Control and Synchronization of a Modified Hyperchaotic Pan System via Active and Adaptive Control Techniques

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Abstract
This paper studies chaos control and chaos synchronization of a modified hyperchaotic Pan system via active and adaptive control techniques based on the Jacobian matrix and Lyapunov stability theory. Although the two nonlinear control techniques are excellent and perform the control and synchronization. But, we found that the active control is less complex and short from than the adaptive control. Numerical simulations are given to illustrate and verify the results.

1. Introduction
Since chaos phenomenon was observed by Lorenz firstly in 1963, a large number of chaos phenomena and chaos behavior have been discovered in natural and social system [7]. Among them, chaos control is one of the chaos phenomena which contains two aspects, namely, chaos control and synchronization. Chaos control attempts to eliminate chaotic behaviors while synchronization means to control a chaotic system (called response system) to follow another chaotic system (called drive system)[5]. Chaos control was first presented by Ott, Grebogi and Yorke in 1990, and the pioneering work on the synchronization was introduced by Pecora and Carroll [4, 7, 9].

Chaos control has attracted a great deal of attentions from various fields including secure communication, information science, chemical reactions, biological systems and some other fields [4, 6, 7, 8, 9, 10]. Especially, the subject of chaotic synchronization has received considerable attentions [5, 8]. Many techniques for chaos control and synchronization have been developed, such as linear feedback method, active control approach, adaptive technique, time delay feedback approach, and back stepping method [3, 4, 6, 7, 9, 10].

Recently, many authors have studied the control and synchronization for the chaotic and hyperchaotic systems. Sundarapandian V. in 2012 [9] studied the control and synchronization of chaotic Cai system by adaptive controllers, Ahmad I. et al. in 2015 [1] designed nonlinear controllers to synchronize two novel chaotic systems.

This letter studies control and synchronization of a modified hyperchaotic pan system via active and adaptive control techniques based on the Jacobian matrix and Lyapunov stability theory, the rest of the letter is organized as follows. Section 2 gives a brief description of a modified hyperchaotic pan system, in section 3, we present chaos control via active and adaptive control, Section 4, we present chaos synchronization between two identical a modified hyperchaotic pan systems via active and adaptive control. Finally, conclusions are given in section 5.
2. System Descriptions

The new hyperchaotic system [2,3] is given by

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= cx - xz + w \\
\dot{z} &= xy - bz \\
\dot{w} &= -dy
\end{align*}
\]

(1)

where \((x,y,z,u) \in R^4\), and \(a,b,c,d \in R\) are constant parameters. When parameters \(a = 10\), \(b = 8/3\), \(c = 28\) and \(d = 10\), system (1) is hyperchaotic and has two Lyapunov exponents i.e. \(LE_1 = 0.38352\), \(LE_2 = 0.12714\), and hyperchaotic attractors are shown in Fig.1. The system (1) has only one equilibrium \((0,0,0,0)\), and the equilibrium is an unstable under these parameters.

![Fig 1. The attractor of system (1) in x-y-z space.](image)

3. Controlling the Modified Hyperchaotic Pan System to Equilibrium Point

In this section, active and adaptive control of modified hyperchaotic pan system with unknown parameters are achieved based on the Jacobian matrix and Lyapunov stability theory respectively.

In order to control the modified hyperchaotic pan system to zero, the feedback controllers of \(u_1, u_2, u_3\), and \(u_4\) are added to the hyperchaotic system (1). Then the controlled hyperchaotic system is given by

\[
\begin{align*}
\dot{x} &= a(y - x) + u_1 \\
\dot{y} &= cx - xz + w + u_2 \\
\dot{z} &= xy - bz + u_3 \\
\dot{w} &= -dy + u_4
\end{align*}
\]

(2)

in which \(a,b,c\) and \(d\) are unknown parameters, and \(u_1, u_2, u_3, u_4\) are feedback controllers to be designed.

Now, in the following theorem, we design active control law for controlling system (2) based on the Jacobian matrix.

**Theorem 1.** If the active controllers are designed as

\[
\begin{align*}
u_1 &= (a - 1)x - ay \\
u_2 &= (z - c)x - y - w \\
u_3 &= -xy + (b - 1)z \\
u_4 &= dy - w
\end{align*}
\]

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

**Proof.** Substituting the active controller (3) from system (2), yields

\[
\begin{align*}
\dot{x} &= -x \\
\dot{y} &= -y \\
\dot{z} &= -z \\
\dot{w} &= -w
\end{align*}
\]

(4)

Then the characteristic values of the Jacobian matrix of system (4) are -1, -1, -1 and -1. So, the system (4) is asymptotically stable at the origin with the controller (3). Consequently, the controlling of system (2) is achieved via active control technique, the proof is completed.

Also, the adaptive control is employed to suppress the hyperchaotic system (2) to the unstable equilibrium based on Lyapunov stability theory by the following theorem.

**Theorem 2.** Let the adaptive control law be defined as

\[
\begin{align*}
u_1 &= (\overline{a} - 1)x - \overline{ay} \\
u_2 &= (\overline{z} - c)x - y - w \\
u_3 &= -xy + (\overline{b} - 1)z \\
u_4 &= d\overline{y} - w
\end{align*}
\]

(5)

and the parameters estimation updates law as follows

\[
\begin{align*}
\dot{\overline{a}} &= xy - x^2 \\
\dot{\overline{b}} &= -z^2 \\
\dot{\overline{c}} &= xy \\
\dot{\overline{d}} &= -yw
\end{align*}
\]

(6)

where \(\overline{a} = \overline{a} - a, \overline{b} = \overline{b} - b, \overline{c} = \overline{c} - c, \overline{d} = \overline{d} - d\) and \(\overline{a}, \overline{b}, \overline{c}, \overline{d}\) are the estimate values of these unknown parameters respectively. Then the controlled system (2) can asymptotically converge to the unstable equilibrium.

**Proof.** According to the Lyapunov stability theory, we construct the following Lyapunov candidate function

\[
V = 1/2(x^2 + y^2 + z^2 + w^2 + \overline{a}^2 + \overline{b}^2 + \overline{c}^2 + \overline{d}^2)
\]

and the time derivation of the Lyapunov candidate function is
\[
\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + a\dot{a} + b\dot{b} + c\dot{c} + d\dot{d} \\
= x[a(y-x) + u_1] + y[cx - xz + w + u_2] + z[xy - bz + u_3] + w[-dy + u_4] \\
+ a\dot{a} + b\dot{b} + c\dot{c} + d\dot{d} \\
\tag{7}
\]

Substituting the adaptive controller (5) and update law (6) into (7), yields

\[
\dot{V} = -x^2 - y^2 - z^2 - w^2 < 0
\]

It is clear that \(V\) is positive definite and \(\dot{V}\) is a negative definite. Therefore, based on the Lyapunov stability theory, the controlled system (2) can asymptotically converge to the unstable equilibrium with the controllers (5) and the parameter estimation update law (6). Consequently, the controlling of system (2) is achieved via adaptive control technique. This completes the proof.

Numerical simulations are used to investigate the controlled hyperchaotic system (2) using fourth-order Runge-Kutta scheme with time step 0.001. We choose the parameters \(a = 10, b = 8/3, c = 28, d = 10\) with active controller (3) and adaptive controller (5), and the initial values are taken as \((-5, -3, 20, 10)\) and the initial values of the parameter update law as \((1, 2, 0, 2)\). Fig. 2 show the state of hyperchaotic system (1) before the control. From Fig. 3 and Fig. 4, we can see the state of hyperchaotic system (2) with controller (3) and (5) respectively.

**4. Synchronization of a Modified Hyperchaotic Pan System**

In this section, active and adaptive synchronization of two identical hyperchaotic systems with unknown parameters are achieved based on the Jacobian matrix and Lyapunov stability theory respectively.

We choose the modified hyperchaotic pan system (1) as the drive system, and the controlled modified hyperchaotic pan system (2) as the response system.

Subtracting system (1) from the system (2), we obtain the error dynamical system between the drive system and the response system which is given by

\[
\begin{align*}
\dot{e}_1 &= a(e_2 - e_3) + u_1 \\
\dot{e}_2 &= (c - z)e_1 + e_4 - e_2 - xe_3 + u_2 \\
\dot{e}_3 &= -be_3 + e_1 e_2 + xe_1 + xe_3 + u_3 \\
\dot{e}_4 &= -de_1 + u_4
\end{align*}
\tag{8}
\]

where \(e_1 = x_1 - x, e_2 = y_1 - y, e_3 = z_1 - z\) and \(e_4 = w_1 - w\).

The goal of synchronization is to find active control for system (8) based on the Jacobian matrix such that the response system (2) and the drive system (1) are globally asymptotically synchronized. Then we obtain the following theorem.
Theorem 3. Let the active control law be defined as
\[
\begin{align*}
    u_1 &= -ae_1 \\
    u_2 &= (z-c)e_1 - e_4 + e_1 e_4 + x e_2 - y e_i \\
    u_3 &= (b-1)e_3 - e_4 e_2 - x e_2 - y e_i \\
    u_4 &= de_2 - e_i
\end{align*}
\] (9)
then the response system (2) is globally synchronization with the drive system (1).

Proof. Substituting the active controller (9) from system (8), yields
\[
\begin{align*}
    \dot{e}_1 &= -ae_1 \\
    \dot{e}_2 &= -e_1 \\
    \dot{e}_3 &= -e_3 \\
    \dot{e}_4 &= -e_4 
\end{align*}
\] (10)

Then the characteristic values of the Jacobian matrix of system (10) are -a, -1, -1 and -1. So, the system (1) and the system (2) are globally asymptotically synchronized. Consequently, the synchronization of the drive system (1) and the response system (2) is achieved via active control technique. The proof is completed.

Also, the adaptive control method is employed to synchronize between the hyperchaotic system (1) and the controlled hyperchaotic system (2) based on Lyapunov stability theory by the following theorem

Theorem 4. System (1) and (2) will approach global asymptotical synchronization for any initial condition with the following adaptive control and the parameters estimation updates law respectively:
\[
\begin{align*}
    u_1 &= (\bar{a}-1)e_1 - \bar{a}e_2 \\
    u_2 &= (z-c)e_1 - e_4 + e_1 e_4 + x e_2 + ye_i \\
    u_3 &= -e_4 e_2 + (\bar{b}-1)e_3 - x e_2 - ye_i \\
    u_4 &= \bar{d}e_2 - e_i
\end{align*}
\] (11)
\[
\begin{align*}
    \dot{a} &= e_4 e_2 - e_1^2 - \bar{a} \\
    \dot{b} &= -e_4^2 - \bar{b} \\
    \dot{c} &= e_4 e_2 - \bar{c} \\
    \dot{d} &= -e_4 e_2 - \bar{d}
\end{align*}
\] (12)

Proof. we construct the following Lyapunov candidate function
\[
V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \bar{a}^2 + \bar{b}^2 + \bar{c}^2 + \bar{d}^2)
\]
and the time derivation of the Lyapunov candidate function is
\[
\dot{V} = e_1 [a(e_2 - e_1) + u_1] + e_2 [(c-z)e_1 + e_4 - e_4 e_3 - x e_2 + u_2] \\
+ e_3 [-b e_1 + e_4 e_2 + x e_2 + y e_i + u_3] + e_4 [-d e_2 + u_4]
\] (13)

Substituting the adaptive controller (11) and updates law (12) into (13), yields
\[
\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - \bar{a}^2 - \bar{b}^2 - \bar{c}^2 - \bar{d}^2 < 0
\]

It is clear that \( V \) is positive definite and \( \dot{V} \) is a negative definite. Hence, synchronization between two identical hyperchaotic system (1) and (2) is achieve via adaptive control technique, This completes the proof.

Numerical simulations are used to investigate the controlled hyperchaotic system (2) using fourth-order Runge-Kutta scheme with time step 0.001. we choose the parameters \( a = 10, \ b = 8/3, \ c = 28, \) and \( d = 10 \) with active controller (9) and adaptive controller (11), the initial values for the drive system and the response system are \((-5,-3,20,10)\) and \((5,3,35,-10)\), respectively, the initial values of the parameter update law as \((1,2,0,2)\). From Fig.5, we can see that synchronization between two identical hyperchaotic systems with controller (9). Fig.6 show the state of the system (1) and system (2) before and after synchronization between them with controller (11).

Fig 5. The synchronization between the drive system (1) and the response system (2) via active control technique.
5. Conclusions

In this study, the control and synchronization of a modified hyperchaotic Pan system are performed through active and adaptive control based on the Jacobian matrix and Lyapunov stability theory to stabilize the system and error dynamics. It has been show that the active control technique has speed and easy performance from that the adaptive control although both methods have excellent and we explain this result analytically as well as graphically.

References


