Analysis of Time Distribution of Immittance Spectral Frequencies and Technique for Their Calculation

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Citation

Abstract
The properties of inter-frame placement for immittance spectral frequencies (ISF) are studied. As a result, a novel efficient method of ISF calculation is proposed. It is shown that average number of operations is reduced in 2.3 in comparison with standard grid approach. Besides, the observed maximum number of operations is 1.7 times lower than the minimum operations number of AMR-WB approach. The performance in different frame error rate (FER) environments is analyzed. The method was implemented in fixed-point wideband vocoder and showed stable performance.

1. Introduction

The development of speech compression methods requires for simple and effective parametric models of speech based on knowledge of speech production and perception mechanisms. A majority of modern speech processing methods are based on autoregressive (AR) model where the spectral information is represented by means of AR coefficients \( a_k, k = 1,2,\ldots, p \). It is evident that wideband speech coding requires higher AR model orders in comparison with narrow-band one [1].

However, AR coefficients are not directly used in speech coding devices due to their high spectral sensitivity and need to be transformed to some alternative set of parameters for which efficient quantization procedures can be applied. The most efficient alternative set of AR parameters became line spectral frequencies (LSF) [2] and immittance spectral frequencies (ISF) introduced some later due to their attractive quantization properties [3, 4].

Computation of ISF and LSF is connected with rootfinding procedures which are undesired for a majority of computational devices (especially fixed-point DSPs) because of unpredictable delays and errors accumulation. The most widely used approaches are the grid methods [5] and decimation-in-degree methods [6]. In work [7] a predictions of LSF values were used to accelerate performance of grid methods. In [8] the possibility to enhance this approach on the base of introduced LSF/ISF inter-frame ordering property was shown.

The main goals of this paper are: 1) to continue study of ISF inter-frame placement which are important for their localization task; 2) to design ISF calculation procedure which unites features of their inter-frame placement and principles for the solution of non-linear equations [9], [10].

In the second section of paper we investigate and discuss the mutual ISF location on adjacent frames and discuss how it can be used for their localization. In the third section a new algorithm of ISF calculation is proposed. In the experimental section we analyze
the performance of method in terms of number of calculations and performance at packet loss conditions.

2. Investigation of ISF Inter-Frame Placement

2.1. Preliminary Results

As is known [3], calculation of ISF \( \omega_k, k = 1, 2, \ldots, p-1 \) can be reduced to the search of roots of polynomials \( P_1(x) \), \( P_2(x) \), where

\[
P_1(x) = \sum_{k=0}^{p-1} x^{k} = 0,
\]

\[
P_2(x) = \sum_{k=0}^{2p-1} x^{2k} = 0.
\]

The roots of these equations are equal to the ISF cosines and satisfy to intra-frame ordering property:

\[
x_{i} < x_{j} < x_{i+1}, i = 2, \ldots, p-2,
\]

where odd cosines correspond to \( P_1(x) \) polynomial, and even - to \( P_2(x) \).

Traditionally the rootfinding procedures consist of roots’ localization and refinement. Our strategy of ISF calculation consists of following stages:

- Localization of roots of \( P_2(x) \) polynomial;
- Refinement of roots of \( P_2(x) \) polynomial;
- Refinement of roots of \( P_1(x) \) polynomial (their automatic location is guaranteed by property (1)).

Let’s consider ISF properties which are important for their localization task.

2.2. Results

Fig. 1 shows time plot and corresponding ISF plot for a speech fragment pronounced by male speaker. The record was digitized with sampling frequency of 16000 Hz and then resampled to 12800 Hz. The calculation of ISF was performed on windowed 20 ms frames for the order of AR model \( p = 16 \).

It can be immediately observed that in a lot of situations inter-frame ordering property [7] takes place:

\[
x_{i+1}^{(n-1)} < x_{i}^{(n)} < x_{i+1}^{(n)}, i = 2, \ldots, p-2,
\]

where the upper indices denote frame numbers.

However, from the localization viewpoint another questions are of high importance

1. May an interval \( [x_{i-1}^{(n-1)}, x_{i+1}^{(n-1)}], i = 2,4, \ldots, p-2 \) contain more than two roots of \( P_2(x) \)?

2. How often an interval \( [x_{i-1}^{(n-1)}, x_{i+1}^{(n-1)}], i = 2,4, \ldots, p-2 \) contains a unique root of \( P_2(x) \)?

To answer these questions we studied immittance spectral frequencies of wideband speech signals (resampled from 16000 Hz to 12800 Hz). The database of nine speakers with general duration of 16 minutes was used. AR model orders from 14 to 20 were considered. The following conclusions were made.

1. As for the first question, there were only about 0.002% of situations when more than two roots of \( P_2(x) \) were located between roots of \( P_1(x) \) at previous frame.

2. The results for the second question are shown in Tab. 1. It shows that investigated property is true for a majority of cases. Corresponding percentage lies from 95.3% at \( p = 20 \) to 97.2% at \( p = 14 \).
2.2. Discussion of Established ISF Properties

Established properties have crucial importance for the ISF localization task. According to the second property, localization of polynomial \( P_n(x) \) roots can be mainly reduced to the comparison of \( P_2(x) \) signs in points \( \{-1, x_5^{(n-1)}, x_5^{(n-1)}, \ldots, x_p^{(n-1)}, 1\} \). If the signs of \( P_2(x_5^{(n-1)}) \) and \( P_2(x_5^{(n-1)}) \) do not coincide, then a unique root is contained at this interval (the presence of any other odd amount of roots is excluded due to the first of established properties). The example is shown at Fig. 2. To establish the presence of roots at intervals \( [x_5^{(n-1)}, x_5^{(n-1)}] \) and \( [x_5^{(n-1)}, x_5^{(n-1)}] \) the method for the localization of non-linear equation roots [9] can be applied.

3. Resulting Algorithm of ISF Computation

According to the above mentioned investigation of ISF mutual placing at adjacent frames, the following algorithm for their computation is proposed.

**Initial data.** AR coefficients of current frame \( a_z, k=1,2,\ldots,p \); values of ISF cosines \( x_1^{(n-1)}, x_2^{(n-1)}, \ldots, x_p^{(n-1)} \) calculated at previous frame.

**Step 1. Calculation of \( P_2(x) \) coefficients.** AR coefficients \( \{a_z\} \) are transformed to coefficients of polynomial \( P_2(x) \) by Chebyshev polynomial mechanism [5].

**Step 2. Localization of even ISF cosines.** The location of roots of \( P_2(x) \) is verified by comparison of \( P_2(x) \) signs in points \( \{-1, x_5^{(n-1)}, x_5^{(n-1)}, \ldots, x_p^{(n-1)}, 1\} \).

If \( \text{sign}(P_2(x_5^{(n-1)})) = \text{sign}(P_2(x_5^{(n-1)})) \), the presence of root at interval \( [x_5^{(n-1)}, x_5^{(n-1)}] \) is verified by the method for the solution of nonlinear equations [9].

**Step 3. Refinement of even roots.** After the localization of ISF cosines, their exact values are refined as in [1].

**Step 4. Calculation of odd ISF cosines.** The odd ISF cosines are refined on intervals \( [-1, x_2^{(n)}], [x_2^{(n)}, x_2^{(n)}], \ldots, [x_p^{(n)}, 1] \).

**Step 5. Transformation to \( \omega \)-domain.** ISF are finally calculated by transformation \( \omega = \arccos(x) \).

Note. In case of previous frame loss, the odd cosines of the last correct frame must be used as a reference points at Step 2.

4. Experimental Results

The effectiveness of the proposed ISF computation method was verified for AR model order \( p = 16 \). For this purpose we used a database described in section 2. Tab. 2 shows average computational expenses in terms of WMOPS and polynomial function evaluations per frame (\( N \)) corresponding to proposed method and the AMR-WB approach. At the same time, it is also instructive to test method performance in packet loss conditions. For this purpose frame error rates (FER) of 5% and 20% were also considered.

<table>
<thead>
<tr>
<th>( N_{\text{av}} )</th>
<th>( N_{\text{max}} )</th>
<th>WMOPS( N_{\text{av}} )</th>
<th>WMOPS( N_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMR-WB method</td>
<td>136.6</td>
<td>142</td>
<td>0.80</td>
</tr>
<tr>
<td>Prop. method</td>
<td>58.8</td>
<td>82</td>
<td>0.34</td>
</tr>
<tr>
<td>Prop. method (FER = 5%)</td>
<td>58.8</td>
<td>82</td>
<td>0.34</td>
</tr>
<tr>
<td>Prop. method (FER = 20%)</td>
<td>59.5</td>
<td>82</td>
<td>0.35</td>
</tr>
</tbody>
</table>

From Tab. 2 it can be seen that proposed algorithm provides a reduction of computational expenses in approximately 2.3 times in comparison with grid approach. Another important property is that maximum value \( N_{\text{av}} \) of the proposed method is 1.7 times lower than the minimum \( N_{\text{av}} \) value for the AMR-WB method.

Another important fact is that frame errors almost do not influence the performance of the proposed method (see Table 2). Fig. 3 shows the example of polynomial evaluations number (\( N \)) distribution for AMR-WB approach and proposed method in case of speech signal pronounced by female speaker with duration of 3.5 sec. The minimum computational expenses take place in pauses filled by
stationary background noise, while the small peaks in the
distribution of operations take place at transitions from one
sound to another as well as in the cases of frame loss.

Figure 3. Top: spectrogram of test signal; middle and bottom: distribution of
polynomial evaluations at FER=0% and 20% respectively.

The accuracy of proposed method is bounded only by
calculation errors and does not depend on a priori distance
between roots as in the case of grid methods. At the same
time, the method does not introduce accumulation of errors
as decimation-in-degree approaches [6]. The method was
implemented on fixed-point ADSP 2191 wideband vocoder
(4800 bit/sec) and showed stable performance.

5. Conclusions

The main goal of this paper was to study properties of ISF
inter-frame placement and to design efficient ISF calculation
procedure which optimally exploits features of ISF time
distribution.

1. As a result of ISF behavior study on the large wideband
speech database, it was stated that in a majority of situations
(from 95.3% to 97.2% for different AR model orders) the
roots of second ISF polynomial are interlaced with the roots
of the first polynomial at the previous frame. At the same
time, there were practically no situations when more than two
roots of \( P_2(x) \) occurred between a pair of \( P_1(x) \) roots at the
previous frame.

2. On the base of established properties the algorithm for
ISF calculation was proposed. In the case when two roots
occur between a pair of previous frame roots, they are
separated by proposed earlier method for the localization of
roots of non-linear equations.

3. It was shown that average number of operations is
reduced in 2.3 times in comparison with AMR-WB ISF
calculation method. Besides, the observed maximum number
of operations is 1.7 times lower than the minimum operations
number of AMR-WB approach. This suggests the advantage
of application of proposed method in real-time systems.

4. It was shown that the performance of the proposed
method practically does not depend on the frame loss. Even
at FER=20% the computational characteristics of proposed
method remain almost the same as in perfect channel
conditions.

5. In oppose to grid approach, the performance of
proposed method does not depend on a priory data on
minimal distance between roots. It also does not introduce
accumulating errors typical for decimation-in-degree
approach. The accuracy of proposed method is only
conditioned by the calculation errors specified by computing
platform.

6. The method was implemented into fixed-point wideband
vocoder and showed stable work.

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