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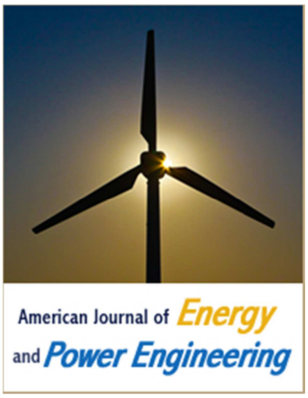

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Work Output and Efficiency of Non-Endoreversible Otto Heat Engine Cycle Using Generalized Redlich-Kwong Gas as Working Substance

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Abstract
The non-endoreversibility factor arising from the Clausius inequality is introduced to analyze the influence of two adiabats and two non-adiabats processes on the performance characteristics of the Otto cycle using the generalized Redlich-Kwong gas as the working substance. The work output and efficiency of the cycle are calculated and the results obtained here are very general and useful, from which the work output and efficiency of the Otto heat engine cycle using Redlich-Kwong, Van der Waals, and ideal gases can also be directly derived.

1. Introduction

Application of finite time thermodynamic optimization techniques to the Otto heat engine cycle has been reported by some scholars [1-6]. Mozurkewich et al.[1] used optimal control theory to optimize piston movement for the ideal Otto cycles. Angulo-Brown et al. [2] optimized a non-endoreversible Otto cycle model which took into account a non-endoreversibility factor arising from the Clausius inequality. Chen et al. [3] considered the irreversible adiabatics, finite-time processes and heat loss through the cylinder wall in the Otto cycle. Ye et al. [4] established an irreversible cycle model of the Otto heat engine, in which the variable heat capacities of the working fluid, the heat leak losses, the irreversibility of working substance resulting from the adiabatic processes and the internal friction loss are taken into account. Other studies [5,6] analyzed the reversible Otto heat engine cycle with non-ideal gas as the working substance. These results are closer to the reality than the classical thermodynamic theory. But the actual heat engine cycle generally is irreversible and the working substance is not the ideal gas.

In this paper, a non-endoreversible Otto heat engine cycle model using generalized Redlich-Kwong gas as working substance is established. The non-endoreversibility factor arising from the Clausius inequality is introduced to analyze the influence of two adiabats and two non-adiabats processes on the performance characteristics of the Otto cycle. The results obtained here are general, from which the work output and efficiency of the Otto heat engine cycle using Redlich-Kwong, Van der Waals, and ideal gases can be also directly derived in the present paper.

2. Thermodynamic Properties of the Generalized Redlich-Kwong Gas

The equation of state for a mole of the generalized Redlich-Kwong gas is[7-9].

$$p=\frac{RT}{V-b}-\frac{a}{T^{i}V(V+B)} \tag{1}$$

Where p, V, and T are the pressure, volume and temperature of the gas, respectively. a and b are two fixed constants for a gas, in which consider the mutual attraction between the molecules and the inherent volume of gas molecules, respectively. R is the gas constant. B and i are two parameters.

According to the first law of thermodynamics and equation (1), the specific heat of generalized Redlich-Kwong gas at constant volume is [7,9-11]

$$C_V=C_V^0+\int T(\frac{\partial^2 p}{\partial T^2})_V dV=C_V^0-\frac{i(i+1)a}{BT^{i+1}}\ln(\frac{V}{V+B}) \tag{2}$$

Where C_V^0 is the specific heat of ideal gas at constant volume.

Furthermore, using the first law of thermodynamics and equation (2), the quasi-static adiabatic process equation of generalized Redlich-Kwong gas can be given[12] as follows.

$$\frac{dT}{dV}=\frac{\frac{RT}{V-b}+\frac{ia}{V(V+B)T^{i}}}{\frac{ai(i+1)}{BT^{i+1}}\ln(\frac{V}{V+B})-C_V^0} \tag{3}$$

It is a transcendental equation. In particular, when parameters a, b, B, and i have different values, equation (3) has a concrete form that can be used to describe the quasi-static adiabatic process equation of the Redlich-Kwong, Van der Waals, and ideal gases and so on.

3. The Non-Endoreversible Otto Heat Engine Cycle Model

The temperature-entropy (T-S) diagram of a non-endoreversble Otto heat engine cycle is shown in Figure 1, where Q_{in} and Q_{out} are, respectively, the heats added to and rejected by the working substance. This heat engine cycle consisting of two irreversible adiabatic processes 2-3 and 4-1, and two isochoric processes 1-2 and 3-4. Processes 2-3S and 4-1S are two reversible adiabats. The temperatures of the working substance at state points 1, 2, 3, and 4 are represented by T_i ($i=1,\ 2,\ 3,\ 4$) , V_1 and V_2 are the volumes of the working substance in the two isochoric processes.

The heats added to and rejected by the working substance in the two isochoric processes are, respectively, given by

$$Q_{in}=\int_{T_1}^{T_2}C_V dT=C_V^0 r_T T_4(1-\frac{1}{x})+\frac{(i+1)a(1-x^{i})}{B(r_T T_4)^{i}}\ln(\frac{V_1}{V_1+B}) \tag{4}$$

and

$$|Q_{out}|=\int_{T_4}^{T_3}C_V dT=C_V^0 T_4(y-1)+\frac{(i+1)a}{BT_4^{i}}(\frac{1}{y^{i}}-1)\ln(\frac{V_2}{V_2+B}) \tag{5}$$

where $x=T_2/T_1$ and $y=T_3/T_4$ are, respectively, the temperature ratios in the isochoric processes 1-2 and 3-4, and $r_T=\frac{T_2}{T_4}$ is the high-low temperature ratio of the cycle.

According to the Clausius inequality $\oint\frac{dQ}{T}\le 0$, we have

$$C_V^0\ln x+\frac{ia}{B}\frac{1}{(r_T T_4)^{i+1}}(1-x^{i+1})\ln(\frac{V_1}{V_1+B})-[C_V^0\ln y+\frac{ia}{B}\frac{1}{T_4^{i+1}}(\frac{1}{y^{i+1}}-1)\ln(\frac{V_2}{V_2+B})]\le 0 \tag{6}$$

equation (6) indicates the entropy change of the working substance is zero in the endoreversible situation. However, if any internal irreversibility is considered, equation (6) must be less than zero. In order to describe the internal irreversibility, we may introduce an internal irreversibility parameter.

$$I=\frac{C_V^0\ln y+\frac{ia}{B}\frac{1}{T_4^{i+1}}(\frac{1}{y^{i+1}}-1)\ln(\frac{V_2}{V_2+B})}{C_V^0\ln x+\frac{ia}{B}\frac{1}{(r_T T_4)^{i+1}}(1-x^{i+1})\ln(\frac{V_1}{V_1+B})}\ge 1 \tag{7}$$

Obviously in the endoreversible limit, I=1. Otherwise, I>1.

Combine equations(4), (5),and (7), the expressions of the work output and efficiency can be derived as

$$W = Q_{in} - |Q_{out}| = C_V^0 T_4 [r_T (1-\frac{1}{x}) - (y-1) + \frac{(i+1)A}{i} \ln(\frac{y}{x^I}) + \frac{(i+1)a}{BC_V^0 T_4^{i+1}} (\frac{1}{y^{i+1}} - 1)(A-1)\ln(\frac{V_2}{V_2+B})] \tag{8}$$

and

$$\eta = \frac{W}{Q_{in}} = \frac{r_T (1-\frac{1}{x}) - (y-1) + \frac{(i+1)A}{i} \ln(\frac{y}{x^I}) + \frac{(i+1)a}{BC_V^0 T_4^{i+1}} (\frac{1}{y^{i+1}} - 1)(A-1)\ln(\frac{V_2}{V_2+B})}{r_T (1-\frac{1}{x}) + \frac{(i+1)A}{i} \ln(\frac{y}{x^I}) + \frac{(i+1)aA}{BC_V^0 T_4^{i+1}} (\frac{1}{y^{i+1}} - 1)\ln(\frac{V_2}{V_2+B})} \tag{9}$$

Where $A = \frac{r_T (1-x^i)}{I(1-x^{i+1})}$.

4. Some Special Cases

(i) When $B = b$ and $i = 1/2$, equation (1) becomes the equation of state for a mole of the Redlich-Kwong (abbr. RK) gas[8-12].

$$p = \frac{RT}{V-b} - \frac{a}{T^{1/2}V(V+b)} \tag{10}$$

At this time, put $B = b$ and $i = 1/2$ to equations (8) and (9), we get

$$W_{RK} = C_V^0 T_4 [r_T (1-\frac{1}{x}) - (y-1) + 3A\ln(\frac{y}{x^I}) + \frac{3a/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)(A-1)\ln(\frac{V_2}{V_2+b})] \tag{11}$$

and

$$\eta_{RK} = \frac{r_T (1-\frac{1}{x}) - (y-1) + 3A\ln(\frac{y}{x^I}) + \frac{3a/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)(A-1)\ln(\frac{V_2}{V_2+b})}{r_T (1-\frac{1}{x}) + 3A\ln(\frac{y}{x^I}) + \frac{3aA/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)\ln(\frac{V_2}{V_2+b})} \tag{12}$$

(ii) When $B = -b$ and $i = 1/2$, equation (1) becomes the equation of state for a mole of the Dieterici gas as a first approximation[9,11,12]. equations (8) and (9) may be simplified as

$$W_D = C_V^0 T_4 [r_T (1-\frac{1}{x}) - (y-1) + 3A\ln(\frac{y}{x^I}) - \frac{3a/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)(A-1)\ln(\frac{V_2}{V_2-b})] \tag{13}$$

and

$$\eta_D = \frac{r_T (1-\frac{1}{x}) - (y-1) + 3A\ln(\frac{y}{x^I}) - \frac{3a/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)(A-1)\ln(\frac{V_2}{V_2-b})}{r_T (1-\frac{1}{x}) + 3A\ln(\frac{y}{x^I}) - \frac{3aA/2}{bC_V^0 T_4^{3/2}} (\frac{1}{y^{3/2}} - 1)\ln(\frac{V_2}{V_2-b})} \tag{14}$$

(iii) When $B = 0$ and $i = 0$, equation (1) becomes the equation of state for a mole of the Van der Waals (abbr. VDW) gas [8-12]. equations (8) and (9) may be simplified as

$$W_{VDW} = C_V^0 T_4 [r_T (1-\frac{1}{x}) - (y-1) + \ln(\frac{y}{x^I}) - \frac{a}{C_V^0 T_4} (\frac{1}{y} - 1)] \tag{15}$$

and

$$\eta_{VDW} = 1 - \frac{(y-1)(1+\frac{a}{yC_V^0 T_4})}{r_T (1-\frac{1}{x}) + \ln(\frac{y}{x^I})} \tag{16}$$

Furthermore, the cycle becomes endoreversible when I=1. That is, State point 1 tends to 1S , and 3 tends to 3S in Figure 1. Here, equation (3) can be simplified as [5,12,13]

$$T(V-b)^{R/C_V^0} = C \tag{17}$$

Where C is the constant.

In addition, from equation (7) we can obtain

$$x=y \tag{18}$$

Combining equations (15)-(18), the expressions of work output and efficiency of endoreversible Otto heat engine cycle using Van der Waals gas as working substance can be derived

$$W_{VDW} = C_V^0 T_4 (x-1)(r_V^{R/C_V^0} - \frac{a}{C_V^0 T_4 x} - 1) \quad (19)$$

and

$$\eta_{VDW} = 1 - \frac{1 + \frac{a}{x C_V^0 T_4}}{r_V^{R/C_V^0}} \quad (20)$$

Where $r_V = \frac{V_2 - b}{V_1 - b}$ is the compression ratio of the cycle using Van der Waals gas as working substance.

(iv) When $a = 0$ and $b = 0$, equation (1) becomes the equation of state for a mole of the ideal gas [8-12]. equations (8) and (9) may be simplified as

$$W_{ideal} = C_V^0 T_4 [r_T (1 - \frac{1}{x}) - (y-1) + \ln(\frac{y}{x^I})] \quad (21)$$

and

$$\eta_{ideal} = 1 - \frac{(y-1)}{r_T (1 - \frac{1}{x}) + \ln(\frac{y}{x^I})} \quad (22)$$

Similarly, we can obtain the expressions of work output and efficiency of endoreversible Otto heat engine cycle using ideal gas as working substance.

$$W_{ideal} = C_V^0 T_4 (y-1)(r_V^{\gamma-1} - 1) \quad (23)$$

and

$$\eta_{ideal} = 1 - \frac{1}{r_V^{\gamma-1}} \quad (24)$$

Where $r_V = \frac{V_2}{V_1}$ is the compression ratio the cycle using ideal gas as working substance and $\gamma = \frac{C_p^0}{C_V^0}$ is the specific heat ratio of ideal gas.

Equations (23) and (24) are also the results of the existing literature[5,14] and have appeared in many textbooks.

5. Numerical Calculations

Equations (8) and (9) show clearly that the work output and efficiency depend on a series of parameters such as the temperature ratios r_T, x and y, the internal irreversibility parameter I, the maximum volume V_2 and so on. Below numerical calculations are carried out based on the parameters $V_2 = 0.001\ m^3 \cdot mol^{-1}$, $T_4 = 300\ K$, $C_V^0 = 20.775\ J \cdot mol^{-1} \cdot K^{-1}$, x=2 and other parameters summarised in Table 1[9].

Table 1. *The performance-related parameters in Figures 2-5.*

Parameter	RK gas	Dieterici gas	VDW gas	Ideal gas
a ($Pa \cdot m^6 \cdot K^i \cdot mol^{-2}$)	6614.11×10^{-5}	6614.11×10^{-5}	6614.11×10^{-5}	0
b ($m^3 \cdot mol^{-1}$)	1.8372×10^{-5}	1.8372×10^{-5}	1.8372×10^{-5}	0
B ($m^3 \cdot mol^{-1}$)	1.8372×10^{-5}	-1.8372×10^{-5}	0	
i	0.5	0.5	0	

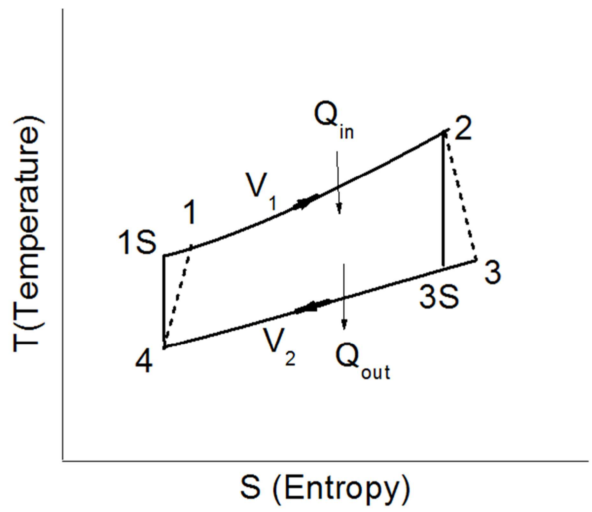

Figure 1. *The T-S diagram of a non-endoreversible Otto heat engine cycle.*

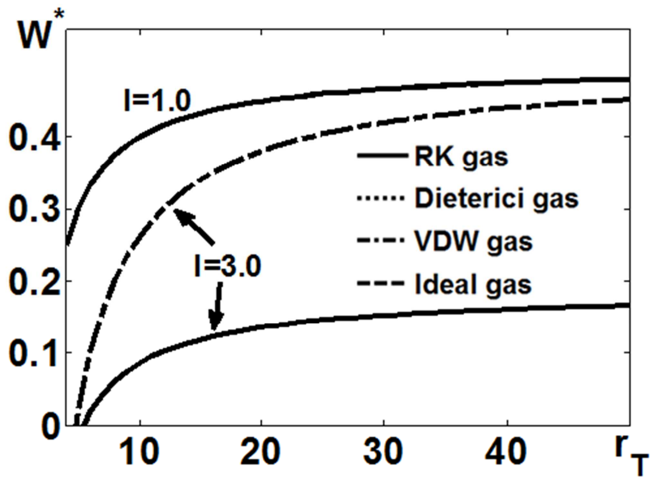

Figure 2. *The W^*-r_T curves of the heat engines using RK (solid line), Dieterici (dot line), VDW (dash-dot line) and Ideal gas (dash line).*

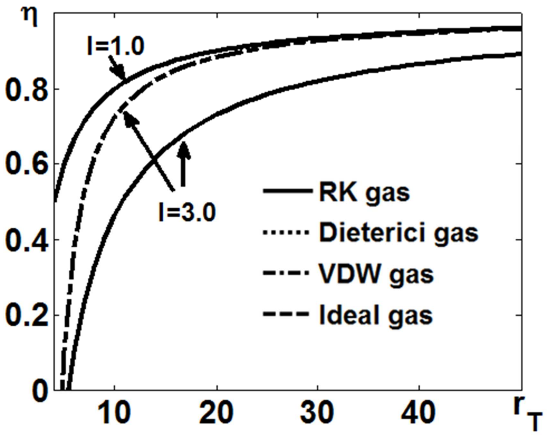

Figure 3. *The η-r_T curves of the heat engines using RK (solid line), Dieterici (dot line), VDW (dash-dot line) and Ideal gas (dash line).*

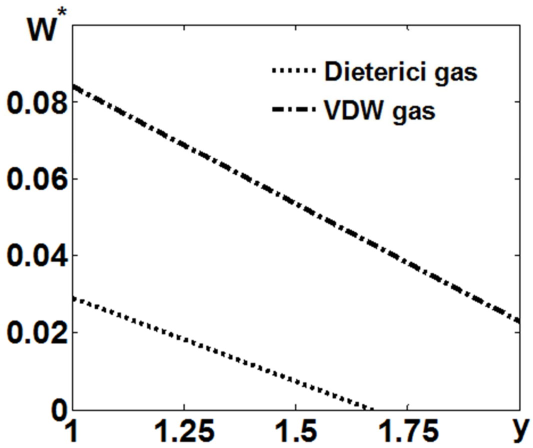

Figure 4. *The W^*-y curves of the heat engines using Dieterici (dot line) and VDW gas (dash-dot line).*

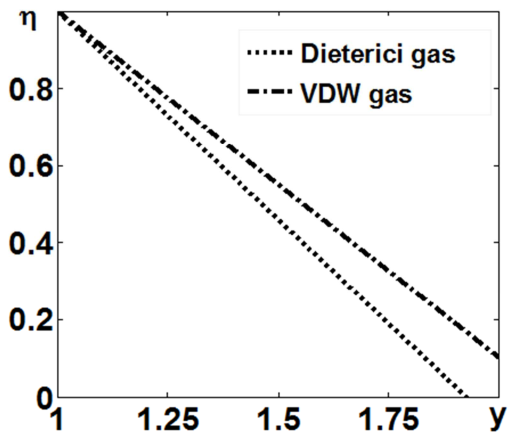

Figure 5. *The η-y curves of the heat engines using Dieterici (dot line) and VDW gas (dash-dot line).*

Using equations (8) and (9), we can plot the curves of the dimensionless work output $W^* = W/(C_V^0 T_4 r_T)$ and efficiency η varying with the high-low temperature ratio, as shown in Figures 2 and 3, where y=2 and the internal irreversibility parameter I=1.0 or 3.0. Obviously, The bigger the parameter I, the greater the irreversiblity of the cycle. So the dimensionless work output and efficiency decrease when the internal irreversibility parameter increases. Furthermore, the dimensionless work output and efficiency increase when the high-low temperature ratio r_T increases.

Similarly, we can also plot the curves of the dimensionless work output and efficiency varying with the parameter y, as shown in Figures 4 and 5, where $r_T = 5$ and the internal irreversibility parameter I= 3.0. From the Figures 4 and 5,we can easily find out the dimensionless work output and efficiency decrease when the parameter y increases.

6. Summary

We have established the non-endoreversible Otto heat engine cycle using the generalized Redlich-Kwong gas as the working substance, which may include the Redlich-Kwong, Van der Waals, ideal gases and so on. The work output and efficiency of the cycle are derived and the performance characteristics of the cycle are obtained by detailed numerical examples. The influence of several important parameters on the performance of the cycle is discussed. It is found that the temperature ratios r_T , x and y, the internal irreversibility parameter I and the maximum volume V_2 show obvious influence on the cycle performance. Obviously, the results obtained here are general, and consequently, can be directly used to derive the work output and efficiency of the Otto heat engine cycle using several interesting gases mentioned above.

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