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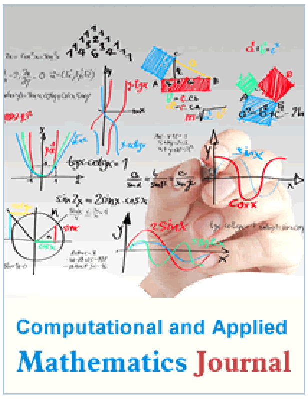

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Discrete-Time Hyperchaotic Systems Under Forced Two Levels Hierarchical Structure for Securing Communication

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Abstract
Based on putting dynamic discrete-time hyperchaotic systems under forced two levels hierarchical structure, to derive new sufficient conditions of asymptotic stability, a new strategy of control is formulated for hyperchaos synchronization of two identical 3D Hénon maps. The designed state feedback controller ensures that the state variables of both controlled hyperchaotic slave system globally synchronize with the state variables of the hyperchaotic master system. Numerical simulations are carried out to assess the performance of the proposed contributions in the important field of encryption and decryption through hyperchaotic synchronization phenomenon.

1. Introduction

Chaos and its applications in the field of secure communication have stimulated intense attentions during the last two decades. Indeed, the pioneering work done in the synchronization of chaotic systems, that was initiated by Pecora and Carroll [1-2], and the random-like behaviour of chaotic signals provide the potential for many applications. Among them, the introduction of chaos into secure communication field. In recent years, a growing number of cryptosystems based on chaos synchronization have been proposed such as: chaotic masking, chaotic modulation, chaotic shift keying [3-8].

The main purpose of this work is to determine necessary and sufficient conditions for the asymptotic stability of the error states between two identical hyperchaotic d iscrete-time processes. In fact, these processes can not only reach chaos synchronization, starting with different initial conditions but also can be applied to two secure communication channels based on chaotic systems. The proposed stabilizing conditions for nonlinear discrete-time two levels hierarchical systems are based on the Borne and Gentina practical criterion for stability study [14-17] associated to the forced arrow form matrix for system description [18-23].

The paper is organized as following. Hierarchical nonlinear systems structure and properties of arrow form matrices are presented in Section II. In Section III, is proposed an approach to design a linear state feedback, effective and systematic in achieving synchronization of discrete-time hyperchaotic systems, guarantying the asymptotic stability for the synchronization errors, characterized in the state space, by a forced arrow form matrix. The implementation of the proposed synchronization approach to two secure chaotic communication channels, using two identical discrete-time hyperchaotic Hénon systems will be performed in Section IV. In Section V, numerical simulations are carried out using this kind of discrete-time hyperchaotic systems, and the proposed secure communication scheme is provided in Section VI. Then, some concluding remarks are given.

2. Studied Two Levels Hierarchical Nonlinear Systems Description

2.1. Hierarchical Nonlinear Systems Structure

The studied hierarchical nonlinear systems (S), composed of r subsystems (S_i), Fig. 1., are described by the following differential equation:

$$x(k+1) = A\big(k, x(k)\big)\,x(k) \tag{1}$$

where A is the instantaneous characteristic matrix of (S) and A_{ii} of subsystems (S_i), $\forall i = 1,\dots,r$ [5-6,14-23].

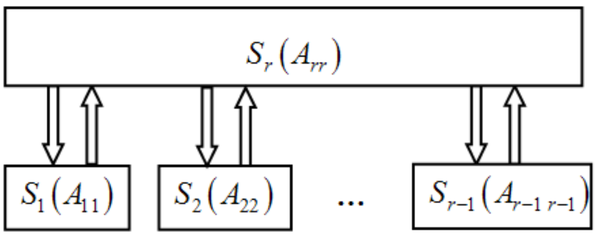

Fig. 1. *Two levels hierarchical nonlinear systems structure.*

2.2. Models Formulation

Three general kinds of the matrix A, called later arrow form matrices are introduced to represent the two levels hierarchical systems structure:

- the Thick Arrow Form matrix A_{TAF} (2), with A_{ii}, A_{rr}, A_{ir} and A_{ri}, respectively, an $(n_i \times n_i)$, $(n_r \times n_r)$, $(n_i \times n_r)$ and $(n_r \times n_i)$ matrices, such that, $\sum_{i=1}^{r} n_i = n$:

$$A = \begin{bmatrix} A_{11} & & & & A_{1r} \\ & \ddots & & & \vdots \\ & & A_{ii} & & A_{ir} \\ & & & \ddots & \vdots \\ A_{r1} & \dots & A_{ri} & \dots & A_{rr} \end{bmatrix} \tag{2}$$

- the Generalized thin Arrow Form matrix A_{GtAF} (3), where A_{ii} are scalar elements designed by: a_{ii}, $\forall i = 1,\dots r-1$, A_{rr}, A_{ir} and A_{ri} $\big((n-r+1)\times(n-r+1)\big)$, $\big(1\times(n-r+1)\big)$, $\big((n-r+1)\times 1\big)$ and $\big((n-r+1)\times(n-r+1)\big)$ matrices, respectively:

$$A_{GtAF} = \begin{bmatrix} a_{11} & & & a_{1r} & \cdots & a_{1n} \\ & \ddots & & \vdots & & \vdots \\ & & a_{r-1,r-1} & a_{r-1,r} & \cdots & a_{r-1,n} \\ a_{r1} & \cdots & a_{r,r-1} & a_{rr} & \cdots & a_{rn} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{n,r-1} & a_{nr} & \cdots & a_{nn} \end{bmatrix} \tag{3}$$

- and the thin Arrow Form matrix A_{tAF}, where A_{ii}, A_{ni} and A_{in}, $\forall i = 1,\dots,n$, are scalar elements.

Many physical linear or nonlinear systems can be directly described by characteristic matrices in arrow forms; for a large class of other systems, it is possible to introduce this form, as shown in Section III [22].

2.3. Arrow Form Matrices Properties

The computation of the determinant is so easy for the matrix A_{tAF}, noted $|A_{tAF}|$:

$$|A_{tAF}| = \Big(f_{nn} - \sum_{i=1}^{n-1} f_{ni}\, f_{ii}^{-1} f_{in} \Big) \prod_{i=1}^{n-1} f_{ii} \tag{4}$$

Then, for the matrix A_{TAF}, it comes out:

$$|A_{TAF}| = \Big| F_{rr} - \sum_{i=1}^{r-1} F_{ri}\, F_{ii}^{-1} F_{ir} \Big| \prod_{i=1}^{r-1} |F_{ii}| \tag{5}$$

The principal minors of order i of the matrix A_{GtAF}, noted $\Delta_i(A_{GtAF})$ and defined by:

$$\Delta_i(A_{GtAF}) = A_{GtAF}\begin{pmatrix} 1 & 2 & \dots & i \\ 1 & 2 & \dots & i \end{pmatrix} \quad \forall i = 1,\dots,n \tag{6}$$

can be computed easily, with the use of the following notations.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{7}$$

$$A_{11} = \mathrm{diag}\{a_{ii},\ \forall i = 1,\dots,r-1\} \tag{8}$$

$$A_{12} = \{a_{ij},\ \forall i = 1,...,r-1,\ \forall j = r,...,n\} \tag{9}$$

$$A_{21} = \{a_{ij},\ \forall i = r,...,n,\ \forall j = 1,...,r-1\} \tag{10}$$

and:

$$A_{22} = \{a_{ii},\ \forall i = r,...,n\} \tag{11}$$

Let $A_{12}^{(i)}$, $A_{21}^{(i)}$ and $A_{22}^{(i)}$, respectively, composed by the i first columns of A_{12}, the i first rows of A_{21} and the i first rows and the i first columns of A_{22}.

Then, the first $(r-1)$ principal minors are:

$$\Delta_i(A_{GtAF}) = a_{ii} \ \forall i = 1,\ldots,r-1 \quad (12)$$

and the followings defined by:

$$\Delta_i(A_{GtAF}) = \left|A_{22}^{(i)} - A_{21}^{(i)}A_{11}^{-1}A_{12}^{(i)}\right| \prod_{j=1}^{r-1}(a_{jj}) \quad \forall i = 1,\ldots,n-r+1 \quad (13)$$

Borne and Gentina criterion, based on matrix determinant computation and matrix principal minor computation, as far as the comparison system matrix is concerned, is then well adapted for systems described by characteristic matrices in arrow forms: A_{tAF}, A_{GtAF} and A_{TAF}, such that nonlinear elements are isolated in either one row or one column of the associated comparison system matrix.

3. 3D Generalized Hénon Map Description

In this Section, two identical hyperchaotic discrete-time Hénon maps calling master and slave systems are presented [24-27].

The master system is described by:

$$\begin{cases} x_{m1}(k+1) = -x_{m2}^2(k) - bx_{m3}(k) + \mu \\ x_{m2}(k+1) = x_{m1}(k) \\ x_{m3}(k+1) = x_{m2}(k) \end{cases} \quad (14)$$

and the slave system is given by:

$$\begin{cases} x_{s1}(k+1) = -x_{s2}^2(k) - bx_{s3}(k) + \mu \\ x_{s2}(k+1) = x_{s1}(k) + u_1(k) \\ x_{s3}(k+1) = x_{s2}(k) + u_2(k) \end{cases} \quad (15)$$

where $x_m = [x_{m1} \ \ x_{m2} \ \ x_{m3}]^T$ is the state vector of the master system (14), $x_s = \begin{bmatrix} x_{s1} & x_{s2} & x_{s3} \end{bmatrix}^T$ is the state vector of the slave system (15) and $u = [u_1 \ \ u_2]^T$ is the control vector to be designed later for achieving synchronization property.

The chaotic attractor of system (14) for $b = 0.1$ and $\mu = 1.76$, with the following initial conditions $x_m(0) = [1 \ \ -1 \ \ 0.5]^T$ is depicted in the Fig. 2., below.

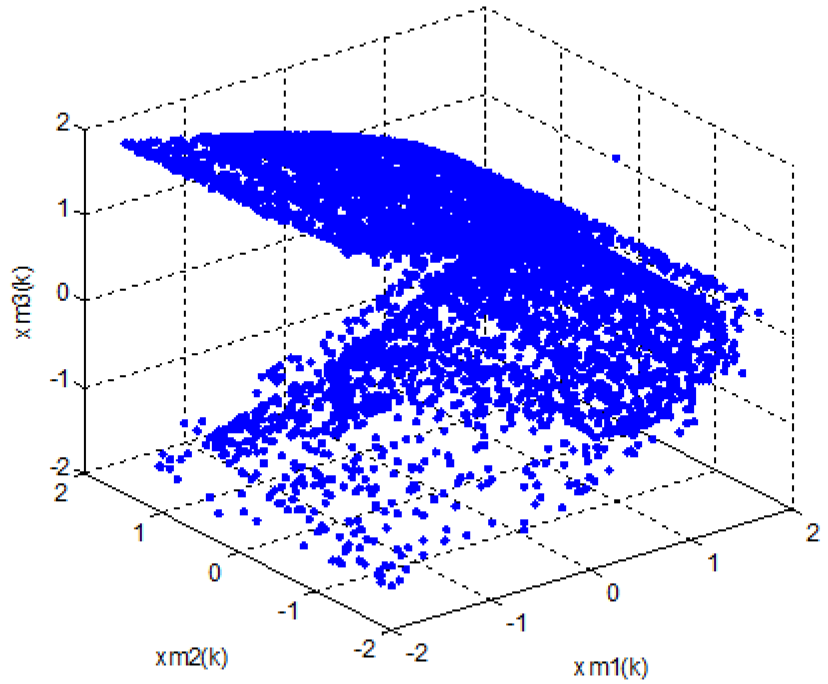

Fig. 2. *3D Chaotic attractor of the generalized Hénon map.*

The obtained responses of systems (14) and (15), when the control is turned off, presented in Fig. 3., show that both states are not yet synchronized. In the numerical simulation results presented through Fig. 3., it is assumed that the initial states of the slave Hénon map are specified as $x_s(0) = [-0.5 \ \ 0.2 \ \ 0.3]^T$.

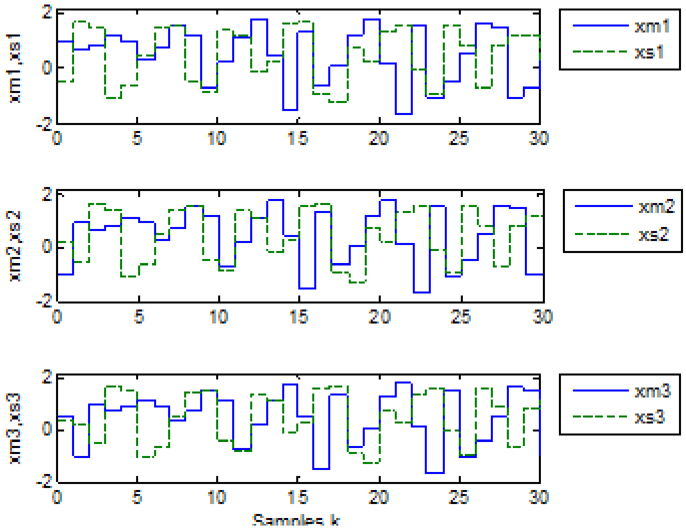

Fig. 3. *Evolutions of the master and slave Hénon maps state responses when controller is switched off.*

4. New Sufficient Conditions of Asymptotic Stability of Error Dynamics for Hyperchaotic Discrete - Time Systems

Let us consider the following dynamical error system:

$$e_i(k) = x_{si}(k) - x_{mi}(k) \ \forall i = 1,\dots,n \tag{16}$$

which leads to the state space description defined by (17):

$$e(k+1) = A(k, x(k))e(k) + Bu(k) \tag{17}$$

When the considered system (17) is stabilized by the state feedback $u(k)$, the errors (16) will converge to zero, i.e.:

$$\lim_{k\to+\infty}(e_i(k)) = 0 \ \forall i = 1,\dots,n \tag{18}$$

implying that the systems (14) and (15) achieve synchronization.

Indeed, in order to assure this main goal, the linear state feedback control law $u(k)$ is conceived as follows:

$$u(k) = -Ke(k) \tag{19}$$

with:

$$K = \{k_{ij}\} \ \forall i, j = 1,\dots,n \tag{20}$$

Consequently, it comes:

$$e(k+1) = A_f(k, x(k))e(k) \tag{21}$$

with:

$$A_f(k, x(k)) = A(k, x(k)) - BK \tag{22}$$

So, by putting in prominent position the application of the classical Borne and Gentina stability criterion, associated to the specific matrix description, namely, the forced arrow form matrix [5-6,14-23], the following theorem is derived.

Theorem. The process, described by (17) is stabilized by the state feedback control law defined by (19), if the characteristic matrix $A_f(k, x(k))$, given by (22), is under the forced arrow form and such that:

i. the nonlinear elements are isolated in either one row or one column of the matrix $A_f(k, x(k))$,

ii. the diagonal elements, $a_{f_{ii}}(k, x(k))$, of the characteristic matrix $A_f(k, x(k))$ are such that:

$$1 - \left|a_{f_{ii}}(k, x(k))\right| > 0 \ \forall i = n, n-1,\dots,2 \tag{23}$$

iii. there exist $\varepsilon > 0$ such that:

$$\left(\begin{array}{l}\left(1 - \left|a_{f_{11}}(k, x(k))\right|\right) \\ -\sum_{i=n}^{2}\left(\begin{array}{l}\left|a_{f_{1n}}(k, x(k))a_{f_{ni}}(k, x(k))\right| \\ \times\left(1 - \left|a_{ii}(k, x(k))\right|\right)^{-1}\end{array}\right)\end{array}\right) \geq \varepsilon \tag{24}$$

Proof. The overvaluing system $M(A_f(k, x(k)))$, associated to the vectorial norm $p(z(k))$ is defined, in this case, by the following system of differential equations:

$$z(k+1) = M(A_f(k, x(k)))z(k) \tag{25}$$

The process, described by (17) is stabilized by the state feedback control law defined by (19), if the matrix $(\mathbb{I} - M(A_f(k, x(k))))$ is the opposite of an $M-$matrix [5], or if, by application of the practical stability criterion of Borne and Gentina [6], we have:

$$\begin{cases} 1-\left|a_{f_{ii}}\left(k,x(k)\right)\right|>0 \ \forall i=n,n-1,\ldots,2 \\ \det\left(\mathbb{I}-M\left(A_f\left(k,x(k)\right)\right)\right)>0 \end{cases} \tag{26}$$

The development of the first member of the last inequality announced by (26):

$$\det\left(\mathbb{I}-M\left(A_f\left(k,x(k)\right)\right)\right)= \left(\begin{array}{l} \left(\begin{array}{l} 1-\left|a_{f_{11}}\left(k,x(k)\right)\right| \\ -\sum_{i=n}^{2}\left(\begin{array}{l}\left|a_{f_{in}}\left(k,x(k)\right)a_{f_{ni}}\left(k,x(k)\right)\right| \\ \times\left(1-\left|a_{f_{ii}}\left(k,x(k)\right)\right|\right)^{-1}\end{array}\right)\end{array}\right) \\ \times\left(\prod_{j=n}^{2}\left(1-\left|a_{f_{ii}}\left(k,x(k)\right)\right|\right)\right)\end{array}\right) \tag{27}$$

achieves easily the proof of the above mentioned Theorem.

Corollary. The process, described by (17) is stabilized by the state feedback control law defined by (19) if the characteristic matrix $A_f\left(k,x(k)\right)$, given by (22), is under the forced arrow form and such that:

i. all the nonlinearities are located in either one row or one column of $A_f\left(k,x(k)\right)$,

ii. the diagonal elements $a_{f_{ii}}\left(k,x(k)\right)$, of the matrix $A_f\left(k,x(k)\right)$, fulfil the constraints (23),

iii. there exist $\varepsilon>0$, such that:

$$a_{f_{in}}\left(k,x(k)\right)a_{f_{ni}}\left(k,x(k)\right)\geq\varepsilon,\ \forall i=n,n-1,\ldots,2 \tag{28}$$

iv. the instantaneous characteristic polynomial $P_{A_f}\left(k,x(k),\lambda\right)$ is strictly positive for $\lambda=1$.

Proof. The proof of this Corollary is inferred from the previous Theorem by taking into account the new added hypothesis (iii) of this corollary, which guarantee, through a simple transformation, the identity of the matrix $A_f\left(k,x(k)\right)$ and its associated overvaluing matrix $M\left(A_f\left(k,x(k)\right)\right)$; this specific case, clearly, agrees to the satisfaction of the linear Aizerman conjecture [5].

5. Synchronization of Two Coupled 3D Generalized Hénon Maps

In this section, we propose a systematic procedure to guarantee the synchronism property relatively to two identical three dimensional generalized Hénon maps. This approach determines a state feedback vector controller $u(k)=\left[u_1(k)\quad u_2(k)\right]^T$ letting the slave hyperchaotic Hénon system achieves synchronism with the master one.

5.1. Problem Statement

The dynamical error vector is chosen as following:

$$\begin{cases} e_1(k)=x_{s1}(k)-x_{m1}(k) \\ e_2(k)=x_{s2}(k)-x_{m2}(k) \\ e_3(k)=x_{s3}(k)-x_{m3}(k) \end{cases} \tag{29}$$

So, by referring to (14) and (15), the equations (29) lead to the following explicit description:

$$\begin{cases} e_1(k+1)=-\left(x_{m2}+x_{s2}\right)e_2(k)-0.1e_3(k) \\ e_2(k+1)=e_1(k)+u_1(k) \\ e_3(k+1)=e_2(k)+u_2(k) \end{cases} \tag{30}$$

In fact, the previous equations (30) can be rewritten under the matrix form (31):

$$e(k+1)=A\left(k,x(k)\right)e(k)+Bu(k) \tag{31}$$

with:

$$e(k)=\left[e_1(k)\quad e_2(k)\quad e_3(k)\right]^T \tag{32}$$

$$A\left(k,x(k)\right)=\begin{bmatrix} 0 & -\left(x_{m2}(k)+x_{s2}(k)\right) & -0.1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{33}$$

and:

$$B=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{34}$$

5.2. Hybrid Synchronization of the Coupled 3D Generalized Hénon Maps Via State Feedback Control Law

At this stage, the state feedback control law intended to stabilize the error dynamics, defined by (30), will be designed as proposed in (35):

$$u(k)=-Ke(k) \tag{35}$$

or, equivalently:

$$u(k)=-\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}e(k) \tag{36}$$

Then, it comes the controlled error dynamical system described, in the state space, by:

$$e(k+1)=A_f\left(k,x(k)\right)e(k) \tag{37}$$

where the instantaneous characteristic matrix $A_f\left(k,x(k)\right)$ is defined by (22), by respect to the illustrative and explicit expressions given in this Section.

Specifically, we have:

$$A_f(k, x(k)) = \begin{bmatrix} 0 & -(x_{m2}(k) + x_{s2}(k)) & -0.1 \\ 1-k_{11} & -k_{12} & -k_{13} \\ -k_{21} & 1-k_{22} & -k_{23} \end{bmatrix} \quad (38)$$

As long as these state feedback control laws, given by (36), stabilize the dynamical error system (31), $e_1(k)$, $e_2(k)$ and $e_3(k)$ will converge to zero as time tends to infinity, which implies that the synchronization of the two coupled Hénon maps (14) and (15) is reached.

Thus, for achieving this purpose, the elements $k_{ij}(.)$, $\forall i = 1, 2$ and $\forall j = 1, \dots, 3$, of the linear gain matrix K, must fulfil at the same time the inequalities (23) and (24), as well as the hypothesis (i) of the above pre-cited Theorem, already, announced in Section IV.

Therefore, the following linear gain matrix is considered as one optimal solution from many ones:

$$K = \begin{bmatrix} 1 & 0.2 & 0 \\ 2 & 1 & 0.5 \end{bmatrix} \quad (39)$$

Fig. 4. illustrates the effectiveness of the proposed method based on the use of aggregation techniques associated to the forced arrow form matrix for system description.

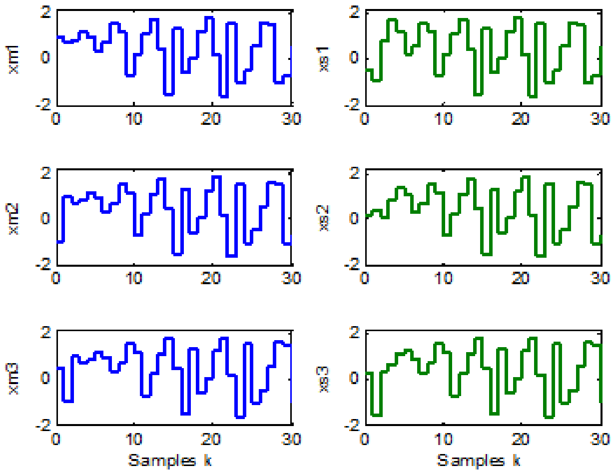

Fig. 4. *Time responses of spatiotemporal chaos synchronization of master and slave Hénon state variables.*

6. Application in Secure Information Transmission

6.1. Basic Idea

Since each chaotic system is able to reproduce the same signal, chaos begins to have practical applications. A secret signal can be embedded with the chaos of one system. The resulting signal appears to be only chaos and noise, and is useless. However, with the synchronizing system producing the same signal, the chaos can be extracted from the transmitted information, leaving only the secret signal transmitted. Thus, chaos can be used as a form of information encryption.

At the transmitter terminal, the useful message is secretly embedded in the parameter of the transmitter chaotic system and the chaotic receiver system is designed to successfully recuperate the former message. So, under some structural assumptions, the recovered signal can exponentially approximate the source signal.

6.2. Case of Discrete-Time Hyperchaotic Systems Synchronization for Secure Color Image Transmission

In this Subsection, the problem of synchronization between two identical hyperchaotic Hénon systems is applied to a new chaos-based image cryptosystem, in order to illustrate the feasibility of the theoretical proposed approach. The input of the considered cryptosystem is the plain image which will be encrypted.

Firstly, we form a vector with three layers in the RGB format containing the image colors. After that, the hyperchaotic signal of the master transmitter system is added to the image, to further enhance the complexity of the considered cryptosystem and thereby improving the security of the image transmission process. Subsequently, the image is successfully recovered through the subtraction between the encrypted image and the slave receiver hyperchaotic signal. At last, the three layers are joined in order to form the color image, as illustrated in Fig. 5.

(a) (b) (c)

Fig. 5. (a). *Original image, (b) Encrypted image, (c) Decrypted image.*

More to the point, it is obvious that the security is compromised even without precise knowledge of the hyperchaotic systems used.

7. Conclusion

In this work, the secure communication problem based on the synchronization of hyperchaotic systems is investigated. The asymptotic convergence of the errors between the states of the master system and the states of the slave system is proven using aggregation techniques associated to forced arrow form matrix properties. The scheme of secure transmission implies the use of the Hénon hyperchaotic system, to encrypt and decrypt the useful information. The emitted signal is modulated into the parameter information of the transmitter system, and the resulting system is still hyperchaotic. The corresponding receiver is designed so that it is able to retrieve, secretly, the former signal. From simulation results, it can be concluded that the developed theoretical approaches are feasible and efficient, since they are fruitfully exploited to confidentially transmit and get back one chosen colour image.

References

[1] *L*. M. Pecora, T. L. Carroll, Synchronization in chaotic systems, Physics Revue Letters, vol. 64, n° 8, pp. 821-824, 1990.

[2] *T. L. Carroll, L. M. Pecora*, Synchronizing chaotic *circuits*, *IEEE Transactions on Circuits and Systems,* vol. *38*, n° 4, *pp. 453-456*, *1991.*

[3] *G*. Chen, X. Dong, From chaos to order: methodologies, perspectives and applications, World Scientific, Singapore, 1998.

[4] *T*. L. Liao, S. H. Tsai, Adaptive synchronization of chaotic systems and its application to secure communication, Chaos, Solitons & Fractals. vol. 11, n° 9, pp. 1387-1396, 2000.

[5] S. Hammami, K. Ben Saad, M. Benrejeb, On the synchronization of identical and non-identical 4-D chaotic systems using arrow form matrix, Chaos, Solitons & Fractals, vol. 42, n° 1, pp. 101-112, 2009.

[6] S. Hammami, M. Benrejeb, M. Feki, P. Borne, Feedback control design for Rössler and Chen chaotic systems anti-synchronization, Physics Letters A, vol. 374, n° 28, pp. 2835-2840, 2010.

[7] M. G. Rosenblum, A. S. Pikovsky, J. Kurths, Phase synchronization of chaotic oscillators, Physics Revue Letters, vol. 76, n° 11, pp. 1804-1807, 1996.

[8] M. G. Rosenblum, A. S. Pikovsky, J. Kurths, From phase to lag synchronization in coupled chaotic oscillators, Physics Revue Letters, vol. 78, n° 22, pp. 4193, 1997.

[9] N. F. Rulkov, M. M. Sushchik, L. S. Tsimring, H. D. I. Abarbanel, Generalized synchronization of chaos in directionally coupled chaotic systems, Physics Revue E, vol. 51, n° 2, pp. 980-994, 1995.

[10] R. Mainieri, J. Rehacek, Projective synchronization in three-dimensional chaotic systems, Physics Revue Letters, vol. 82, n° 15, pp. 3042-3045, 1999.

[11] X. S. Yang, Concepts of synchronization in dynamical systems, Physics letters A, vol. 260, n° 5, pp. 340-344, 1999.

[12] H. C. Juhn, K. C. Hsien, K. L. Yu, Synchronization and anti-synchronization coexist in Chen-Lee chaotic systems, Chaos, Solitons & Fractals, vol. 39, n° 2, pp. 707-716, 2009.

[13] X. Huang, Z. Wang, Y. Li, Hybrid synchronization of hyperchaotic Lü system based on passive control, in the International Workshop on Chaos-Fractal, Theories and Applications, pp. 34-38, 2010.

[14] S. Hammami, M. Benrejeb, Coexistence of synchronization and anti-synchronization for chaotic systems via feedback control, Chaotic Systems, Croatia: Editions INTECH, pp. 203-224, 2011.

[15] S. Hammami, M. Benrejeb, P. Borne, New nonlinear output feedback controller for stabilizing the Colpitts oscillator, International Journal on Sciences and Techniques of Automatic control and computer engineering, vol. 3, n° 2, pp. 996-1011, 2009.

[16] S. Hammami, M. Benrejeb, P. Borne, On nonlinear continuous systems stabilization using arrow form matrices, International Review of Automatic Control, vol. 3, n° 2, pp. 106-114, 2010.

[17] S. Hammami, M. Benrejeb, New stabilizing controller nonlinear system design using the generalized thin arrow form matrix. Application to the chaos synchronization, in the 12th Int Federation of Automatic Control Large Scale Systems Symposium: Theory and Applications, Lille, Villeneuve d'Ascq, France, 2010.

[18] S. Hammami, Nonlinear observer design for chaotic systems synchronization with known or unknown parameters, International Journal of Emerging Science and Engineering, vol. 1, n° 8, pp. 32-38, 2013.

[19] S. Hammami, Synchronization of identical chaotic systems using new state observers strategy, International Journal of Innovative Technology and Exploring Engineering, vol. 3, n° 2, pp. 144-149, 2013.

[20] S. Hammami, Secure image transmission via nonlinear observer-based chaotic synchronization, Archives Des Sciences, vol. 66, n° 7, pp. 100-115, 2013.

[21] S. Hammami, Feedback 4-D chaotic systems synchronization for secure image encoding, in the IEEE World Congress on Multimedia and Computer Science, in the International Conference on Computer Science and Applications, Hammamet, Tunisia, 2013.

[22] S. Hammami, Hybrid synchronization of discrete-time hyperchaotic systems based on aggregation techniques for image encryption, in the IEEE 14th International conference on Sciences and Techniques of Automatic control & computer engineering, Sousse, Tunisia, pp. 325-330, 2013.

[23] S. Hammami, Security analysis of high dimensional chaotic-based cryptosystem via its key sensitivity study, Journal of Information Security Research, vol. 4, n° 4, pp. 183-194, 2013.

[24] G. Baier, M. Klein, Maximum hyperchaos in generalized Hénon circuit, Physics Letters A, vol. 151, n° 67, pp. 281-284, 1990.

[25] D. A. Miller, G. Grassi, A discrete generalized hyperchaotic Hénon map circuit, in the 44th IEEE Midwest Symposium on Circuits and Systems, vol. 1, pp. 328-331, Dayton, Ohio, August 2001.

[26] G. Grassi, D. A. Miller, Theory and experimental realization of observer-based discrete-time hyperchaos synchronization, IEEE Transactions on CAS-I, vol. 49, n° 3, pp. 373-378, 2002.

[27] E. Li, G. Li, G. Wen, H. Wang, Hopf bifurcation of the third-order Hénon system based on an explicit criterion, Nonlinear Analysis: Theory, Methods & Applications, vol. 70, n° 9, pp. 3227-3235, 2009.