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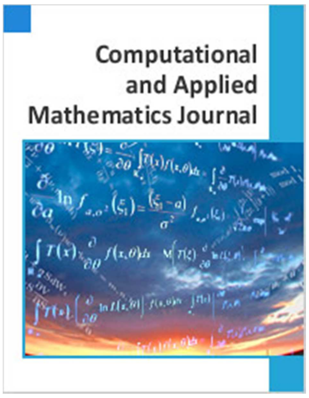

Steady Blood Flow Through a Porous Medium with Periodic Body Acceleration and Magnetic Field

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Abstract
The present study is concerned with a mathematical model for steady flow of blood through a porous medium in a rigid straight circular tube under the influence of periodic body acceleration and magnetic field by considering blood as a couple stress fluid. The physiological parameters that affect human body such as axial velocity, shear stress and flow rates have been computed analytically as an exact solution in the Bessel's Fourier series form by the finite Hankel transform techniques. It is of interest to note that as the magnetic effect is increased the velocity of the blood decreased whereas it increases as permeability of the porous media and body acceleration increases. The effects of shear stress and other parameters are shown through graphs. Finally, the applications of this mathematical model to some biological and cardiovascular diseases by the magnetic effect have been indicated.

1. Introduction

Human body may encounter with vibrations or accelerations that can be transmitted through the seat or feet (known as whole-body vibration or WBV). Drivers of some mobile machines, including certain tractors, operating a dumper truck, fork lift trucks can cause fatigue, insomnia, headache and shakiness with symptoms similar to those that many people experience after a long car or boat trip. After daily exposure over a number of years these same whole-body vibrations can result in number of health disorders affecting the entire body including permanent harm to internal organs, muscles, joints and bone structure. Other work factors such as posture and heavy lifting are also known to contribute to back pain problems for drivers. Many researchers have studied the effect of periodic body acceleration by considering blood as non- Newtonian, third grade fluid and visco-elastic fluid [1-3].

The biological systems in general are greatly affected by the application of external magnetic field. Kollin [4] has introduced the electromagnetic theory in the field of medical research for the first time in the year 1936.The heart rate decreases by exposing biological systems to an external magnetic field, reported by Barnothy [5]. The application of magnetic field for regulating the blood movement in the human system was discussed by Korcheuskii and Marochnik [6]. Gold [7] investigated the effect of uniform transverse magnetic field to a non-conducting circular pipe. The effect of transverse magnetic field on a physiological type of flow through uniform circular pipe has been studied by Rao and Deshikachar [8]. The application of magnetic field deaccelerates the blood flow was mentioned by Vardanyan [9].

When the fatty and fibrous tissues are clotted in the wall lumen, its distribution acts like a porous medium. The general equation of motion for the flow of a viscous fluid through a porous medium has been derived by Ahmadi and Manvi [10]. The porous material containing the fluid is in fact a non-homogenous medium. For the sake of analysis it is possible to replace it with a homogenous fluid which has dynamical properties equivalent to the local averages of the original non-homogenous medium. The distribution of fatty cholesterol and artery-clogging blood clots in the lumen of the coronary artery in some pathological situations can be considered as equivalent to a fictitious porous medium by Dash et al [11]. Mishra [12] studied the effect of porous parameter and stenosis height on the wall shear stress of human blood flow. Gaur and Gupta [13] also analyzed the magnetic effects on steady blood flow through an artery under axisymmetric stenosis.

Even though the flow in the human circulatory system is unsteady, particularly at the pre-capillary level, steady flow models provide some insight into the aspects of flow through the arteries and some applications can be found using the steady flow models. As can be expected, steady flow models are also simpler to use because of the absence of the time variations in the governing equations. Steady flow models also avoid the complexity of the moving interface between the blood and the vessel wall as the artery distends with the pulse pressure. The effects of body acceleration and magnetic field on human body have been investigated as a steady flow model by considering blood as couple stress fluid [14, 15].The steady flow of blood as Casson fluid through a cylindrical artery with porous walls has been studied by Gaur and Gupta [16]. Gaur and Gupta [17] also analyzed the magnetic effect on steady blood flow through an artery under axisymmetric stenosis. An effort has been made to study the Steady blood flow through a porous medium with periodic body acceleration in the presence of uniform transverse magnetic field by considering blood as a couple stress fluid in a uniform straight and rigid circular tube.

2. Formulation of the Problem

Blood is assumed as an electrically conducting, incompressible and couple stress fluid, magnetic field is acting along the radius of the tube and flow is axially symmetric.

The pressure gradient and body acceleration are given by:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \tag{2.1}$$

$$G = a_0 \cos(\varphi) \tag{2.2}$$

Where A_0 is the steady – state part of the pressure gradient, A_1 is the amplitude of the oscillatory part, a_0 is the amplitude of body acceleration, φ is its phase difference, z is the axial distance. Based on Stokes [18] model, a one-dimensional steady blood flow through a uniform, straight, rigid and non-conducting tube under body acceleration through a porous medium in the presence of magnetic field has been formulated as

$$\begin{gathered}\eta\nabla^2(\nabla^2 u) - \mu\nabla^2 u + \sigma B_0^2 u = -\frac{\partial p}{\partial z} + \rho G - \frac{\mu}{K}u \\ where \quad \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\end{gathered} \tag{2.3}$$

where u(r) is velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field and r is the radial coordinate.

Let us introduce the following dimensionless quantities:

$$\begin{gathered}u^* = \frac{u}{\omega R},\ r^* = \frac{r}{R},\ A_0^* = \frac{R}{\mu\omega}A_0,\ A_1^* = \frac{R}{\mu\omega}A_1, \\ a_0^* = \frac{\rho R}{\mu\omega}a_0,\ z^* = \frac{z}{R},\ K^* = \frac{K}{R^2}\end{gathered} \tag{2.4}$$

In terms of these variables, equation (2.3) [after dropping the stars] becomes

$$\nabla^2\left(\nabla^2 u\right) - \bar{\alpha}^2\,\nabla^2 u - \bar{\alpha}^2(A_0 + A_1 + a_0\cos\varphi) + \bar{\alpha}^2(H^2 + h^2)u = 0 \tag{2.5}$$

Where $\bar{\alpha}^2 = \dfrac{R^2\mu}{\eta}$ - Couple stress parameter, $H = B_0 R\left(\dfrac{\sigma}{\mu}\right)^{\frac{1}{2}}$ is the Hartmann number and R is the radius of the pipe.

The boundary conditions for this problem are:

$$\begin{aligned}&u \ and\ \nabla^2 u \quad are\ all\ finite\ at\ r = 0 \\ &u = 0,\quad \nabla^2 u = 0 \qquad at\quad r = 1\end{aligned} \tag{2.6}$$

3. Required Integral Transform

If $f(r)$ satisfies Dirichlet conditions in closed interval [0, 1] and if its finite Hankel transform Sneddon [19, page 82] is defined to be:

$$f^*(\lambda_n) = \int_0^1 r\,f(r)\,J_0(r\lambda_n)\,dr\,, \tag{3.1}$$

where the λ_n are the roots of the equation $J_0(r) = 0$. Then at each point of the interval at which $f(r)$ is continuous:

$$f(r) = 2\sum_{n=1}^{\infty} f^*(\lambda_n)\frac{J_0(r\lambda_n)}{J_1^2(\lambda_n)} \tag{3.2}$$

where the sum is taken over all positive roots of $J_0(r) = 0$, J_0 *and* J_1 are Bessel function of the first kind.

Now applying finite Hankel transform (3.1) to eqn. (2.5) in the light of (2.6) we obtain:

$$u^* = \frac{J_1(\lambda_n)\bar{\alpha}^2}{\lambda_n} \frac{\left(A_0 + A_1 + a_0 \cos\varphi\right)}{\left(\lambda_n^4 + \bar{\alpha}^2\left(\lambda_n^2 + H^2 + h^2\right)\right)} \tag{3.3}$$

Now the finite Hankel inversion of (3.3) gives the final solution as:

$$u(r) = 2\sum_{n=1}^{\infty} \frac{J_0(r\lambda_n)\bar{\alpha}^2}{\lambda_n J_1(\lambda_n)} \cdot \frac{[A_0 + A_1 + a_0 \cos\varphi]}{[\lambda_n^4 + \bar{\alpha}^2\ (\lambda_n^2 + H^2 + h^2)]} \tag{3.4}$$

The expression for the flow rate can be written as:

$$Q = 2\pi \int_0^1 r\,u\,dr \tag{3.5}$$

then

$$Q(r) = 4\pi \sum \frac{\bar{\alpha}^2}{\lambda_n^2} \cdot \frac{[A_0 + A_1 + a_0 \cos\varphi\ ]}{[\lambda_n^4 + \bar{\alpha}^2\,(\lambda_n^2 + H^2 + h^2)]} \tag{3.6}$$

Similarly the expression for the shear stress τ_w can be obtained from

$$\tau_w = \mu \frac{\partial u}{\partial r} \tag{3.7}$$

$$\tau_w = -2\mu \sum_{n=1}^{\infty} \frac{J_1(r\lambda_n)\bar{\alpha}^2}{J_1(\lambda_n)} \frac{[A_0 + A_1 + a_0 \cos\varphi]}{[\lambda_n^4 + \bar{\alpha}^2(\lambda_n^2 + H^2 + h^2)]} \tag{3.8}$$

4. Results and Discussions

The velocity profile for a steady blood flow computed by using (3.4) for different values of Hartmann number H, body acceleration (a_0), permeability of the porous medium K and couple stress parameter $\bar{\alpha}$ and have been shown through Figs. 1 to 4. It can be observed that as the value of the Hartmann number (H) increases the velocity of the blood decreases, whereas the velocity profile increases with the increase of permeability of the porous medium K and body acceleration a_0 which are shown in Figs 1,. 2. and 3.

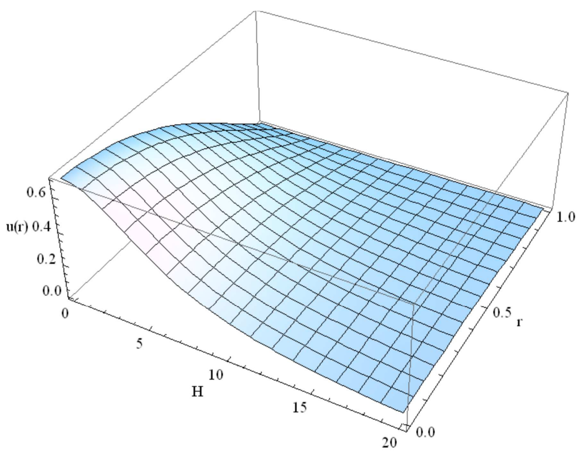

Fig. 1. *Variation of velocity profile with Hartmann number H;* $K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \bar{\alpha} = 1, \varphi = 15^0$

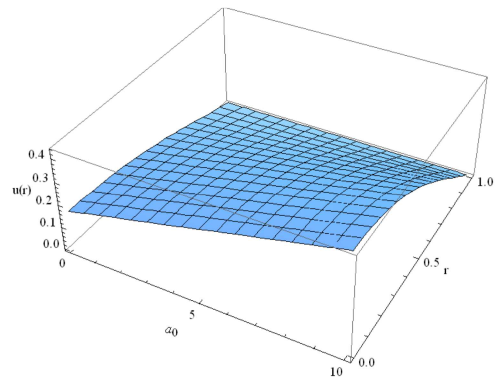

Fig. 2. *Variation of velocity profile with amplitude of body acceleration* a_0*;* $K = 2.5, A_0 = 2, A_1 = 4, \bar{\alpha} = 1, \varphi = 15^0, H = 4$

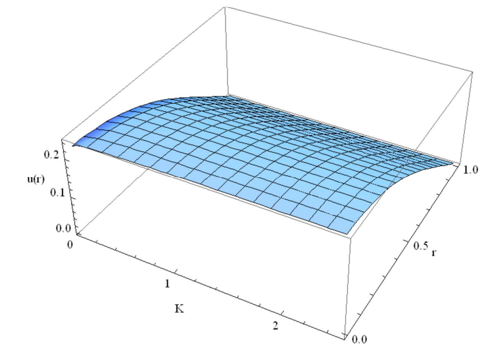

Fig. 3. *Variation of velocity profile for different values of permeability parameter K;* $A_0 = 2, A_1 = 4, a_0 = 3, \bar{\alpha} = 1, \varphi = 15^0, H = 4$

As the couple stress parameter $\bar{\alpha}$ increases, the velocity profile increases initially and slightly decreases at higher values of $\bar{\alpha}$ and is shown in Fig. 4.

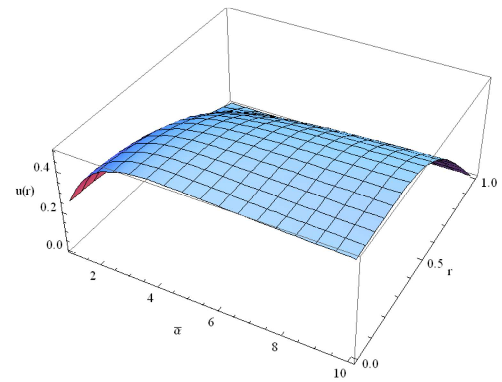

Fig. 4. *Variation of velocity profile for different values of couple stress parameter* $\bar{\alpha}$*;* $K = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \varphi = 15^0, H = 4$

The physiological parameter wall shear stress has been computed by (3.8) for different values of Hartmann number H, body acceleration (a_0) and permeability of the porous media K and are shown through graphs 5 to 7. The wall shear stress decreases with an increase of Hartmann number H and a reverse flow is observed which is shown in Fig. 5. The wall shear stress increases with an increase of amplitude of body acceleration (a_0) and permeability of the porous media K, a reverse flow is observed which are shown through Figures 6 to 7.

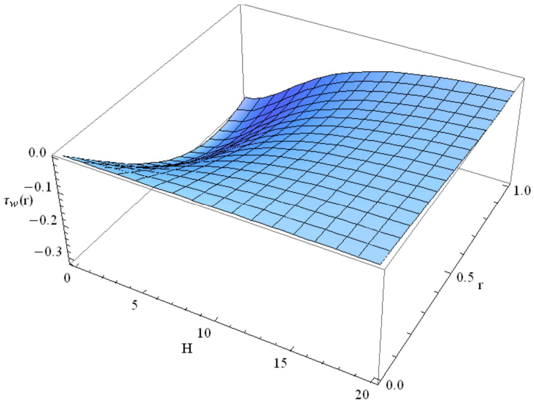

Fig. 5. *Variation of wall shear stress with Hartmann number H;* $\mu = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \bar{\alpha} = 1, \varphi = 15^0$.

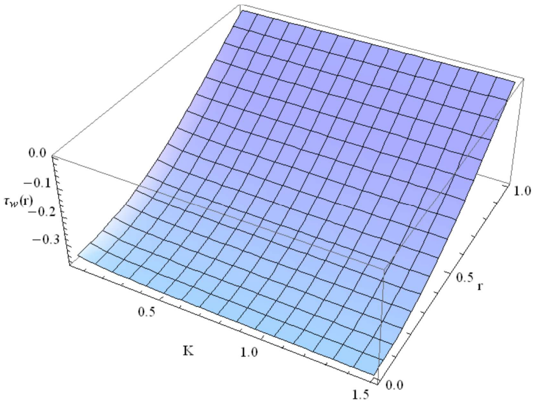

Fig. 6. *Variation of wall shear stress with permeability of the porous media K;* $\mu = 2.5, A_0 = 2, A_1 = 4, a_0 = 3, \bar{\alpha} = 1, \varphi = 15^0, H = 4$.

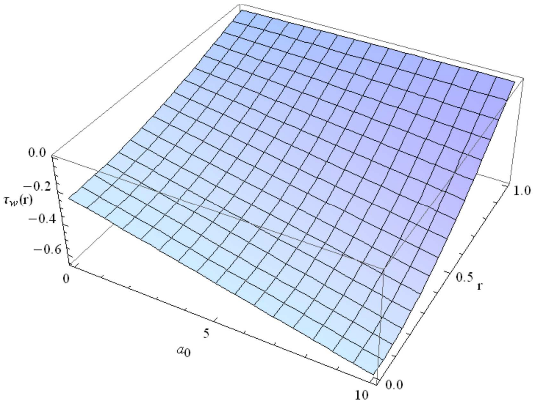

Fig. 7. *Variation of wall shear stress with amplitude of body acceleration* a_0 *;* $\mu = 2.5, A_0 = 2, A_1 = 4, K = 2.5, \bar{\alpha} = 1, \varphi = 15^0, H = 4$.

The present model gives a most general form of velocity expression from which the other mathematical models can easily be obtained by proper substitution. It is of interest to note that the velocity expression (3.4) obtained for the present model includes various velocity expressions for different mathematical models such as

- The velocity expression for steady blood flow with periodic body acceleration through a porous medium in the absence of magnetic field can be obtained by substituting H=0 in (3.4).
- The velocity expression for steady blood flow with periodic body acceleration in the absence of magnetic field and porous medium can be obtained by substituting H=0, h=0 in (3.4) which gives the result of Rathod et al [20].
- The velocity expression for steady blood flow with periodic body acceleration and magnetic field in the absence of porous medium can be obtained by substituting h=0 in (3.4)which is the result of Rathod et al [21].

5. Conclusions

The present mathematical model gives a most general form of velocity expression for the blood flow and can be utilized to some pathological situations of blood flow in coronary arteries when fatty plaques of cholesterol and artery-clogging blood clots are formed in the lumen of the coronary artery and also the application of magnetic field for therapeutic treatment of certain cardiovascular diseases.

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