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The Organizational Features of Good Mathematical Cognitive Structure Based on the Flop-Map Method

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Abstract: This study aims to find out the organizational features of mathematics knowledge in good mathematical cognitive structure (GMCS) based on the previous studies with the help of flop-map method. The results indicated the GMCS: (1) contained a relatively larger amount of more inclusive and abstract knowledge, such as propositions about connotation, domain, range, odd-even and monotone properties of the general function, as well as the relatively more knowledge about core concepts, such as propositions about connotation, domain, and range of function. (2) contained relatively more knowledge processed by higher-order conditional inferring and comparing and contrasting, as well as the radical defining, besides the majority processed by describing. (3) contained a larger amount of relatively more exact knowledge. (4) contained more connections and it was more compact and likely to be activated. (5) contained not any misconceptions about the connotation of the core concepts. (6) the connections contained in GMCS were the ones between parallel knowledge as well as between inclusive knowledge. These new findings undoubtedly enriched the existing results about GMCS and deepened the research about GMCS.

Keywords: Mathematics Proposition, Mathematical Cognitive Structure, Outstanding Students, Flop-Map

1. Introduction

Even though the mathematical cognitive structure (MCS) was an internal hypothetical structure which meaning the contents and connection of mathematics knowledge in mind, the existing research had demonstrated that it played a vital role in individual mathematical activities [1, 2]. Especially, it could influence the individual's understanding, mastering as well as applying mathematics knowledge completely [3-6]. Hence, almost all teachers expected to help their students to form a good mathematical cognitive structure (GMCS), that is the one that could lead an individual to understand the mathematics knowledge correctly and quickly and master it firmly and extract it flexibly to solve problems also. To achieve it, many relevant research works had been conduced [7-10]. Nevertheless, reviewing them, it could be found that little of them did it with the quantitative method [11, 12]. Therefore, it is significant to conduct a further study on GMCS, especially with the quantitative method. It will not only enrich the results in this field and help to reveal the whole picture of GMCS but also help to understand the GMCS exactly and deeply.

2. Methodology

2.1. Participants

24 first-grade outstanding students in mathematics coming from eight senior high schools in China were chosen as participants. The reason about that was it was believed they usually owned GMCS [5]. Besides, another 24 first-grade middle and general level students in the same senior high schools were chosen as references. The distinction between the three levels of students was mainly according to their daily performances and achievements. Outstanding students generally behaved positively, efficiently and effectively, and they usually made high and steady achievements with reasonable learning methods [13-15].

2.2. Material

The concept of function was selected as materials. The reason was that function was the thread running through the whole of mathematics at the stage of senior high school, it could inspire students remind more relevant propositions to facilitate the research [16].

2.3. Method

The flow-map method was adopted in this study. This method was first brought forward by Anderson and Demetrius in 1993 and used for probing and quantitative analyzing of students' cognitive structures [17]. It was different from ever before methods for studying cognitive structure, such as concept map, cards sorting and words association, etc. [18, 19] and (1) It adopted audio-taping to collect data beginning with an interview. The interview had two steps. The first step was to let the student narrate what they had thought focused on the interview questions and the second step was to let he/she amend or supplement what he/she had narrated based on his/her audio-taping. The data from both above steps were used to draw a flow map representing the student's cognitive structure. (2) It could be used for probing the connections between complex ideas such as propositions, besides absolute concepts in a student's cognitive structure. (3) The flow map of a certain student could represent both sequential and network features of knowledge in his/her cognitive structure. (4) A flow map represented the student's natural stream of thoughts without presupposing the amount of knowledge in his/her mind or imposing a predetermined hierarchical, network or other variety of structure. (5) The student needed not any relevant training for performing flow map or to draw a flow map by his/her own so that it might easily perform and relatively improve the reliability.

2.4. Data Collection

Based on the requirements of flow-map method, this study began with eliciting the student's ideas in his/her mind. In order to better elicit relatively complete knowledge about function in participants' cognitive structures, we decided to conduct the one-on-one flow-map interview at the end of the second term when they had almost finished curriculum concerning function. The interview included two parts as mentioned above. The questions of the first part were as follows. (1) Which concepts or knowledge points should be contained in function as you think? (2) Could you please elaborating on the concepts or knowledge points that you have just mentioned? (3) Could you tell me the relationship among the ideas you have mentioned?

The interview process was audio-taped by a recording pen. Then the interviewer asked the student listen to the narrative of his/her tape-recording and try his/her best to add more. This process was audio-taped by another recording pen. It was the second part of the interview, namely meta-listening period called by Tsai [20].

2.5. Data Analyses

According to the audio data above, all interviewed students' mathematical cognitive structures were represented via flow maps constructions based on the requirements of flow-map method. Each student had a unique flow map.

3. Results

3.1. The Quantitative Analysis of Mathematical Cognitive Structure

The quantitative dimensions of students' cognitive structure include cognitive structure variables and knowledge processing strategies. The former included six aspects: extent, richness, integratedness, misconception, information retrieval rate and flexibility. And the latter included five types: defining, describing, comparing and contrasting, conditional inferring and explaining. The meaning of these concepts was as shown in table 1.

Table 1. *The meaning of relevant concepts.*

Dimensions	Categories	Connotations
Cognitive variables	Extent	The total number of statements shown in the flow map.
	Richness	The total number of recurrent arrows, representing the number of recurrent linkages.
	Integratedness	The proportions of recurrent linkages, equal to number of recurrent linkages/(number of statements + number of recurrent linkages).
	Misconceptions	The number of inexact or incorrect statements in the flow map.
	Knowledge retrieval rate	Statements the student narrated per second, equal to number of statements elicited in the first part of the interview/ total time used in that interview period.
	Flexibility	The number of statements the student narrated in the meta-listening period, indicating he/she idea change as a result of that period.
Knowledge processing strategies	Defining	Defining a concept or scientific term.
	Describing	Depicting a phenomenon or a real thing.
	Comparing and Contrasting	Showing the similarities or differences between different things.
	Conditional Inferring	Inferring what will happen under a supposing condition.
	Explaining	Providing two things or causal relationship between them with proving.

The quantitative analysis of three levels of students' mathematical cognitive structures was conducted respectively. The results were shown in Table 2.

Table 2. *The results of quantitative analyses of cognitive structures.*

Dimensions	Categories	OS a	MLS b	GLS c	AS d
Cognitive Structure Variables	Extent	12.54	8.58	4.46	8.53
	Richness	14.75	7.75	1.88	8.13
	Integratedness	0.50	0.37	0.19	0.35
	Misconception	0.25	0.54	1.08	0.63
	Knowledge Retrieval Rate	0.09	0.08	0.07	0.08
	Flexibility	1.21	0.42	0.08	0.57
Knowledge Processing Strategies (Percentages of knowledge)	Defining	21.93	16.99	20.56	20.03
	Describing	56.48	66.02	67.29	61.56
	Comparing and Contrasting	6.98	4.85	4.67	5.86
	Conditional Inferring	14.29	11.65	7.48	12.21
	Explaining	0.33	0.49	0.00	0.33

a Outstanding Students, b Middle Level Students,
c General Level Students, d All Students. The same below.

It could be seen from Table 2, that the average extent value of outstanding students' mathematical cognitive structures was about 1.5 times as large as that of middle level students' and about 3 times as large as that of general level students'.

The mean richness value of outstanding students' mathematical cognitive structures was 1.9 times larger than that of middle level students' and 7.8 times larger than that of general level students'.

The mean integratedness value of both outstanding and middle level students' mathematical cognitive structures were larger than that of all students'. However, the mean integratedness value of general level students' mathematical cognitive structures was smaller than that of all students'.

The average misconception of outstanding students was significantly fewer than that of all students, and the average misconception of middle level students was slightly fewer than that of all students. But the average misconception of general level students was significantly larger than that of all students.

The mean knowledge retrieval rate of outstanding students was higher than that of both middle and general level students, and the knowledge retrieval rate of middle level students was higher than that of general level students. However, there was no obvious differentiation.

The average flexibility value of outstanding students' mathematical cognitive structures was significantly larger than that of all students'. However, the flexibility value of middle level students' was slightly fewer than that of all students' and that of general level students' was obviously smaller than that of all students'.

The top three knowledge processing strategies used by all three levels of students were orderly describing, defining and conditional inferring. As for knowledge processed by describing, general level students' mathematical cognitive structures possessed them most, followed by middle level and outstanding students'. Moreover, the amount of knowledge processed by describing in middle and general level students' mathematical cognitive structures were respectively larger than that in all students'. But the case of outstanding students was against that. As for defining, it was a kind of knowledge processing strategy used quite frequently by all three levels of students. Compared with middle and general level students, outstanding students narrated most frequently with defining strategy. As for conditional inferring, outstanding students used it most frequently, followed by middle and general level students. Moreover, the use frequency of outstanding students was higher than that of all students. But the cases of middle and general level students were all against that. The case of use of comparing and contrasting was similar. Last but significantly, whether outstanding or middle or general level students who used relatively less explaining.

3.2. The Knowledge Details About Function in STUDENTS' Mathematical Cognitive Structures

According to the flow maps representing visually students' cognitive structures, this research counted and summarized knowledge details recalled by most students. This might help with detecting which types of knowledge were focused in students' cognitive structures. The details were shown in Table 3. It must be said that some similar statements about specific functions were just replaced by a general form (e.g. "sine function was symmetric about origin" "cosine function was symmetric about y-axis" were all replaced by "** function was symmetric about **") because of too many of them.

From Table 3, it could be seen that all three levels of students in the flow-map interview highlighted mostly specific elementary functions, analytical formula, image, monotonicity, odevity, domain, range, and max-minimum, etc., which are usually required in tests and examinations. However, another knowledge points about function, such as zero point and period, etc. were stated by few students. Specifically, regarding outstanding students, 70 percent of them whose mathematical cognitive structures contained elementary knowledge, such as what is function, domain and range, etc., no less than 50 percent of them whose mathematical cognitive structures contained knowledge about odevity and monotonicity of general function, and more than 50 percent of them whose mathematical cognitive structures contained propositions about domain, analytical formula, and image of various specific functions. As for middle level students, respectively more than 80, 50 and 45 percent of them whose mathematical cognitive structures contained propositions about analytical formula, monotonicity and domain of specific functions, and more than 30 percent of them whose mathematical cognitive structures contained propositions about range, image, and period of specific functions. Moreover, nearly 30 percent of middle level students whose mathematical cognitive structures contained propositions about odevity of specific functions. As for general level students, more than 60 percent of them whose cognitive structures contained knowledge about analytical formula of specific function, more than 20 percent of them whose cognitive structures contained knowledge about odevity and monotonicity of general function, and less than 20 percent of them whose cognitive structures contained any other knowledge.

Table 3. *The knowledge details about function recalled by students in the flow-map interview.*

Modules	Knowledge Details	OS (%)	MLS (%)	GLS (%)	AS (%)
General statements	Function contains analytical formula, image, and properties etc.	75.00	58.33	45.83	59.72
	Function has three elements: domain, range, and corresponding law.	20.83	12.50	0.00	11.11
	Functions include liner, quadratic function, etc.	50.00	83.33	70.83	68.06
	Trigonometric functions include cosine, sine and tangent functions, etc.	16.67	20.83	8.33	15.28
Connotation and properties of General function	Function means for each number in domain, according to the definite law, its corresponding number in range is unique.	75.00	25.00	16.67	38.89
	Function means dependent variable varies as independent variable varies.	0.00	16.67	0.00	5.56
	Function can be expressed as y=f(x).	45.83	29.17	12.50	29.17
	Domain is a set made up of all x.	70.83	12.50	0.00	27.78
	Range is a set made up of all y.	70.83	12.50	0.00	27.78
	Odevity means a certain function is an odd or even function.	58.33	29.17	20.83	36.11
	The image of even function is symmetric about y-axis, while the image of odd function is symmetric about origin.	37.50	16.67	8.33	20.83
	Even function meets f(-x)=f(x), while odd function meets f(-x)=-f(x).	37.50	16.67	8.33	20.83
	Monotonicity means a certain function is an increasing or a decreasing function.	54.17	33.33	20.83	36.11
	Increasing function means y increases as x increases, while decreasing function means y decreases as x increases.	41.67	12.50	8.33	20.83
	Max-minimum values include maximum value and minimum value.	8.33	4.17	0.00	4.17
	If f(a)=0 exists, the constant a is a zero point of f(x).	4.17	0.00	0.00	1.39
	For each *x* in domain, if f(x+T)=f(x) always exists, f(x) is a periodic function and T is its period.	8.33	0.00	0.00	2.78
Connotation, image and properties of specific functions	** (The analytical formula of) function is y=**x.	54.17	87.50	62.50	68.06
	The domain of ** function is **.	50.00	45.83	8.33	34.72
	The range of ** function is **.	45.83	37.50	4.17	29.17
	The image of **function is **.	54.17	33.33	12.50	33.33
	**function is an odd (even) function.	25.00	29.17	4.17	19.44
	**function is symmetric about **.	4.17	20.83	8.33	11.11
	**function is increasing (decreasing) at a certain interval.	25.00	54.17	8.33	29.17
	The maximum (minimum) of ** function is**.	0.00	16.67	4.17	6.94
	The period of ** function is **.	33.33	33.33	0.00	22.22
	The zero point of ** function is **.	4.17	8.33	16.67	9.72

3.3. The Analyses of Recurrent Linkages Between Statements Recalled by Students

According to flow maps visually representing students' cognitive structures, this study respectively counted and analyzed recurrent linkages between statements recalled by three levels of students in the flow-map interview. After analyzing, we classified these connections into five relatively large categories. The first meant the connections between some elementary knowledge about core concepts, which majorly meant connotation, domain, range, the express y=f(x) ("analytical formula" called by many students), and image. The second meant the connections between statements about general properties and statements about core concepts. General properties meant odevity, monotonicity, max-minimum, period, and zero point, etc. Although domain and range were generally classified into properties, general properties in this case excluded them lest repetition. The third meant the connections between statements about specific functions and statements about core concepts and general properties, majorly made up of two parts, namely the connections between statements about specific functions and statements about core concepts and the connections between statements about specific functions and statements about general properties. The fourth meant the connections between statements about general properties, mainly made up of two parts: the connections between statements about odd and even functions and the connections between statements about monotone functions. The fifth meant the connections between statements about specific functions, mainly trigonometric, quadratic, and linear functions. The details were shown in table 4.

Table 4. *The details of recurrent connections between statements.*

Recurrent connections	OS (%)	MLS (%)	GLS (%)	AS (%)
between statements about core concepts	18.93	4.81	8.89	13.65
between statements about general properties and statements about core concepts	17.51	0	0	10.58
between statements about specific functions and statements about core concepts and general properties	10.73	3.74	0	7.68
between statements about specific functions and statements about core concepts	7.06	2.67	0	5.12
between statements about specific functions and statements about general properties	3.67	1.07	0	2.56
between statements about general properties	20.06	9.63	26.67	17.24
between statements about odd and even functions	14.41	9.09	22.22	13.31
between statements about monotone functions	5.37	0.53	4.44	3.75
between statements about specific functions	32.20	81.82	64.44	50.51
between statements about trigonometric functions	14.41	47.59	4.44	24.23
between statements about quadratic functions	9.60	12.83	17.78	11.26
between statements about linear functions	0.85	0.53	6.67	1.19
between statements about logarithmic and exponential functions	4.80	12.30	28.89	9.04

From table 4, generally, it could be seen that connections between statements about various specific functions accounted for 50.51 percent with the largest proportion, in which the most was between statements about trigonometric functions accounting for 24.23 percent. Followed by connections between statements about general properties (17.24 percent), between statements about core concepts (13.65 percent), and between statements about general properties and core concepts (10.58 percent). The connections between statements about specific functions and statements about core concepts and general properties just accounted for 7.68 percent. Specifically, the top three recurrent connections between statements recalled by outstanding, middle and general level students were respectively the ones between specific functions, between general properties and between core concepts. But their percentages were significantly different from each other.

The connections between statements about specific functions recalled by outstanding students just accounted for 32.20 percent, while recalled by middle and general level students respectively ran to 81.82 and 64.44 percent. The connections between statements about general properties recalled by outstanding students accounted for 20.06 percent, while recalled by middle and general level students accounted respectively for 9.63 and 26.67 percent. The majority was connections between statements about even and odd functions. The connections between statements about core concepts recalled by outstanding students accounted for 18.93 percent, while recalled by middle and general level students just accounted respectively for 4.81 and 8.89 percent. Moreover, it should be noted that the connections between statements about general properties and statements about core concepts recalled by outstanding students accounted for 17.51 percent. But, identical types of statements recalled by middle or general level students did not produce any linkage. The connections between statements about specific functions and statements about core concepts and general properties recalled by outstanding students accounted for 10.73 percent, while recalled by middle level students just accounted for 3.74 percent. By the same token, the identical types of statements recalled by general level students did not produce any connection.

3.4. The Analyses of Misconceptions stated by Students in the Flow-Map Interview

According to flow maps visually representing students' cognitive structures, this study conducted specifically analyses of misconceptions in the flow maps. The details were shown in table 5.

Table 5. *The misconception details recalled by students.*

Modules	Misconceptions	OS (%)	MLS (%)	GLS (%)	AS (%)
Connotation of general function	Function means for any element in A, its corresponding unique element always exists in B. A and B were nonempty sets.		4.17		1.39
	Function means for each independent variable, its corresponding dependent variable always exists.		4.17		1.39
	The essence of function is dependent variable varies as independent variable varies.		4.17		1.39
	Function is a figure such as a line or curve.			4.17	1.39
	Function is a set.			4.17	1.39
	Function is a combination of figure and image, namely function not only is a figure, but also contains allure of image.			4.17	1.39
	The analytical formula of function is y=f(x).	8.33	4.17		4.17
	The corresponding relation of function is its analytical formula.		4.17		1.39
	The corresponding relation of function means linear or quadratic function, etc.			4.17	1.39
Properties of general function	The extent of domain (range) is R (i.e. the set of all real numbers).			4.17	1.39
	When f(-x)=f(x) exists, f(x) is an even function. Or when f(-x)=-f(x) exists, f(x) is an odd function.	4.17	4.17	4.17	4.17
	Odd (even) functions satisfy f(x)=-f(x).		4.17	8.33	4.17
	Odd functions satisfy f(x)=0.	4.17		4.17	2.78
	Functions can be compared in size and applied for calculating.			4.17	1.39

Modules	Misconceptions	OS (%)	MLS (%)	GLS (%)	AS (%)
Specific functions and their properties	The analytical formula of quadratic function is y=ax2+bx+c.			12.50	4.17
	The analytical formula of linear function is y=kx+b.			8.33	2.78
	The analytical formula of proportional function is y=kx.			4.17	1.39
	The analytical formula of inverse proportional function is y=k/x.			4.17	1.39
	The proportional function becomes linear function with an additional b.			4.17	1.39
	When the exponent is larger (smaller) than 0, a certain exponential function is an increasing (decreasing) function.			4.17	1.39
	When the base number is larger (smaller) than 0, a certain logarithmic function is an increasing (decreasing) function.			4.17	1.39
	The intersection point of the quadratic curve and x-axis is $-b\pm\sqrt{b^2-4ac}/2a$.			4.17	1.39
	The intersection point where the image of logarithmic function cuts x-axis is 1.			4.17	1.39
	The zero point of cosine function is (π/2+kπ, 0).		4.17		1.39

From Table 5, it could be seen that all three levels of students had misconceptions in modules of both general and specific functions. As for misconceptions about radical connotation of general function, respectively 8.33 and 4.17 percent of outstanding and middle level students mistook the express y=f(x) for analytical formula. It was seriously incorrect because y=f(x) is just a symbol for function while analytical formula is one of representations of function. And respectively 4.17 percent of middle and general level students misunderstood the corresponding relation for analytical formula and specific functions such as linear and quadratic function, etc. Moreover, respectively 4.17 percent of middle level students whose cognitive structures contained inexact function relation to varying degree, overlooking the relation between function and mapping, no limiting the unique corresponding function value of each independent variable, and blurring the relation between definite function and indefinite correlation. And 1.41 percent of general level students whose cognitive structures contained functions replaced by images, sets or figures. As for misconceptions about properties of general function, respectively 4.17 percent of outstanding, middle and general level students who overlooked the limit of domain when stating propositions about even and odd functions. Moreover, respectively 4.17 percent of middle level students and 8.33 percent of general level students blurred the relational expression used for judging odd and even functions. 4.17 percent of general level students whose cognitive structures contained functions could be compared in size and calculated like concrete figures. 4.17 percent of general level students mistook domains and ranges of all functions for the set of all real numbers. As for misconceptions about specific functions and their properties, respectively 4.17 percent of middle level students and 8.34 percent of general level students who blurred zero point, root of corresponding equation and intersection point where x-axis cutting image of a certain function. More than 29 percent of general level students did not limit non-zero coefficient when stating propositions about connotations or analytical formulas of quadratic, linear or proportional function. Moreover, 4.17 percent of general level students blurred base number of exponential (logarithmic) function and exponent.

4. Discussion

Many recent studies in this field showed MCS played an important role in individual mathematics activities. So, almost all mathematics teachers expected to help their students fostering a GMCS. In order to achieve this, quite a few of existed researches had involved two significant aspects of mathematical cognitive structures, namely content and organization. However, most of these researches focused on absolute mathematical concepts and mainly adopted theoretical and qualitative methods. There still existed a gap in the field of researching on mathematical propositions in good cognitive structures with quantitative methods. So we chose the function as material and 72 random senior one students as participants and conducted a quantitative study on GMCS with the flop-map method. This study mainly involved knowledge variables, processing strategies, main types of proposition knowledge, recurrent connections between propositions and misconception details in good mathematical cognitive structures.

As for cognitive structure variables, firstly, extent meant the amount of statements in the flow map of a certain student, representing the amount of ideas in his or her MCS. From date analyses above, it could be seen the extent of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', exactly almost respectively 1.5 and 3 times as large as the latter two, which indicated outstanding students' cognitive structures contained a larger number of mathematical ideas. Secondly, richness meant the number of recurrent arrows in the flow map, representing the number of recurrent connections between ideas in his or her MCS. From analyses above, it could be seen the richness of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', exactly respectively 1.9 and 7.8 times larger than the latter two, indicating more connections between ideas in outstanding students' cognitive structures. Thirdly, integratedness meant the ratio of the number of recurrent arrows to the sum of statements and recurrent arrows in the flow map, representing integration degree of his or her cognitive structure. From results above, it could be seen the integratedness of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', almost respectively 1.4

and 2.6 times as large as the latter two, indicating a more compact proposition network in outstanding students' cognitive structures. Fourthly, misconception meant the number of incorrect or inexact statements in the flow map, representing the correctness of his or her MCS. From date analyses above, it could be seen the number of misconceptions in outstanding students' mathematical cognitive structures was smaller than that of middle and general level students', exactly respectively less than a half and quarter of the latter two, which indicated outstanding students' cognitive structures contained less inexact ideas, namely with a high correctness. Fifthly, knowledge retrieval rate meant the number of statements per second recalled by the student during the first part of interview. From results above, it could be seen the knowledge retrieval rate of outstanding students was higher than that of middle and general level students, which indicated outstanding students' cognitive structures were more likely to be activated. And sixthly, flexibility is defined as number of statements elicited in the second part of the interview, namely the meta-listening period. From data analyses above, the flexibility of outstanding students' cognitive structures was larger than that of middle and general level students', exactly near 3 times as large as middle level students' and 15 times larger than general level students'. This indicated outstanding students' mathematical cognitive structures were more likely to be activated with further self-prompts.

As for the amount of knowledge processed by each processing strategy, from results above, the majority of ideas in all three levels students' mathematical cognitive structures were the ones processed by describing. But, outstanding students' cognitive structures contained more knowledge processed by defining, comparing and contrasting as well as conditional inferring than middle and general level students'. This indicated outstanding students' mathematical cognitive structures contained relatively more ideas processed by radical defining as well as relatively higher-order conditional inferring and comparing and contrasting, besides most ideas processed by describing.

As for knowledge details, from results above, ideas contained in outstanding students' mathematical cognitive structures were majorly about odd and even functions, analytical formulas, domains and images of specific functions as well as connotation, domain and range of general function. Ideas contained in middle level students' cognitive structures were mainly analytical formulas, monotonicity, domains, ranges, images, period and odevity of various specific functions such as trigonometric and quadratic functions, etc. Ideas in general level students' cognitive structures were mainly about analytical formulas of specific functions. These indicated outstanding students' cognitive structures contained relatively more ideas about connotation, domain, range, monotone and odd-even qualities of general function and ideas about analytical formulas, domains and images of specific functions that were more abstract and general and closer to the core concept "function".

As for recurrent connections between ideas in students' mathematical cognitive structures, from results above, outstanding students' cognitive structures contained connections not only majorly between specific functions, between core concepts and between general properties, but also relatively more between general properties and core concepts and between specific functions and general properties as well as core concepts. While connections in both middle and general level students' cognitive structures were majorly the ones between specific functions and between odd-even functions. That was, relatively more connections contained in outstanding students' mathematical cognitive structures from inclusive knowledge as well as parallel knowledge. While connections in both middle and general level students' cognitive structures were from main parallel knowledge and little inclusive knowledge.

As for misconceptions, from results above, all students had misconceptions in aspects of connotations and properties of both general and specific functions to varying degrees. It seemed to indicate unreal and unstable connections between the above and existed knowledge in their cognitive structures. Nevertheless, outstanding students had few misconceptions, particularly no misconception about the essence of function, namely corresponding relationship. While both middle and general level students had relatively more misconceptions, particularly the ones about connotation of function. Misunderstanding and even rote learning might mostly account for their frequent misconceptions.

It was universally believed that outstanding students had a GMCS [19]. So GMCS should have several characteristics as follows. (1) It should contain a relatively larger number of ideas about specific functions focused on core concepts (e.g. the ones about domains, images and analytical formulas of various specific functions) as well as more abstract and inclusive ideas (e.g. the ones about connotation, domain, range, ode-even and monotone properties of general function). (2) It should contain relatively more knowledge processed by higher-order strategies (i.e. conditional inferring and comparing and contrasting) as well as the radical strategy (i.e. defining), besides the majority processed by describing. (3) It should contain a larger amount of knowledge with relatively higher correctness. (4) It should contain more connections and be more compact and likely to be activated. (5) Connections contained in it should be the ones not only between parallel knowledge (e.g. the ones between ideas about specific functions, the ones between ideas about core concepts, the ones between ideas about general properties), but also between inclusive knowledge (e.g. the ones between general properties and core concepts, the ones between specific functions and core concepts). (6) It should contain not any misconceptions about connotation of core concept.

5. Conclusion

Compared with existed research, this study further analyzed knowledge, particularly the propositions in GMCS from the main perspective of knowledge details, connections between them and misconceptions [21]. Generally, the results were shown as follows. (1) GMCS contained a relatively larger amount of more inclusive and abstract knowledge, such as propositions about connotation, domain, range, odd-even and monotone properties of general function. Moreover, it contained relatively more knowledge about core concepts, such as propositions about connotation, domain and range of function. Even specific knowledge in GMCS focused closer on core concept, such as propositions about analytical formula, domain and image of a certain function. (2) GMCS contained relatively more knowledge processed by higher-order conditional inferring and comparing and contrasting, as well as the radical defining, besides the majority processed by describing. (3) GMCS contained a larger amount of relatively more exact knowledge. (4) GMCS contained more connections and was more compact and likely to be activated. (5) Connections contained in GMCS were the ones between parallel knowledge (e.g. the ones between propositions about specific functions, the ones between propositions about core concepts and the ones between propositions about general properties) as well as between inclusive knowledge (e.g. the ones between general properties and core concepts and the ones between specific functions and core concepts). (6) GMCS contained not any misconceptions about connotation of core concept. These new findings apprantly enriched the existing results and deepened the research about GMCS. Besides, this study reproved that the flow-map method is feasible and effective on the study of educational psychology.

References

[1] Zhang, Ch. W. (2003). The Psychological Meaning of Students' Mathematical Cognitive Structure in Mathematics Teaching. *Research of Mathematics Teaching-Learning*, 12, 2-4.

[2] Yu, P. (2004). *Psychology of Mathematics Education*. Nanning: Guangxi Education Publishing House.

[3] Skemp, R. R. (1971). *The psychology of learning mathematics*. Penguin Books Limited.

[4] Yang, Q. (1993). On the Influence of Cognitive Structure on Mathematics Learning -- One of the Explores about Psychological Factors that Have Influence on Mathematics Learning. *Journal of Mathematics Education*, 1, 66-70.

[5] Zhang, M. L.(2007). How the Knowledge and Cognitive Structure Promote Each Other?. *China Education Daily*, 5, 11-23.

[6] Zhang, Y. (2002). The Functions of Good Mathematical Cognitive structure. *Proceedings of Conference on Psychology and Social Harmony*. Wuhan: Wuhan University Scientific Research Publishing.

[7] Wang, G. M. & Wang, Y. (2004). The Comparison of Top and Ordinary Students' Mathematical Cognitive Structures in Senior High School, the Possible Reasons for difference, and Teaching Suggestions. *Reference for Middle School Mathematical Education*, 12, 1-4.

[8] Han, B. & Wang, G. M. (2005). Role of Cognitive Structure and Reflections on It in the Process of Problem Solving. *Educational Department of Journal of Junior Mathematics*, 6, 5-7.

[9] Jin, X. F. (2011). Again on the Construction of Students' Good Mathematical Cognitive Structure. *New Curriculum Research*, 209, 179-181.

[10] Zhao, Ch. X. (2013). Optimizing and Perfecting Students' Mathematical Cognitive Structure in Inquiry Learning. *Educational Practice and Research*, 5, 53-55.

[11] Sun, D. D., & Yang Z. Zh. (2015a). Research on Good Mathematical Cognitive Structure in Mainland China. Advances in Social and Behavioral Sciences: Proceeding of 2015 3rd Asian Conference on the Social Sciences, 15: 216-219.

[12] Sun, D. D., & Yang Z. Zh. (2015b). Study on Content and Organization of Mathematical Cognitive Structure in Mainland China. Advances in Intelligent Systems Research, 283.

[13] Maker, C. J. (1981). *Curriculum Development for the Gifted*. Aspen Publications.

[14] NCTM (2000). Principles and Standard for School Mathematics, Reston, NJ.

[15] Johnson, D. T. (2000). Teaching Mathematics to Gifted Students in a Mixed-Ability Classroom. *Eric DIGEST E594*, ERIC Publications.

[16] Klein, F. (1989). Elementary Mathematics Under the High Viewpoint. Shijiazhuang: *Hubei Education Publishing House.*

[17] Anderson, O. R. & Demetrius, O. J. (1993). A flow-map method of representing cognitive structure based on respondents' narrative using science content. *Journal of Research in Science Teaching*, 30, 953-969.

[18] Zhang, J. W. & Chen, Q. (2000). Test Method of Cognitive Structure. *Journal of Psychological Science*, 6, 750-751.

[19] Sofia M. V. C., Vitor G. L., Luis M. C. G., & Ricardo L. G. (2017). Evaluation of Changes in Cognitive Structures after the Learning Process in Mathematics. *International Journal of Innovation in Science and Mathematics Education*, *25* (2): 17-33.

[20] Tsai, C. C. (2001). Probing Students' Cognitive Structures in Science: the Use of a Flow Map Method Coupled with a Meta-Listening Technique. *Studies in Educational Evaluation*, 257-268.

[21] Yang Z. Z., Zhu M., Qu Z. H., & Zhang Y. Q. (2018). Research on Organization of Mathematics Knowledge in Good Mathematical Cognitive Structure. *Eurasia Journal of Mathematics, Science and Technology Education, 14* (1), 291-302.