho_2\,(u_1,\nabla)u_2=-\nabla p+\Delta(\rho_1\nu_1 u_1+\rho_2\nu_2 u_2). \tag{26}$$

It is convenient to introduce the notation

$$P=p+\frac{\rho_1}{2}\,(u_2-u_1)^2.$$

In terms of the velocity u_2, and the pressure P, equation (18) has the form:

$$\frac{\partial u_2}{\partial t}+(u_2,\nabla)u_2=-\frac{\nabla P}{\bar{\rho}}+\nu_2\,\Delta u_2. \tag{27}$$

Multiplying both sides of equations (26) and (27) by the test function and using relations (23) - (25), we can obtain the equalities

$$\frac{d(\rho_1 J_1+\rho_2 J_2)}{dt}=\rho_1\int_{\Omega} u_{1,z}^2\, dx+\rho_2\int_{\Omega} u_{2,z}^2\, dx+f_1(t), \tag{28}$$

$$\frac{dJ_2}{dt}=\int_{\Omega} u_{2,z}^2\, dx+f_2(t), \tag{29}$$

$$J_1=\int_{\Omega}(u_1,u)\, dx, J_2=\int_{\Omega}(u_2,u)\, dx,$$

$$f_1(t)=g(t)+\rho_1\,\nu_1\, h_1(t)+\rho_2\,\nu_2\, h_2(t),$$

$$f_2(t)=G(t)+\nu_2 h_2(t),$$

$$G(t)=-\frac{1}{\bar{\rho}}\int_0^R\int_0^{2\pi} r\, L\, P(r,\varphi,0)\, dr d\varphi.$$

Further, the estimation, proved in [16] for $\lambda\in(0,3)$, is written down as

$$\frac{(3-\lambda)^2}{L^{(6-\lambda)}}\, J_1^2\leq 2\int_{\Omega} u_{2,z}^2\, dx.$$

From (28), (29), one can obtain the system of differential inequalities

$$\frac{dJ_1}{dt}\geq k^2\, J_1^2+f(t),$$

$$\frac{dJ_2}{dt}\geq k^2 J_2^2+f_2(t),$$

$$f(t)=f_1(t)-f_2(t)/\rho_1, k^2=\frac{(3-\lambda)^2}{L^{(6-\lambda)}}.$$

From the above differential inequalities it follows that the succeeding theorem is valid.

Theorem. There is no global existence of the classical solution to problem (16)–(21) if the following conditions are fulfilled:

1) let $f(t), f_2(t)\geq 0$, then under the conditions $J_1(0), J_2(0)\ >\ 0$, the lower estimations are valid:

$$J_1(t)\geq\frac{J_1(0)}{1-J_1(0)\, k^2\, t}, J_1(0)=\int_{\Omega} u_{1,0}(x)u\, dx,$$

$$J_2(t)\geq\frac{J_2(0)}{1-J_2(0)\, k^2\, t}, J_2(0)=\int_{\Omega} u_{2,0}(x)\, u\, dx,$$

in this case, there is an estimation for the time of destruction:

$$T_{\infty}\leq\frac{1}{k^2}\min\left(\frac{1}{J_1(0)},\frac{1}{J_2(0)}\right);$$

2) let $f(t)\geq a_1^2>0, f_2(t)\geq a_2^2>0$, then

$$J_1(t)\geq\frac{a_1}{k}\tan(a_1 k\, t+c_1), c_1=\arctan\left(\frac{k\, J_1(0)}{a_1}\right),$$

$$J_2(t)\geq\frac{a_2}{k}\tan(a_2\, k\, t+c_2), c_2=\arctan\left(\frac{k\, J_2(0)}{a_2}\right),$$

with the estimation of the time of destruction $T_{\infty}\leq \frac{1}{k}\min((\pi/2-c_1)/a_1,(\pi/2-c_2)/a_2)$;

3) let $f(t)\geq -a_1^2, f_2(t)\geq -a_2^2$, then under the conditions $k\, J_1(0)>|a_1|, kJ_2(0)\ >|a_2|$ the lower estimations are valid:

$$J_1(t) \geq \frac{a_1}{k}\frac{1+c_1\,\mathrm{e}^{2\,a_1\,k\,t}}{1-c_1\,\mathrm{e}^{2a_1\,k\,t}}, c_0 = \frac{k\,J_1(0)-a_1}{k\,J_1(0)+a_1},$$

$$J_2(t) \geq \frac{a_2}{k}\frac{1+c_2\,\mathrm{e}^{2a_2\,k\,t}}{1-c_2\mathrm{e}^{2\,a_2\,k\,t}}, \tilde{c}_0 = \frac{k\,J_1^2(0)-a_2}{k\,J_2(0)+a_2},$$

and the estimation for the time of destruction holds:

$$T_\infty \leq \frac{1}{2\,k}\min\left(\frac{1}{a}\ln\left(\frac{k\,J_1(0)+a_1}{k\,J_1(0)-a_1}\right), \frac{1}{\tilde{a}}\ln\left(\frac{k\,J_2(0)+a_2}{kJ_2(0)-a_2}\right)\right).$$

4. Conclusion

The equations of motion of Two-Fluid Medium in the case of taking into account dissipations are obtained by adding the corresponding irreversible flows into the non-dissipative equations of motion. It is proved that in some cases depending on the initial and boundary conditions of the problem the equations of the Two-Fluid Medium allow blow-up solutions.

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