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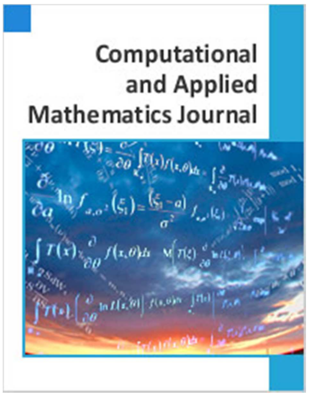

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Inverse Number Sequence Distance of Ordinal Preference Ranking

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Abstract
This paper presents a new definition regarding inverse number sequence distance, proving that it satisfies five of the six conditions put forward by Cook and Seiford that sequence distance needs to meet. Sequence distance based on the absolute value has been separately used to solve minimum violation ranking scheduling problems that contained six variable sequences. Results have shown that the degree of overlap for the solution space of the two-distance scale was high; there were 176 optimal solutions through the use of the inverse number sequence distance. The proportion of the same solutions was 68.8%, and the sequence distance scales based on the absolute value were 228 and 42.0% respectively. The two-distance scale explains the sequence distance from different angles, but it can be seen from the data that the inverse number sequence distance explains the distance of sequence more fully than absolute distance, so the hit rate is relatively higher. Also we considered the sequence distance of n variables. The solution space is n!, the calculation is very large, so we have chosen the gravity optimization algorithm in order to solve it. Results show that using the proposed algorithm saves time and results in a good effect solution.

1. Introduction

In practice, there are many decision-making problems: choices regarding tourism destinations, municipal elections, and sporting events; for many of them it is necessary to weigh multiple criteria in decision-making. In other words, for the same question we will have different preference sequences under the condition of different criteria. For the purpose to find which has the highest degree of consistency with all sequences, it is necessary to determine the sequence from the these preferences [1-5].

For more than two centuries, ranking preference sequence problems have been a hot research topic [6-9]. The original theory, produced during the 18th century's first election, has evolved into today's social choice theory. A number of researchers have conducted extensive studies on the problem: Ali[1], Goddard [2], Moon [3], as well as others have done work researching the ranking problem, which is about the football match cycle back to the game [10-12]. Among them, in Cook and Lawrence [13] put forward six conditions that sequence distance should meet, containing three choice conditions and three optional conditions. Results show that the only sequence distance scale that meets the six conditions is:

$$d_{cs}(A,B)=\sum_{i-1}^{M}|a_i-b_i|,\ A \text{ and } B \text{ represent two different sequences}.$$

The sequence distance is a way to measure an individual's ranking difference in the two sequences, and then we need to calculate the sum of all the individual ranking deviations, which is then used to measure the distance of two sequences.

This paper presents a new definition and proofs regarding the inverse number sequence distance. We have been solved minimum violation ranking scheduling problems through a contrastive analysis of the solution space, which the inverse number sequence distance and the absolute value sequence distance method are applied. We have continued the analysis of the advantages and disadvantages, and finally used a gravity optimization algorithm to calculate the optimal solution.

2. Data Representation

Sequence distance can take many forms, such as matrix expression or vector expression.

2.1. Pairwise Comparison Matrix Expression Form

The distance of matrix expression between the two objects is a kind of common sequence data expression. Through the comparison between the two matrix, then we use 0, 1 to represent the relationship of the two respectively. As shown below:

$$a_{ij} = \begin{cases} 1, & \text{represent that } i \text{ is better than } j \\ 0, & \text{represent that } j \text{ is better than } i \end{cases}.$$

If there are n variables, there are $n \times n$ pairs to compare, which will make up the comparison matrix $A = (a_{ij})$.

2.2. Vector Expression

For vector expression $A = (a_1, a_2, ..., a_n)$, a_i represents the i-th variable location in the sequence. For example, let's assume that the variables of sequence A are: a, b, c, and d in turn; then $A_1 = (2,1,4,3)$ represents that a is in second place, b is in first place, c is in fourth place, and is in third place. Yet $A_2 = (1,4,2.5,2.5)$ represents that a is in first place, b is in fourth place, and c and d are in second and third place together.

3. Sequence Model of the Preference

3.1. Inverse Number Definition

In advanced algebra [14-16], arrangement and inverse number is defined as following:

Arrangement: a sequence array that consists of 1, 2,..., n is called an n-level arrangement.

Inverse number: if the position sequence of two numbers is in contrast to their size order in a sequence, it is an inverse order. The total number of inverse order in a sequence is called the inverse number of sequence.

We have a new understanding of the above two definitions. In actuality, an inverse number is the sum of the inverse order that compares the position of each number in a random arrangement relative to the natural order n (1, 2,..., n). An inverse order can be eliminated by a data exchange. Suppose now we get the inverse number of the two random arrangements n1, n2 relative to the natural order n(notes for $\tau(n1,n)$ and $\tau(n2,n)$). If $\tau(n1,n) < \tau(n2,n)$, then n1 requires less change than n2 to become a natural number sequence. So the inverse number of two sequences can be called as their distance. Consequently, it can be said that the distance between n1 and n is closer than the distance between n2 and n.

Obviously, the arrangement of the data defined above does not contain the same data. We have promoted a method that calculates the inverse number of the number sequence: a random number sequence for which the length is n and a natural number sequence that is then applied to any two number sequence. We established a natural number sequence firstly according to the element position information of one array; then we have built an number sequence according to the relative information of the element location for the other array. The result that occurs is a situation like a random number sequence for which the length is n and a natural number sequence; we then used this method to calculate their inverse number. A concrete example is shown in Figure 1: (M represents the inverse number sequence distance of two arrangements).

Based on the natural number sequence

1	2	3	4	1	2	3	4
2	3	4	1	2	1	4	3
0	0	0	3	0	1	0	1
M=0+0+0+3=3				M=0+1+0+1=2			

Based on the random number sequence

5	6	3	8	7	2	9	1
3	8	6	5	2	9	7	1
0	0	2	3	0	0	2	1
M=0+0+2+3=5				M=0+0+0+1=1			

Figure 1. *Relativeinverse number concept map.*

3.2. Based on the CS Vector Mode Preference Sequence Distance Meet the Conditions

According to the different ordinal preference expression forms, Cook and Seiford have provided several measurement methods of the ordinal preference distance when working with group decision-making problems, and put forward some conditions that the sequencing vector distance measurement function must satisfy. These conditions are called the CS mode:

Suppose the sorting vectors are as follows:

$$A = (a_1, ..., a_M), \quad B = (b_1, ..., b_M), \quad C = (c_1, ..., c_M)$$

Where $a_1, a_2 ..., a_M$, $b_1, b_2 ..., b_M$ and $c_1, c_2 ..., c_M$ are both a full permutation for which that number is between 1 and M; a_m, b_m, c_m all represent the ranking position of the scheme S_m ; $d(A,B)$ is a distance measure function of sorting vector A and sorting vector B . Cook and Seiford think $d(A,B)$ should meet the following conditions:

Condition 1: $d(A,B) \geq 0$. $d(A,B) = 0$ if and only if $A = B$.

Condition 2: $d(A,B) = d(B,A)$.

Condition 3: $d(A,B) + d(B,C) \geq d(A,C)$.

Condition 4: $d(A', B') = d(A,B)$, when A' and B' are the new formation of the sequencing vector after A and B make the same ranking transfer.

Condition 5: $d(A^*, B^*) = d(A,B)$, when

$A^* = (a_1, ..., a_M, M+1)$, $B^* = (b_1, ..., b_M, M+1)$.

Condition 6: The minimum effective distance between the sequences is 1.

Seiford and Cook proved that only sequence distance measure that meets the sixth conditions is:

$$d_{cs}(A,B) = \sum_{i=1}^{M} | a_i - b_i | .$$

3.3. Proof Process of the Inverse Number Sequence Distance Satisfies the Conditions

Condition 1: $d(A,B) \geq 0$. $d(A,B) = 0$ if and only if $A = B$.

Proof: Obviously, according to the definition of inverse number we can get: $d(A,B) > 0$, for all the A and B;

$d(A,B) = 0$ if and only if $A = B$.

Sufficiency: When A = B, the elements in the sequence A and B have the same location; then $d(A,B) = 0 + 0 + ... + 0 = 0$.

Necessity: proof by contradiction. Suppose A≠B, where $A = (a_1, ..., a_M)$, $B = (b_1, ..., b_M)$, elements a_i of sorting A are integers between 1 and M, and elements b_i of sorting B are integers between 1 and M.

When A≠B, there are at least two numbers in A whose positions change relative to B, so $d(A,B) \geq 1$, and $d(A,B) = 0$ contradictions. Therefore, when $d(A,B) = 0$, A=B.

Condition 2: $d(A,B) = d(B,A)$.

Proof: Let's use a mathematical induction to prove that the relative inverse number of any two sorting in stances is equal.

When $n = 1$, it is clearly established.

When $n = 2$, if A, B are the same, then the inverse number is 0.

If A, B are different, assume $A:(1,2)$, $B:(2,1)$, then $B\{A\} = A\{B\} = 1$; $A:(2,1)$, $B:(1,2)$.

Suppose when the series is $n-1$, a conclusion is established. Namely, the inverse number of $A: i_1 i_2 i_3 ... i_{n-1}$ and $B: j_1 j_2 j_3 ... j_{n-1}$ for each other is k_1 .

When the series is n,we added element t; its position in A is x and in B is y, then:

$$A: i_1 i_2 i_3 ... i_{x-1} t i_{x+1} ... i_{n-1} \quad B: j_1 j_2 j_3 ... j_{y-1} t j_{y+1} ... j_{n-1} .$$

If $x = y$, $B'\{A'\} = A'\{B'\} = k_1$. Do not break general, assume $y > x$,

(1) B' relative to A' , t_y and t、t_{x+1}、...、t_{y-1} constitutes an inverse order, then $B'\{A'\} = k_1 + (y - x - 1)$.

(2) A' relative to B' , t_x and t_{x+1}、t_{x+2}、...、t_y constitutes an inverse order, then $A'\{B'\} = k_1 + (y - x - 1)$.

Therefore, proposition $B'\{A'\} = A'\{B'\} = k_1 + (y - x - 1)$ is founded.

Condition 3: $d(A,B) + d(B,C) \geq d(A,C)$.

Proof: Assume $A = (a_1, ..., a_M)$, $B = (b_1, ..., b_M)$, $C = (c_1, ..., c_M)$. For the three sequences contain in different elements, we ruled the sorting of elements in A as a standard order. Then:

The inverse orders of $B = (b_1, ..., b_M)$ are $\{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\}$.

The inverse orders of $C = (c_1, ..., c_M)$ are $\{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\}$.

So $d(A,B) = n$; $d(A,C) = m$.

(1) When $\{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\} \cap \{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\} = \phi$, $d(B,C) = n + m$.

(2) When $\{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\} \subseteq \{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\}$, $d(B,C) = m - n$.

(3) When $\{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\} \subseteq \{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})$, $d(B,C) = n - m$.

(4) When $\{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\} \cap \{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\} \neq \phi$,

$$d(B,C) = | \{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\} | + | \{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\} | - | \{(b_{i1}, b_{j1})(b_{i2}, b_{j2})...(b_{in}, b_{jn})\} \cap \{(a_{i1}, a_{j1})(a_{i2}, a_{j2})...(a_{im}, a_{jm})\} | .$$

So we have obtained: $d(B,C) \leq | n - m |$.

Therefore, when $n \geq m$, $d(A,B) + d(B,C) \geq d(A,C)$; when $n \leq m$, $d(A,B) + d(B,C) \leq n + | n - m | = n - n + m = m = d(A,C)$. Namely $d(A,B) + d(B,C) \geq d(A,C)$ is founded.

Condition 5: $d(A^*, B^*) = d(A,B)$, when $A^* = (a_1, ..., a_M, M+1)$, $B^* = (b_1, ..., b_M, M+1)$.

Proof: $d(A^*,B^*)=d(A,B)$, when $A^*=(a_1,...,a_M,M+1)$, $B^*=(b_1,...,b_M,M+1)$.

Assume: $A=(a_1,...,a_M)$, $B=(b_1,...,b_M)$; we ruled the sorting of elements in A as a standard order. Then the inverse number of B is n, and its set is $\{(b_{i1},b_{j1})(b_{i2},b_{j2})...(b_{in},b_{jn})\}$.

We added an element of M + 1 in the end of sequence A, then $A^*=(a_1,...,a_M,M+1)$. We also ruled the sorting of elements in A as a standard order. In addition, we added an element of M + 1 to the end of sequence B, then $B^*=(b_1,...,b_M,M+1)$. Because the position of the previous elements had not changed, the inverse orders of B^* were also $\{(b_{i1},b_{j1})(b_{i2},b_{j2})...(b_{in},b_{jn})\}$, but there was no inverse order for M + 1 and the rest of the elements. Therefore, $d(A^*,B^*)=n=d(A,B)$.

Condition 6: Minimum effective distance between the sequences is 1.

Proof: Assume $A=(a_1,...,a_M)$; clearly, there must be a sequence $X=(a_2,a_1,...,a_M)$. Then the inverse order of sequence X on the basis of sequence A is (a_2,a_1), so there must be a minimum effective distance d = 1 in any sequence.

4. Application Example

4.1. Data Form of Expression

We consider a minimum violations ranking problem that contains six variable sequences. In order to express this more easily, this example's data uses the vector expression form. Now suppose there are six variables: a, b, c, d, e, f, and there are two sortings: $A_1=(2,5,3,6,1,4)$ and $A_2=(1,4,6,5,3,2)$. In order to stay in line with the following expression, the sorting sequence is shown in the table below:

Table 1. *Exampleinitial sequence.*

2	5	3	6	1	4
1	4	6	5	3	2

4.2. Preference Distance to Find the Optimal Solution

We have found that the solution of the inverse number sequence distance and Euclidean distance, respectively, by using the traversal method. As shown in the figure below, the X-axis represents the sequence, and the y-axis represents the sum of the distance between the sequence and the above two sequences.

Through the two images above, it is evident that two of the solution spacesin the distance of the overlap degree are high.An inverse number sequence distance can be obtained from the optimal solution for 176, the absolute value distance ofthe optimal solution for 228. Among them, the common optimal solution number is 96. Yet, the inverse number sequence distance in common proportion to the optimal solution is 68.8%, and the absolute distance is 42.0%. The two distances explain the sequence distance from different angles, but it can be seen from the data that the inverse number sequence distance explains the distance of the sequence more fully than the absolute distance; the hit rate is relatively higher.

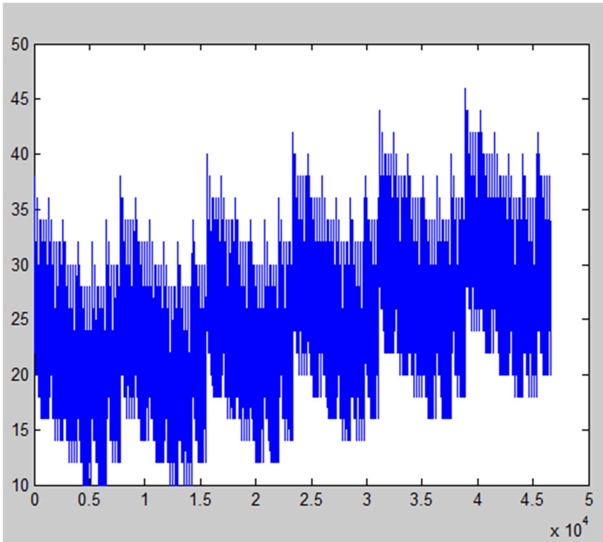

Figure 2. *Inverted number distance solution space diagram.*

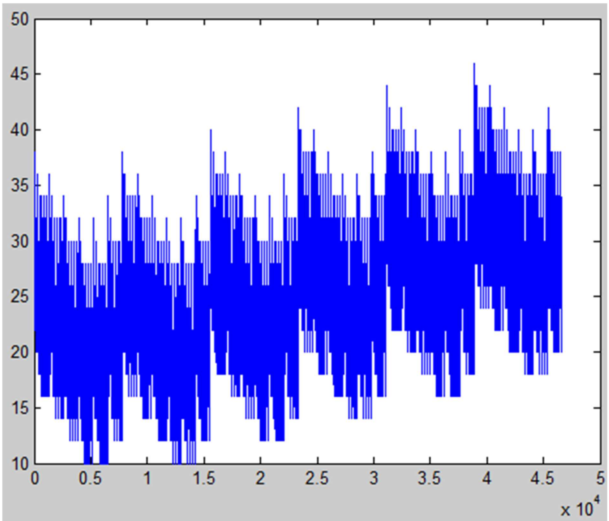

Figure 3. *Absolute value distance solution space diagram.*

4.3. Borda Method

This method was put forward by Borda and includes a detailed analysis by Kendall; the main idea is to calculate the sum of each variable assigned to the sorting order. If there are three variables, a, b, c, and there are 60 people voting to sort these three variables, the final results in Table 2 shows:

Table 2. *The Borda algorithm example data.*

	Variable scores		
Votes	**a**	**b**	**c**
23	1	2	3
17	3	1	2
2	2	1	3
10	2	3	1
8	3	2	1

Each variable total score should be:

$$a: 23\times1+17\times3+2\times2+10\times2+8\times3=122$$

$$b: 23\times2+17\times1+2\times1+10\times3+8\times2=111$$

$$c: 23\times3+17\times2+2\times3+10\times1+8\times1=127$$

So the result is $A^* = (2,1,3)$ using the Borda method. The absolute value obtains 176 optimal solutions, but we can use the Borda method to get an optimal solution. The obtained results are as follows: $A^* = (1,5,4,6,2,3)$.

5. Gravitation Search Algorithm

5.1. The Implementation Process of the Algorithm

Firstly, we randomly initialized the particle's position and speed to calculate each particle's fitness function, and then calculate the inertial mass according to the fitness of each particle, calculating the best value of inertial mass and the worst value. Secondly, we calculated the gravity on each particle in each dimension, solving for gravitational acceleration. Finally, we updated the particle's position and speed in circulation to find the final result. Specific steps are as follows:

(1) Identify the search space.

(2) Random initialization.

(3) Calculate the particle's fitness.

(4) Update the gravity coefficient G(t), the best value best(t), the worst value worst(t), and the inertial mass of particles.

(5) Calculate the sum of the force in all directions.

(6) Calculate the acceleration and speed.

(7) Update the positions of the particles.

(8) Return to step (3), loop iteration until it meets either the requirements of cycles or precision.

(9) End of cycle, output the result.

5.2. Application of Gravitational Search Algorithm

In the process of using a method to solve the inverse number distance, the optimal solution that we search is the sequence for which the sum of the distance is the shortest among all other sequences. When lesser elements are in the sequence, however, we can use a traversing method, by gradually ordering of all possible traverses to find the optimal solution. However, the complexity of the algorithm is O(n!), so we cannot use this method to find the optimal solution when n is too large.

We have used a gravitational search algorithm (this algorithm can determine one of the optimal solution, but does not determine all optimal solutions like the traverse method). After a certain number of sequences, though, we can use the search algorithm to solve the inverse number distance.

However, the gravitational search algorithm is used in continuous data, and the inverse number sequence distance that we have defined is discrete data, which is between 1 and n; consequently, the algorithm should be improved once more while we are in the application. The improved aspects are as follows:

(1) When initializing the solution space, the random number is between 1 and n.

(2) An update particle is given a serial number according to the size of its location in solving the process of iteration.

(3) A new position matrix carries on the iteration as well.

Through these improvements, eventually we will obtain the optimal solution.

5.3. Numerical Example

Let's assume that we have three initial sequence as following, and the sorting is from 1 to 10, respectively.

Table 3. *Example data of gravitation algorithm.*

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
2	5	7	8	1	3	9	4	10	6
1	3	9	2	6	4	5	8	7	10
5	6	2	9	3	8	4	1	7	10

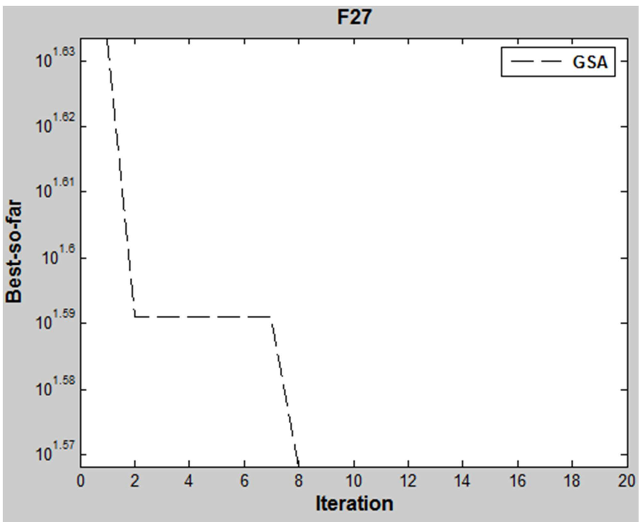

Figure 4. *Gravitation solving iterative process.*

One of the most optimal sequences is obtained by using the above methods as shown in Table 4, The optimal solution is obtained by using the iterative method as shown in Table 5.

Table 4. *Optimal solution of gravitation algorithm.*

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	3	6	9	4	2	7	8	5	10

Table 5. *Optimal results of Iteration.*

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	4	7	9	3	2	6	8	5	10

The gravity algorithm only needs 8 iterations get the optimal solution; it is accurate that here there is only areversal of two pairs of numbers, compared with the optimal solution of the iterative method. Thus, the gravity optimization algorithm not only demonstrates superior performance in computing time, but also provides precise results.

6. Conclusion

This paper established a sequence distance that was defined based on minimum violation rankings. Relative to the absolute value, when using the distance to solve the problem of the real preference, this definition of the distance is better than Cook and others. But there are still some problems that leave room for subsequent improvement: inverse number distances consider the position between the two variables to be opposite from each other without considering to what extent the inverse order compares the gap between the two, and cannot reflect completely information regarding voter preference (e.g., degree of preference), consequently providing no quantitative consideration.

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