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Approximation Properties of Seemingly Unrelated Unrestricted Regression Equation Model

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Abstract: This paper is to derive the bias & variance covariance matrix of Feasible Seemingly Unrelated Regression Model Estimator by using the methodology of Nagar's expansion for the moment of estimator, derivation consider the order of magnitude criteria, derived biased estimators to order $O_P(T^{-1})$ "that refer to big (O) in probability where (T) being the number of observations", the Covariance matrix of the Feasible (SURE) estimator to order $O_P(T^{-2})$. This approximation in view of Nagar's procedure.

Keyword: Approximation Properties, (SUUR) Model, Covariance Matrix, (FSUR) Model, Nagar's Expansion

1. Introduction

We study Approximation Properties of Seemingly Unrelated Unrestricted Regression Equation Model, then we derive the bias to order (T^{-1}), derive covariance matrix of the Feasible SUR Model Estimator to order (T^{-2}) under general condition

The organization of the present paper is the following

In Section (2) we present Model and Assumptions with some required notation that will be used throughout this paper, in section (3) we derive the bias to order (T^{-1}), and in Section (4) we derive covariance matrix of the Feasible Seemingly Unrelated Regression Model Estimator to order (T^{-2}) by using Nagar's expansion for the moment of econometric.

2. Model and Assumptions

In (1962) Zellner suggested a system of equations regression model and proposed the model contain two equations as a multi. regression

The basic model we are concerned is the following system of two-equation of (SURE) model proposed by Zellner (1962, 1963)

$$\underset{(Tx1)}{Y_i} = \underset{(Txk_i)}{X_i}\underset{(K_ix1)}{\beta_i} + \underset{(Tx1)}{u_i}, \; i = 1,2 \tag{1}$$

Where

$$K = k_1 + k_2$$

This can be written as

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}_{(2Tx1)} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}_{(2TxK)} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}_{(Kx1)} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_{(2Tx1)} \tag{2}$$

Where

y_i is a (Tx1) vector of observations on the i^{th} dependent variable (the variable to be explained by the i^{th} regression equation).

x_i is a (Txk_i) block diagonal matrix of observations on (K_i) nonstochastic independent variables, each column of which consists of T observation on a regressor 'explanatory variables' in the i^{th} equation of the model, with rank (K_i) that (x_1) , (x_2) are the matrices of fixed elements.

β_i is a $(k_i x1)$ vector of regression coefficients with unknown parameters in the i^{th} equation of the model.

u_i is the corresponding (Tx1) vector of random disturbances terms in the i^{th} regression equation.

Throughout this paper the following assumptions on the system of equations (2) are displayed

The disturbance terms have zero mean vectors

$$E\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}_{(Tx1)} \tag{3}$$

And variance covariance matrix

$$E(u_1 u_2') = E\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\begin{pmatrix} u_1' & u_2' \end{pmatrix} = \begin{pmatrix} \sigma_{11}I & \sigma_{12}I \\ \sigma_{21}I & \sigma_{22}I \end{pmatrix} = (\Sigma \otimes I)_{(T.T)}$$

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \tag{4}$$

2-1 σ_{ii} is scalar and represents the variance of the random disturbance in the i^{th} equation for each observation in the sample,

I_T is $(T.T)$ identity or unit matrix of order T, and $E(.)$ denotes the usual expectation operation, $\Sigma = (\sigma_{ij})$, $i,j = 1,2$ represents the covariance between the disturbances of i^{th} equations and i^{th} equation for each observation in the sample, (Σ) is known as variance covariance matrix,

$(\otimes)$ is the usual kronecker product.

2-2. The contemporaneous disturbances of the two equations are not linearly dependent, so (Σ) is positive definite matrix

2-3 Contemporaneous elements of (u_i, u_j), $i,j = 1,2$ have a normal distribution with zero mean and variance covariance matrix $(\Sigma \otimes I)$

2-4. Orthogonal condition

$$x_1' x_2 = 0,\ x_2' x_1 = 0 \tag{5}$$

3. Bias Estimators to Order $O_P(T^{-1})$

The order of magnitude of function is the big (O) which is Order of magnitude or order of convergence it is important investigating limiting behavior of random variables.

Big (O) notation with a capital letter (O) also called Landau's symbol, or big Omicron. It is a symbolism used in complexity theories, to describe the asymptotic behavior of functions and explain that how fast a function grows or declines.

Then we can say that big (O) notation represent the description of limiting behavior of a function.

Refer to the basic model and its assumptions in (1), (2), applying Aitken generalized least square approach to estimate the seemingly unrelated unrestricted regression (SUUR) model which contains two regression equations,

Then we have the Aitken (GLS) estimator as $\underset{GLS}{\widehat{\beta}}$

$$\underset{GLS}{\widehat{\beta}} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} (x_1'x_1)^{-1}x_1'y_1 - \dfrac{\sigma_{12}}{\sigma_{22}}(x_1'x_1)^{-1}x_1'y_2 \\ (x_2'x_2)^{-1}x_2'y_2 - \dfrac{\sigma_{12}}{\sigma_{11}}(x_1'x_1)^{-1}x_2'y_1 \end{pmatrix} \tag{6}$$

When replacing σ_{ij} by it's estimator S_{ij} in (5.1), we have feasible Aitken (GLS) estimator of β

$$\underset{FGLS}{\widehat{\beta}} = (X'(S^{-1} \otimes I)X)^{-1} X'(S^{-1} \otimes I)Y$$

Then

$$\underset{FGLS}{\widehat{\beta}} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} s^{22}x_1'x_1 & -s^{12}x_1'x_2 \\ -s^{12}x_2'x_1 & s^{11}x_2'x_2 \end{pmatrix}^{-1} \begin{pmatrix} s^{22}x_1'y_1 - s^{12}x_1'y_2 \\ -s^{21}x_2'y_1 + s^{11}x_2'y_2 \end{pmatrix}$$

Estimated of (S_{ij}) dependence on residuals obtained by the application of Aitken least square method, this procedure yield "Unrestricted Residuals"., Then the Feasible Aitken (GLS) estimator $\underset{FGLS}{\widehat{\beta}}$ is

$$\underset{FGLS}{\widehat{\beta}} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} (x_1'x_1)^{-1}x_1'y_1 - \dfrac{s_{12}}{s_{22}}(x_1'x_1)^{-1}x_1'y_2 \\ (x_2'x_2)^{-1}x_2'y_2 - \dfrac{s_{12}}{s_{11}}(x_1'x_1)^{-1}x_2'y_1 \end{pmatrix}$$

We are going to derive the bias estimators to order $O_P(T^{-1})$

Theorem (1)

The bias of order $O_P(T^{-1})$ is zero under the assumptions of the disturbance term that follow normal distribution

Proof:

We have feasible generalized least square estimator $\left(\underset{FGLS}{\widehat{\beta}}\right)$

$$(b_1) = \hat{\beta}_1 - \frac{s_{12}}{s_{22}}\hat{\beta}_2$$

Then

$$(b_1) = (x_1'x_1)^{-1}x_1'y_1 - \frac{s_{12}}{s_{22}}(x_1'x_1)^{-1}x_1'y_2, \text{ Where } y = x\beta + u$$

Then we get

$$(b_1) = (x_1'x_1)^{-1}x_1'x_1\beta_1 + (x_1'x_1)^{-1}x_1'u_1 - \frac{s_{12}}{s_{22}}(x_1'x_1)^{-1}x_1'x_2\beta_2 - \frac{s_{12}}{s_{22}}(x_1'x_1)^{-1}x_2'u_2$$

Using orthogonal condition

$$x_1'x_2 = 0, x_2'x_1 = 0$$

Then we have

$$\left(b_1-\beta_1\right)=(x_1'x_1)^{-1}x_1'u_1-\frac{s_{12}}{s_{22}}(x_1'x_1)^{-1}x_2'u_2 \tag{7}$$

Thus
Let

$$s_{12}=\left(\varepsilon_{12}+\sigma_{12}\right)_{O(T^{-\frac{1}{2}})} \tag{8}$$

Let

$$A_{\frac{-1}{2}}=(x_1'x_1)^{-1}x_1'u_1.......O(T^{\frac{-1}{2}}) \tag{9}$$

Let

$$B_{\frac{-1}{2}}=(x_1'x_1)^{-1}x_2'u_2.........O(T^{\frac{-1}{2}}) \tag{10}$$

From the above results we get

$$\left(b_1-B_1\right)=A_{\frac{-1}{2}}-\left(\varepsilon_{12}+\sigma_{12}\right)\left(\varepsilon_{22}+\sigma_{22}\right)^{-1}B_{\frac{-1}{2}}$$

$$=A_{\frac{-1}{2}}-\sigma_{12}\left(\frac{\varepsilon_{12}}{\sigma_{12}}+1\right).\sigma_{22}{}^{-1}\left(\frac{\varepsilon_{22}}{\sigma_{22}}+1\right)^{-1}B_{\frac{-1}{2}}$$

Then

$$\left(b_1-\beta_1\right)=A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}\left(1+\frac{\varepsilon_{12}}{\sigma_{12}}\right).\left(1+\frac{\varepsilon_{22}}{\sigma_{22}}\right)^{-1}B_{\frac{-1}{2}}$$

Expand the second brackets

$$=A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}\left(1+\frac{\varepsilon_{12}}{\sigma_{12}}\right).\left(1-\frac{\varepsilon_{22}}{\sigma_{22}}+\frac{\varepsilon_{22}^2}{\sigma_{22}^2}-\frac{\varepsilon_{22}^3}{\sigma_{22}^3}+....\right)B_{\frac{-1}{2}}$$

Then

$$\left(b_1-\beta_1\right)=\begin{pmatrix}\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-1}{2}})}\\+\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^2}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)_{O(T^{-1})}\\+\left(\frac{\varepsilon_{12}\varepsilon_{22}}{\sigma_2^2}\beta_{\frac{-1}{2}}-\frac{\sigma_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-3}{2}})}\\+\left(\frac{\sigma_{12}\varepsilon_{22}^3}{\sigma_{22}^4}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{-2})}\\+\left(\frac{\varepsilon_{12}\varepsilon_{22}^3}{\sigma_{22}^4}B_{\frac{-1}{2}}\right)_{O(T^{\frac{-5}{2}})}\end{pmatrix}$$

Take the expectation then we get

$$E\left(b_1-\beta_1\right)=\begin{pmatrix}E\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-1}{2}})}\\+E\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^2}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)_{O(T^{-1})}\\+E\left(\frac{\varepsilon_{12}\varepsilon_{22}}{\sigma_2^2}\beta_{\frac{-1}{2}}-\frac{\sigma_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-3}{2}})}\\+E\left(\frac{\sigma_{12}\varepsilon_{22}^3}{\sigma_{22}^4}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{-2})}\\+E\left(\frac{\varepsilon_{12}\varepsilon_{22}^3}{\sigma_{22}^4}\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-5}{2}})}\end{pmatrix}$$

$$Let\quad C_{\frac{-1}{2}}=\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-1}{2}})}$$

$$Let\quad D_{-1}=\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^2}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)_{O(T^{-1})}$$

$$Let\quad F_{\frac{-3}{2}}=\left(\frac{\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^2}\beta_{\frac{-1}{2}}-\frac{\sigma_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-3}{2}})}$$

$$Let\quad H_{-2}=\left(\frac{\sigma_{12}\varepsilon_{22}^3}{\sigma_{22}^4}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}\varepsilon_{22}^2}{\sigma_{22}^3}\beta_{\frac{-1}{2}}\right)_{O(T^{-2})}$$

Then

$$\left(b_1-B_1\right)=\left(C_{\frac{-1}{2}}\right)+\left(D_{-1}\right)+\left(F_{\frac{-3}{2}}\right)+\left(H_{-2}\right)$$

To derive the bias estimator to order of magnitude in probability $O_P\left(T^{-1}\right)$, take the expectation of the values

Then

$$E\left(b_1-\beta_1\right)=E\left(C_{\frac{-1}{2}}\right)+E\left(D_{-1}\right)+E\left(F_{\frac{-3}{2}}\right)+E\left(H_{-2}\right) \tag{11}$$

$$E\left(C_{\frac{-1}{2}}\right)=E\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)_{O(T^{\frac{-1}{2}})}$$

Then

$$E\left(C_{\frac{-1}{2}}\right)=0 \tag{12}$$

$$E\left(D_{-1}\right)=\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^2}E\left((x_1'x_1)^{-1}x_2'u_2\right)-\frac{\varepsilon_{12}}{\sigma_{22}}E\left((x_1'x_1)^{-1}x_2'u_2\right)\right)_{O(T^{-1})}$$

$$E\left(D_{-1}\right)=0 \tag{13}$$

$$E\left(F_{\frac{-3}{2}}\right)=\left(\frac{\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^{2}}E\left((x_1'x_1)^{-1}x_2'u_2\right)-\frac{\sigma_{12}\varepsilon_{12}^{2}}{\sigma_{22}^{3}}E\left((x_1'x_1)^{-1}x_2'u_2\right)\right)_{O(T^{-1})}$$

Then we get

$$E\left(F_{\frac{-3}{2}}\right)=0 \tag{14}$$

$$E\left(H_{-2}\right)=\left(\frac{\sigma_{12}\varepsilon_{12}^{3}}{\sigma_{22}^{4}}E\left((x_1'x_1)^{-1}x_2'u_2\right)-\frac{\varepsilon_{12}\varepsilon_{12}^{2}}{\sigma_{22}^{3}}E\left((x_1'x_1)^{-1}x_2'u_2\right)\right)_{O(T^{-1})}$$

Then we get

$$E\left(H_{-2}\right)=0 \tag{15}$$

Then

$$E\left(b_1-B_1\right)=E\left(C_{\frac{-1}{2}}\right)+E\left(D_{-1}\right)+E\left(F_{\frac{-3}{2}}\right)+E\left(H_{-2}\right)+.....=0$$

Then

$$E\left(b_1\right)=\left(\beta_1\right).$$

$\left(b_1\right)$ is an unbiased estimator of $\left(\beta_1\right)$

4. Covariance Matrix of the Feasible (SUR) Estimators to Order (T^{-2})

We are going to drive the moment matrix of Feasible Seemingly Unrelated Regression Estimator $\left(b_{1F}\right)$ of order $O(T^{-2})$

We have

$$\left(b_1-\beta_1\right)=\left(C_{\frac{-1}{2}}\right)+\left(D_{-1}\right)+\left(F_{\frac{-3}{2}}\right)+\left(H_{-2}\right)$$

Then

$$E\left(b_1-\beta_1\right)\left(b_1-\beta_1\right)'=\left(C_{\frac{-1}{2}}+D_{-1}+F_{\frac{-3}{2}}+H_{-2}\right)\left(C_{\frac{-1}{2}}+D_{-1}+F_{\frac{-3}{2}}+H_{-2}\right)'$$

Then we have

$$E\left(b_1-\beta_1\right)\left(b_1-\beta_1\right)'=\begin{pmatrix}\left(CC'\right)+\left(CD'+DC'\right)\\+\left(CF'+FC'+DD'\right)\end{pmatrix}+O(T^{\frac{-5}{2}})$$

Where the order of magnitude of these values are

$\left(CC'\right)_{O(T^{-1})}$, $\left(CD'+DC'\right)_{O(T^{\frac{-3}{2}})}$, $\left(CF'+FC'+DD'\right)_{O(T^{-2})}$

Take the expectation of all (6) values we get
First term

$$E\left(CC'\right)=\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)'$$

Using orthogonal condition, then we get

$$E\left(CC'\right)=\left(\sigma_{11}+\frac{\sigma_{12}^{2}}{\sigma_{22}}\right)(x_1'x_1)^{-1} \tag{16}$$

Second term

$$E\left(CD'\right)=E\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^{2}}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)'$$

$$E\left(CD'\right)=\left(\frac{\sigma_{12}\varepsilon_{12}}{\sigma_{22}}-\frac{\sigma_{12}^{2}\varepsilon_{22}}{\sigma_{22}^{2}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1} \tag{17}$$

Third Term

$$E\left(DC'\right)=E\left(\frac{\sigma_{12}\varepsilon_{22}}{\sigma_{22}^{2}}\beta_{\frac{-1}{2}}-\frac{\varepsilon_{12}}{\sigma_{22}}\beta_{\frac{-1}{2}}\right)\left(A_{\frac{-1}{2}}-\frac{\sigma_{12}}{\sigma_{22}}.\beta_{\frac{-1}{2}}\right)'$$

$$E\left(DC'\right)=\left(\frac{\sigma_{22}\sigma_{12}\varepsilon_{12}}{\sigma_{22}^{2}}-\frac{\sigma_{22}\sigma_{12}^{2}\varepsilon_{22}}{\sigma_{22}^{3}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1} \tag{18}$$

Following the previous steps we get

$$E\left(CC'\right)=\left(\sigma_{11}+\frac{\sigma_{12}^{2}}{\sigma_{22}}\right)(x_1'x_1)^{-1}$$

$$E\left(CD'\right)=\left(\frac{-\sigma_{12}^{2}\varepsilon_{22}}{\sigma_{22}^{2}}+\frac{\sigma_{12}\varepsilon_{12}}{\sigma_{22}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}$$

$$E\left(DC'\right)=\left(\frac{\sigma_{12}\varepsilon_{12}}{\sigma_{22}}-\frac{\sigma_{12}^{2}\varepsilon_{22}}{\sigma_{22}^{2}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}$$

$$E\left(CF'\right)=\left(\frac{\sigma_{12}^{2}\varepsilon_{22}}{\sigma_{22}^{3}}-\frac{\sigma_{12}\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^{2}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}$$

$$E\left(FC'\right)=\left(-\frac{\sigma_{12}\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^{2}}+\frac{\sigma_{12}^{2}\varepsilon_{22}^{2}}{\sigma_{22}^{3}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}$$

$$E(DD')=\left(\frac{\sigma_{12}^{2}\varepsilon_{22}^{2}}{\sigma_{22}^{3}}-2\frac{\sigma_{12}\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^{2}}+\frac{\varepsilon_{12}^{2}}{\sigma_{22}}\right)(x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}$$

Then

$$E\left(b_1-\beta_1\right)\left(b_1-\beta_1\right)'=\begin{pmatrix}\left(CC'\right)_{O(T^{-1})}+\left(CD'+DC'\right)_{O(T^{\frac{-3}{2}})}\\+\left(CF'+FC'+DD'\right)_{O(T^{-2})}+O(T^{\frac{-5}{2}})\end{pmatrix}$$

From above equations we get covariance of $\left(b_{1F}\right)$

$$E\left(b_1-\beta_1\right)\left(b_1-\beta_1\right)'=\begin{pmatrix}\left(\sigma_{11}+\dfrac{\sigma_{12}^2}{\sigma_{22}}\right)(x_1'x_1)^{-1} \quad + \\ \begin{bmatrix}\left(\dfrac{2\sigma_{12}\varepsilon_{12}+\varepsilon_{12}^2}{\sigma_{22}}\right)\\+\left(\dfrac{-2\sigma_{12}^2\varepsilon_{22}-4\sigma_{12}\varepsilon_{12}\varepsilon_{22}}{\sigma_{22}^2}\right)\\+\left(\dfrac{3\sigma_{12}^2\varepsilon_{22}}{\sigma_{22}^3}\right)\end{bmatrix}\left((x_1'x_1)^{-1}\left(x_2'x_2\right)(x_1'x_1)^{-1}\right)\end{pmatrix} \quad (19)$$

5. Result

We proof that the bias of order $O_P(T^{-1})$ is zero under the assumptions of the disturbance term that follow normal distribution, Then

$$E\left(b_1\right)=\left(\beta_1\right).$$

And found the Covariance matrix of the Feasible (SUR) estimators to order $O_P(T^{-2})$ is presented in (19)

6. Conclusion

We study Approximation Properties of Seemingly Unrelated Unrestricted Regression Equation Model and using Nagar's expansion for the moment of econometric to derive the bias & variance covariance matrix of Feasible SUR Model Estimator this derivation consider the order of magnitude criteria, derived biased estimators to order $O_P(T^{-1})$ and derive the Covariance matrix of the Feasible (SURE) estimator to order $O_P\left(T^{-2}\right)$ under general condition by using Nagar's expansion and orthogonal condition of Zellner model.

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Biography

Salwa Aly Kamel Hegazy awarded PhD from the Department of Applied Statistic And Econometrics, Institute of Statistical Studies & Research, Cairo University, Egypt.
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