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Technology of Embedding Systems as a Method for Studying the Dynamic Regimes of Complex Electric Systems

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Abstract: Approximately 20-25 years ago, new approaches to the research of automatic control systems based on matrix methods appeared in the literature. New matrix designs (zero divisors, canonizers) have been developed that make it possible to extend the range of solvable problems, including not only minimal-phase, but also non-minimal-phase systems. It is obvious that matrix methods of investigation of linear systems are promising directions for the development of analysis and synthesis of dynamic objects, including studies of the steady–state stability of complex electrical systems. The mathematical formulation of the problem of studying the steady–state stability of electric power systems boils down to the following. Since all processes in the elements of the automatic control system are described by differential equations, the stability analysis reduces to investigating the properties of the solution of linearized equations for small perturbations. When analyzing and synthesizing dynamic systems, it becomes necessary to solve matrix equations. Along with the known methods for solving matrix equations, the article gives a method called the canonization method. Advantages of this method is its analyticity, i.e. this method allows us to carry out analytical studies of the resulting matrix equations. Canonicalization is based on a modified Gauss algorithm, in which the computational procedure is minimized. Here it should be noted that in the electrical system, when perturbations occur, the loss of stability occurs as a result of the synchronous generator leaving the synchronism or in the general case of rotating machines. Static elements affect the stability of electrical systems only by their parameters, which are usually assumed to be constant or slowly changing. Therefore, determining the conditions for the output from synchronism of a particular synchronous generator or their grostaups (stations) in a complex electrical system is the main task. The technology of embedding systems is an effective method for studying the steady–state stability (small oscillations) of an electrical system that makes it possible to determine all possible dynamic and structural properties of the linear matrix system under study.

Keywords: Electrical System, Mathematical Models, Embedding Matrix, Promatrix, Transient Characteristic

1. Introduction

The technology of embedding systems is the universal set of methods and methods for solving problems in the theory of systems, based primarily on modern achievements of algebra and reducing to determining the conditions under which a complexly organized (multidimensional, matrix) system behaves similarly to a relatively simpler (simply connected, well-studied or accessible for in-depth research) system [1]. The technology of embedding systems has the following features [2]:

a. it is focused on analytical research and synthesis of linear multiply connected systems;
b. takes into account a wide range of structural properties (poles, all types of zeros, algebraic features) of the studied or synthesized linear system;
c. provides the possibility of obtaining the whole set of equivalent (indistinguishable by statement of the problem) results of analysis or synthesis of linear stationary dynamical systems if the desired solution exists and is not unique.

Technology of embedding systems involves the sequential execution of three stages [3]. Let's consider the stages of applying the technology of embedding systems.

1. At the first stage, the general structure of the system being studied or synthesized is formalized. This is done by reducing the mathematical models of all the subsystems and the connections between them to a matrix of a special design - the problem matrix (the promatrix) Ω (p) of the problem being solved. The promatrix always has a square form and is reversible. If calculate the matrix inverse to the matrix, obtain a reversible problem matrix (resample matrix) Ω^{-1} (p), which will contain all possible transfer functions of the linear dynamical system. Therefore, the promatrix is the only object of investigation, which exhaustively characterizes all the properties of a linear dynamical system.

2. In the second stage, a so-called embedding identity is formed that establishes selective equivalence of the system under study and some other system, the image of ω(p) that has a known or desired set of properties. Speech are talking about the fragmented identification of the Ω^{-1}(p) and the image of ω(p):

$$\beta\Omega^{-1}\alpha = \omega,$$

which, in fact, gave the name of the technology. Here α and β in the general case are polynomial matrices of the required size, ω-image of the system under investigation.

3. In the third stage, the transition from the identity of the investment to the calculated formulas is carried out. Matrices of equations α, β and ω form matrix equations whose solutions either do not exist (the problem posed is unsolvable), or it requires the fulfillment of those relations (values of matrix coefficients) that are this solution.

Steady–state stability - stability under small perturbations, is investigated on the basis of methods that are based on the analysis of differential equations of the first (linear) approximation [4].

Checking the steady–state stability of power systems consists in determining the possibility of the existence of a stable regime with small perturbations of the parameters of the regime with given values of the parameters of the power system, the mode of generating sources, the load of node points, and the tuning of automatic mode control devices [5-7].

2. Mathematical Model of Transients

The study of small oscillations of a multi-machine electrical system will be carried out on the basis of equations in the state space having the form [3]:

$$\dot{x} = A_\Sigma x + B_\Sigma u, \tag{1}$$

$$y = Cx + D\varepsilon, \tag{2}$$

where x, $\dot{x}$, y - the input vector of the state of the system and its derivatives, the output vector; A_Σ, B_Σ, C, D - functional constants of the matrix, composed of the coefficients of the elements of the object and regulator of the system under study, u is a vector of input and disturbing factors, ε is the output vector.

This model describes the transient process in the electric system, taking into account the balance of the moments (powers) on the shaft of the i-th power plant and has the form [8].

$$\frac{d^2\delta_i}{dt^2} = \frac{\omega_0}{T_{ji}}[P_{Ti} - P_{Gi}] \tag{3}$$

where ω_0 is the synchronous angular frequency; T_{ji}, δ_i, P_{Ti}, P_{Gi} – the inertia constant of the i-th aggregate, the load angle of the i-th generator, the mechanical power of the i-th turbine, the electromagnetic power of the i-th synchronous generator, respectively.

The equation of electromagnetic power of the i-th synchronous generator in the positional idealization has the form [4]:

$$P_{Gi} = E_i^2 y_{ii} \sin\alpha_{ii} + \sum_{j=1, j\neq i}^{n} E_i E_j y_{ij} \sin(\delta_{ij} - \alpha_{ij}), \tag{4}$$

where E_i, E_j - emf. i-th and j-th synchronous generators; y_{ii}, y_{ij} - intrinsic and mutual conductivity of the network; α_{ii}, α_{ij} are complementary angles.

Equation (4) is nonlinear, since the components of the equation are transcendental, in the form of a sinusoidal function. Therefore, in the study of small oscillations of electric power systems (EPS), Taylor series expansions and some trigonometric relations are used, which allow one to linearize the nonlinear differential equation of the aggregate (4) at the initial point of the mode P_0 (the P-mode parameter: power, voltage, etc.), which simplify the studies of the static stability of the electrical system. The method of small oscillations, used in this case, is based on the assumption that the regime parameters that receive deviations of $P = P_0 \pm \Delta P$ for small perturbations in the electrical system change to small values.

Transcendental functions are linearized with the help of the following relations for any i and j:

$$\delta_{ij} = \delta_i - \delta_j,\ \delta_i = \delta_{i0} + \Delta\delta_i,\ \delta_j = \delta_{j0} + \Delta\delta_j,\ \delta_{ij} = -\delta_{ji}, \tag{5}$$

and beyond

$$\begin{aligned} \sin(\delta_{ij} - \alpha_{ij}) &= \sin[(\delta_{i0} + \Delta\delta_i) - (\delta_{j0} + \Delta\delta_j) - \alpha_{ij}] \\ &= \sin[(\Delta\delta_i - \Delta\delta_j) + (\delta_{i0} - \delta_{j0} - \alpha_{ij})] \\ &= \Delta\delta_i \cos\beta_{ij} - \Delta\delta_j \cos\beta_{ij} + \sin\beta_{ij}, \end{aligned} \tag{6}$$

where $\beta_{ij} = \delta_{i0} - \delta_{j0} - \alpha_{ij}$.

It should be noted that the derivation of formula (6) uses the obvious relationships:

$$\sin(\Delta\delta_i - \Delta\delta_j) \cong (\Delta\delta_i - \Delta\delta_j) \text{ and } \cos(\Delta\delta_i - \Delta\delta_j) \cong 1,$$

which are valid for small deviations in the generator load angles.

After the transformations (4), taking into account (5), (6) and the substitution, equation (3) takes the form:

$$\frac{d^2\delta i}{dt^2}=\frac{\omega_0}{T_{ji}}[P_{Ti}-(E_i^2 y_{ii}\sin\alpha_{ii}-\sum_{j=1,j\neq i}^{n} b_{ij}\Delta\delta_i+b_{ii}\Delta\delta_i+c_{ij})], \quad (7)$$

and taking into account the parameters of the initial regime and the relation $\delta_i=\delta_{i0}+\Delta\delta_i$ finally leads to a differential equation in the deviations [9]:

$$\frac{d^2\Delta\delta_i}{dt^2}=\frac{\omega_0}{T_{ji}}[\sum_{j=1,j\neq i}^{n} b_{ij}\Delta\delta_j-b_{ii}\Delta\delta_i], \quad (8)$$

where

$$b_{ij}=a_{ij}\cos\beta_{ij},\ a_{ij}=E_iE_jy_{ij},\ b_{ii}=\sum_{j=1,j\neq i}^{n} b_{ij},\ c_{ij}=\sum_{j=1,j\neq i}^{n} a_{ij}\sin\beta_{ij},$$

$$P_{Ti}-(E_i^2 y_{ii}\sin\alpha_{ii}+c_{ij})=0.$$

If take into account the damper contours of the rotor of the i-th synchronous generator, equation (8) takes the form:

$$\frac{d^2\Delta\delta_i}{dt^2}=\frac{\omega_0}{T_{ji}}[\sum_{j=1,j\neq i}^{n} b_{ij}\Delta\delta_j-b_{ii}\Delta\delta_i-P_{di}\frac{d\Delta\delta_i}{dt}], \quad (9)$$

where P_{di} is the coefficient of the generalized damper moment of the i-th generator.

If the deviation of the emf is taken into account. i-th synchronous generator, equation (9) takes the form:

$$\frac{d^2\Delta\delta_i}{dt^2}=\frac{\omega_0}{T_{ji}}[\sum_{j=1,j\neq i}^{n} b_{ij}\Delta\delta_j-b_{ii}\Delta\delta_i-P_{di}\frac{d\Delta\delta_i}{dt}-\frac{dP_i}{dE_{qi}}\Delta E_{qi}]. \quad (10)$$

A feature of equation (10) is that it is allowed with respect to the absolute angles of the system generators and, for example, for the three-generator electric system has the form [3]:

$$\frac{d^2\Delta\delta_1}{dt^2}=\frac{\omega_0}{T_{j1}}[-b_{11}\Delta\delta_1+b_{12}\Delta\delta_2+b_{13}\Delta\delta_3-P_{d1}\frac{d\Delta\delta_1}{dt}-\frac{dP_1}{dE_{q1}}\Delta E_{q1}],$$
$$\frac{d^2\Delta\delta_2}{dt^2}=\frac{\omega_0}{T_{j2}}[b_{21}\Delta\delta_1-b_{22}\Delta\delta_2+b_{23}\Delta\delta_3-P_{d2}\frac{d\Delta\delta_2}{dt}-\frac{dP_2}{dE_{q2}}\Delta E_{q2}],$$
$$\frac{d^2\Delta\delta_3}{dt^2}=\frac{\omega_0}{T_{j3}}[b_{31}\Delta\delta_1+b_{32}\Delta\delta_2-b_{33}\Delta\delta_3-P_{d3}\frac{d\Delta\delta_3}{dt}-\frac{dP_3}{dE_{q3}}\Delta E_{q3}].$$

The system of equations of the EPS, reflecting transient processes for small deviations, is convenient, both algorithmically and computationally, in particular, in cases of their joint solution with steady-state equations - nodal voltages equations. This is explained by the fact that the result of the nodal voltages equations solution is the voltage module of the i-th node U_i and its argument δ_i, used in the above differential equations, determined with respect to the balancing node.

3. Mathematical Model of the Circuit of Excitation and Automatic Regulator of Excitation of Synchronous Generators

The equations of electromagnetic transient processes in the excitation circuit of the i-th synchronous machine in the deviations were in [3], and in a somewhat modified form have the form:

$$T_{di}^{'}\frac{d\Delta E_{qi}}{dt}=\Delta E_{qi}-\Delta E_{qei},$$

$$T_{ei}\frac{d\Delta E_{qei}}{dt}=\Delta U_{AECi}-\Delta E_{eqi},$$

$$T_{pi}\frac{d\Delta U_{AECi}}{dt}=\Delta e_i-\Delta U_{AECi},$$

where - $T_{di}^{'}, T_{ei}, T_{pi}$ the transition time constant of the excitation winding, the constant times of the exciter, the automatic excitation controller, respectively; ΔE_{qi}, ΔE_{qei}, ΔU_{AECi} - deviations of the synchronous, forced emf. and the voltage at the output of the automatic excitation controller (AEC), respectively.

The generation of signals via the AEC Δe_i channels in an idealized form (provided that the constant times of the differentiating elements of the AEC are considered to be zero) can be represented as:

$$\Delta e=\ (\mathrm{k}_{0P\mathrm{k}}\Delta P_\mathrm{k}+\mathrm{k}_{1P\mathrm{k}}(\mathrm{d}\Delta P_\mathrm{k}/\mathrm{dt})+\mathrm{k}_{2P\mathrm{k}}(\mathrm{d}^2\Delta P_\mathrm{k}/\mathrm{dt}^2),$$

where k_{0Pk}, k_{1Pk}, k_{2Pk} are the gain factors of the AEC on the deflection channels, the first and second derivatives of the regime parameters ΔP_k, respectively, k is the number of adjustable mode parameters.

4. Formation of Equations of Complex Electric System Based on Technology of Embedding Systems

It is necessary to form models of the corresponding equations of EPS and regulating devices, using representations in the state space with respect to the technology of embedding systems. Taking into account the complexity of the equations of interrelations characterizing the multi-machine EPS, it is necessary to form equations and templates of the technology of embedding systems with maximally simplified relationships and at the same time allowing to fully take into account the dynamic properties of the electrical system.

As indicated in [1], for a square matrix (m = n and C = I_n), the model for the object will have the form:

$$\Omega(p) = \begin{bmatrix} pI_n - A & -B \\ 0 & I_s \end{bmatrix}, \tag{11}$$

and for an adjustable dynamic system with a static regulator

$$u = -Kx,$$

the promatrix will be equal to

$$\Omega(p) = \begin{bmatrix} pI_n - A & -B \\ K & I_s \end{bmatrix}, \tag{12}$$

where A, B, C are given numerical matrices, K is the matrix of the regulator coefficients (n is the degree of the mathematical model of the object of the system under study, m is the number of system inputs, s is the number of controller outputs).

In accordance with [2], the technology of embedding in a scalar image

- for an open system:

$$f(p) = \frac{b(p)}{a(p)}, \tag{13}$$

- for the system closed by the regulator:

$$g(p) = \frac{q(p)}{d(p)}, \tag{14}$$

requires the formation of deterministic relationships that solve the problem of finding matrix numerators:

$$a(p) = \det(pI_n - A), \tag{15}$$

$$[b(p)] = \det(pI_n - A + B\overline{\alpha}\overline{\beta}) - a(p), \tag{16}$$

$$d(p) = \det(pI_n - A + BK), \tag{17}$$

$$[q(p)] = \det(pI_n - A + B\overline{\alpha}\overline{\beta} + K) - d(p), \tag{18}$$

The content of the system of matrix equations is as follows: a (p), d (p) are the characteristic polynomials for the model of the object under study - the electrical system and the closed controlled electrical system; [b (p)], [q (p)] are their matrix numerators, respectively, $\overline{\alpha}$ and $\overline{\beta}$ two polynomial imbedding matrices.

Relations (15) - (18), in contrast to the one-dimensional case, allow us to find matrix numerators of matrix transfer functions for the system under study for a multidimensional object with a regulator, which is characteristic of the modern approach to the study of complex dynamical systems [9].

The formation of the matrices A and B of the system (1) - (2) is based on the following considerations. Taking into account the complexity of the interrelations of the equations of the multi-machine electrical system, the self-matrix A is formed from the deviations of the absolute angles and slides of n generators, and the input matrix B from the deviations of the emf. machines. Then for a complex EPS the data of the matrix have the form [3]:

$$A_\Sigma = \begin{bmatrix} 0_{nxn} & I_{nxn} \\ A_{21(nxn)} & A_{22(nxn)} \end{bmatrix}, \; B_\Sigma = \begin{bmatrix} 0_{nxn} \\ B_{nxn} \end{bmatrix},$$

where

$$A_{21(nxn)} = \begin{bmatrix} -\omega_{11} & \omega_{12} \; ... \; \omega_{1n} \\ \omega_{21} & -\omega_{22} \; ... \; \omega_{2n} \\ \multicolumn{2}{c}{\dotfill} \\ \omega_{n1} & \omega_{n2} \; ... -\omega_{nn} \end{bmatrix}, \; A_{22(nxn)} = \begin{bmatrix} -P_{d1} & 0 \; ... \; 0 \\ 0 & -P_{d2} \; ... \; 0 \\ \multicolumn{2}{c}{\dotfill} \\ 0 & 0 \; ... -P_{dn} \end{bmatrix}.$$

Here the vector-column of state parameters, consisting of the parameters of the EPS mode:

$$x = [\Delta\delta_1 ... \Delta\delta_n \;\; | \;\; \Delta s_1 ... \Delta s_n]^T.$$

and the input vector:

$$u = [\Delta E_{q1} ... \Delta E_{qn}]^T.$$

The matrix B_Σ can be formed from two models of the dependence of the deviation of the active power of the i-th generator ΔP_i on the deviation of the emf. other generators of the electrical system:

a) $\Delta P_i = f(\Delta\delta_i ... \Delta\delta_n, \Delta E_{qi})$, i.e. $\Delta E_j = 0$, $j = 1-n$, $j \neq i$. In other words, the deviation of the active power of the generators depends on the deviation of the absolute angles and only the own emf. This generator, while the matrix B has the form:

$$B_{nxn} = \begin{bmatrix} -\frac{dP_1}{dE_{q1}}\frac{\omega_0}{T_{j1}} & 0 & ... & 0 \\ 0 & -\frac{dP_2}{dE_{q2}}\frac{\omega_0}{T_{j2}} & ... & 0 \\ \multicolumn{4}{c}{\dotfill} \\ 0 & 0 & & -\frac{dP_n}{dE_{qn}}\frac{\omega_0}{T_{jn}} \end{bmatrix},$$

b) $\Delta P_i = f(\Delta\delta_i ... \Delta\delta_n, \Delta E_{qi} ... \Delta E_n)$ – the deviation of the power of the i-th generator depends on the deviation of the absolute angles and the emf. of all n generators and the matrix B has the form:

$$B_{nxn} = \begin{bmatrix} -\frac{dP_1}{dE_{q1}}\frac{\omega_0}{T_{j1}} & -\frac{dP_1}{dE_{q2}}\frac{\omega_0}{T_{j1}} & ... & -\frac{dP_1}{dE_{qn}}\frac{\omega_0}{T_{j1}} \\ -\frac{dP_2}{dE_{q1}}\frac{\omega_0}{T_{j2}} & -\frac{dP_2}{dE_{q2}}\frac{\omega_0}{T_{j2}} & ... & -\frac{dP_2}{dE_{qn}}\frac{\omega_0}{T_{j2}} \\ \multicolumn{4}{c}{\dotfill} \\ -\frac{dP_n}{dE_{q1}}\frac{\omega_0}{T_{jn}} & -\frac{dP_n}{dE_{q2}}\frac{\omega_0}{T_{jn}} & ... & -\frac{dP_n}{dE_{qn}}\frac{\omega_0}{T_{jn}} \end{bmatrix}.$$

As can be seen from these matrices, the generalized eigenmode of the dynamics of a complex EPS A_Σ has a size of 2nx2n, and the input matrix B_Σ is a size of 2nxn. These matrices make it possible to carry out in full the studies of the dynamic properties of the EPS under study on the basis of the technology of embedding systems if the matrix of coefficients of the regulator K is known, which is determined on the basis of synthesizing [10].

5. Conclusion

Matrix methods of investigation of linear systems are promising directions for the development of analysis and synthesis of dynamic objects, including studies of the static stability of complex electrical systems. In this case, the speed of computation and the dimension of the systems under study will not be limiting factors, since they are solved as the development of information technology and the base of computer technology.

The matrix equations of the EPS elements and the whole system were compiled on the basis of the most widely obtained equations of state variables, which are small deviations of the mode parameters-the angles of the rotor load of the synchronous generator, busbar voltages, power, and other operating parameters of the EPS. The considered matrix equations are used for analysis of transient processes and steady–state stability of EPS and for the synthesis of optimal parameters of regulators of synchronous machines operating in an electrical system.

The mathematical model of the electrical system, resolved with respect to the deviations of the absolute angles of synchronous generators, can be used independently for studies of small oscillations of complex EPS. This model of small oscillations of complex electrical systems must be used in conjunction with the method of nodal voltages equations that determine the voltage modules U_i of nodes and their arguments representing the absolute angles δ_i with respect to the balancing node.

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Biography

Tokhir Makhmudov was born in Tashkent, Uzbekistan in 1987. He received the B. Sc. and M. Sc. degrees in Electric Power Engineering from the Tashkent State Technical University, Uzbekistan in 2008 and 2011 respectively. From 2011 to 2017, he worked at the Department of Electrical Stations, Networks and Systems, the Tashkent State Technical University as a senior teacher. Since 2018 he is a doctoral student of the Tashkent State Technical University. Areas of research of Makhmudov Tokhir consist in studying transients in electric power systems, analysis of static stability of complex electrical systems.