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The Propagation of High-Frequency Shear Elastic Waves on Interface of Isotropic Elastic Half-Spaces with Canonical Surface Protrusions

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Abstract: The wave phenomena on the contact of two isotropic elastic half-spaces with canonical surface protrusions is investigated. The junction of two half-spaces is modeled as a three-layer waveguide consisting of two homogeneous half-spaces and embedded, periodically inhomogeneous inner layer. The conditions of wave propagation of allowed frequencies are obtained in periodically inhomogeneous layered structure. The problem of wave formation in transversely periodic cells of the composite waveguide is solved. As a particular case, the propagation of high-frequency (shortwave) wave signal along the composite waveguide is numerically investigated. The variety of generated waves through the thickness of composite waveguide are given depending on the relative linear dimensions of the layers and physico-mechanical characteristics of materials of the composite waveguide. The bands of allowed (or forbidden) frequencies are defined for these forms.

Keywords: Wave Signal, Composite Waveguide, Non-smooth Surface, Periodic Heterogeneity, Forbidden Frequencies

1. Introduction

The wave phenomena in bounded and inhomogeneous elastic structures are diverse. In particular, the dispersion and/or dissipation effects on the propagation of normal wave signal in finite, homogeneous elastic medium are strongly depended on the boundary conditions. It is shown in the primary sources about the study of wave propagation, that even in the case of perfectly smooth body surface, as a result of existing dispersion are possible:

a. localization of wave energy of plane deformation in the near-surface zone of mechanically free surface of elastic isotropic half-space Rayleigh J. W. [1] (Rayleigh wave, 1885),
b. localization of wave energy of elastic anti-plane strain in the near-surface area of joining zone of elastic half-space and a soft elastic layer Love A. E. H. [2] (Love wave, 1911),
c. propagation of formed combinations of standing and running waves of complex elastic normal waves in plates with thickness comparable to the wavelength Lamb H. [3] (Lamb wave, 1917),
d. localization of wave energy of elastic strain in the near-surface zone of contact of two isotropic elastic half-spaces, if the density and elastic modulus of adjacent environments vary slightly Stoneley R. [4] (Stoneley wave, 1924)

It is also known that the geometric inhomogeneity of surfaces of elastic bodies or near-surface inhomogeneity of layer material leads to energy dissipation and changes the existing dispersion in composite waveguides on the propagation of wave signal [5-9], etc.

In the case of elastic shear wave propagation in an inhomogeneous half-space with localized inhomogeneity of the material in the near-surface zone, the inhomogeneous piezoelectric half-space A. S. Avetisyan [5] is formally shown in the form of two-layer waveguide. Love electro-elastic problem is solved in the case of thin inhomogeneous piezoelectric layer, when the length of the normal wave and the thickness of the layer of material inhomogeneity are commensurable. The dispersion equation and the conditions of possible localization of wave energy at the surface of inhomogeneity virtual section of the material obtained Valier-Brasier T., Potel C., and Bruneau M., [6].

In [7, 8] Avetisyan A. S. proposes to investigate the joints influence of rough surfaces of constituent layers on the wave propagation in composite waveguides by the method of virtual section and input of MELS hypotheses (hypotheses-Magneto Elastic Layered Systems). The waveguide is modeled as multilayer by inputting virtual cross-sections. Mathematical boundary value problem of contact of rough surfaces is formulated taking into account the thinness of virtual layers, introducing hypotheses *MELS* (hypotheses of Magneto Elastic Layered Systems).

For different levels of *nanometer* roughness on the crystal slices of silicon (001) and (111) C. M. Flannery and H. von Kiedrowski [9] investigated the influence of surface roughness on the dispersion of surface acoustic wave packages of the frequency range 30-200 MHz. It is shown that the effect of frequency dispersion is significant on the induced surface roughness. Although the results obtained by this approach qualitatively matches with the results of the experiment, this theory is not sufficient to predict the real dispersion of surface acoustic waves.

In the recent years intensively studied phenomena of wave propagation in structures with variable, periodic physical-mechanical characteristics, which are characterized by the presence of locking zones of Floquet elastic and electromagnetic waves, where is not available propagating wave. The overview of these works and the analysis of the role of impedance on the existence of forbidden frequency are presented in the papers of A. S. Avetisyan, K. B. Ghazaryan [10, 11], where it is also shown that if the impedance of the periodically inhomogeneous 1D structure is constant, then in that structure do not exist forbidden frequencies.

The overview of prospects, the current state and future direction of development of researches of wave processes in periodic structures is given in M. I. Hussein, M. J. Leamy, M. Ruzzene [12].

First time the presence of zones of locking frequencies in unidirectional elastic periodic structure has been noted in the work of Lord Rayleigh [13].

The papers of E. H. Lee [14] and E. H. Lee and W. H. Yang [15] devoted to the application of Floquet-Lyapunov theory to problems of propagation of elastic waves in periodic structures. In the papers of S. Adams, R. Craster, S. Guenneau [16, 17] the authors study the range of Floquet-Bloch wave in elastic periodic waveguides.

The spectral theory of transverse oscillations of periodic elastic beams is described in V. G. Papanicolaou [18, 19] from a mathematical point of view.

However, little is known about the problem of normal wave propagation in transversely inhomogeneous waveguide with periodically repeated composite layers. In these tasks, the process of wave formation through the thickness of layered waveguide constrains the process of wave transmission of appropriate lengths, or vice versa, the periodically transverse inhomogeneity delays corresponding frequency formation.

Such a challenge comes when studying the propagation of normal shear elastic waves on the interface of elastic half-spaces with periodic surface canonical protrusions. Equation Section (Next)

2. Modeling of Surface Interconnection of Elastic Isotropic Half-Spaces with Canonical Rectangular Protrusions

Let us assume homogeneous isotropic half-spaces with canonical (rectangular, periodic in cut) surface protrusions (pins)

$$\Omega_1\{x;y\}=\left\{|x|<\infty;\ y\le\begin{cases}-h_0 & in \quad -(1+n)b+n(a+b)\le x\le n(a+b)\\ h_0 & in \quad n(a+b)\le x\le(1+n)a+nb\end{cases};\quad |z|<\infty\right\} \tag{1}$$

$$\Omega_2\{x;y\}=\left\{|x|<\infty;\ y\ge\begin{cases}h_0 & in \quad n(a+b)\le x\le(1+n)a+nb\\ -h_0 & in \quad -(1+n)b+n(a+b)\le x\le n(a+b)\end{cases};\quad |z|<\infty\right\} \tag{2}$$

are ideally embedded by each other surface protrusions (Figure 1). In (1) and (2) $n\in\mathbb{N}^+$ is the numbering of protrusions.

It is shown from the formation of half-spaces with protrusions (1) and (2), that for the convenience of the mathematical boundary value problem, the coordinate plane yoz (coordinate plane $x=0$) is conventionally allocated on one of lateral surfaces of the protrusion contact of the half-spaces $\Omega_{10}^*\{x;y\}$ and $\Omega_{20}^*\{x;y\}$, and the coordinate axis oz is parallel to the forming of these projections

$$\Omega_{1n}^*\{x;y\}=\{n(a+b)\le x\le(1+n)a+nb;\ |y|\le h_0;\ |z|<\infty\} \tag{3}$$

$$\Omega_{2n}^*\{x;y\}=\{-(1+n)b+n(a+b)\le x\le n(a+b);\ |y|\le h_0;\ |z|<\infty\} \tag{4}$$

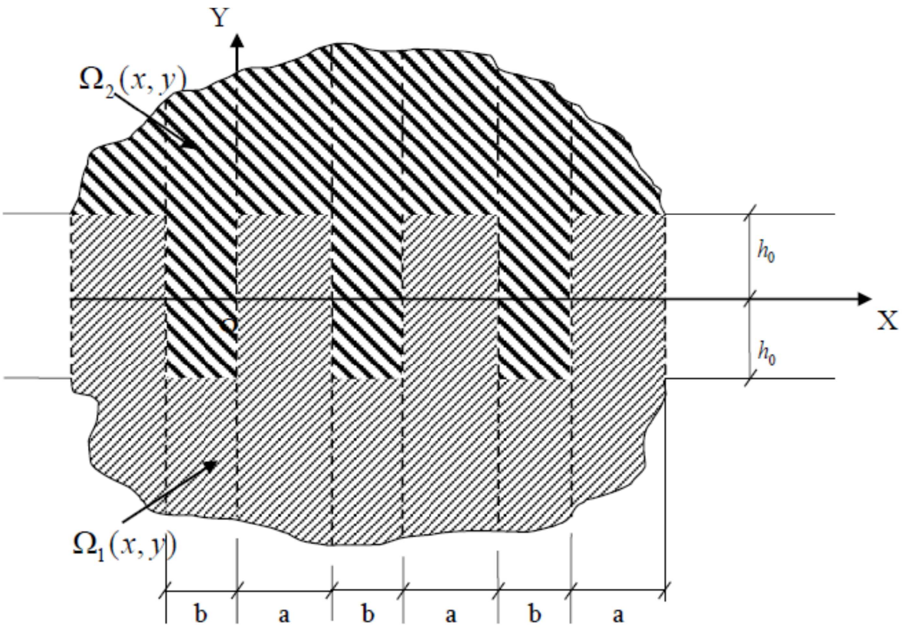

Figure 1. *Connection diagram of two elastic half-spaces with canonical surface pins, as a three-layer waveguide with a periodically inhomogeneous inner layer.*

The canonicity of projections (the forms of pins and their linear dimensions) allows us to provide the full mechanical contact along the all line of contact of half-spaces.

By the input of virtual cross-sections $y = h_0$ and $y = -h_0$, in fact a three-layer waveguide is formed from two homogeneous half-spaces

$$\Omega_{10}\{x;y\} = \{|x| < \infty;\ y \le -h_0;\ |z| < \infty\} \text{ и } \Omega_{20}\{x;y\} = \{|x| < \infty;\ y \ge h_0;\ |z| < \infty\} \tag{5}$$

and virtually separated longitudinally inhomogeneous (piecewise-homogeneous) layer of periodically distributed cells of protrusions (pins) pairs

$$\Omega_n^*\{x;y\} \triangleq \Omega_{1n}^*\{x;y\} \cup \Omega_{2n}^*\{x;y\} \tag{6}$$

The given modeling allows us to consider the wave process of normal wave propagation in periodically inhomogeneous layer $\Omega_n^*\{x;y\}$ located at $y = h_0$ and $y = -h_0$, which is in perfect mechanical contact with homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$.Equation Section (Next)

The mathematical boundary problem on the propagation of normal wave signal *(SH)* of elastic shear is formulated from the equations of the corresponding homogeneous half-spaces (5) and their respective protrusions (3) and (5)

$$\partial^2 \mathrm{w}_1(x;y)/\partial x^2 + \partial^2 \mathrm{w}_1(x;y)/\partial y^2 = -\omega^2/c_{1t}^2 \cdot \mathrm{w}_1(x;y) \text{ in } \Omega_{10}\{x;y\} \text{ and } \Omega_{1n}^*\{x;y\} \tag{7}$$

$$\partial^2 \mathrm{w}_2(x;y)/\partial x^2 + \partial^2 \mathrm{w}_2(x;y)/\partial y^2 = -\omega^2/c_{2t}^2 \cdot \mathrm{w}_2(x;y) \text{ in } \Omega_{20}\{x;y\} \text{ and } \Omega_{2n}^*\{x;y\} \tag{8}$$

In (7) and (8) ω is the frequency of wave process, $c_{nt} \triangleq \sqrt{G_n/\rho_n}$ is the velocity of shear bulk wave, G_n is the shear modulus, and ρ_n is the density of the respective adjacent materials $n = 1;2$.

For normal elastic shear wave signal

$$U = \{0;\ 0;\ \mathrm{w}(x;y;t) = \mathrm{w}_0(x;y) \cdot \exp(-i\omega t)\} \tag{9}$$

one group of boundary conditions of full mechanical contact is satisfied on the virtual cross-sections $y = h_0$ and $y = -h_0$ along the widths of surface protrusions, respectively.

Along the width of each protrusion $\Omega_{1n}^*\{x;y\}$, the continuity surface conditions of mechanical fields will be satisfied on intervals $n(a+b) \le x \le (1+n)a + nb$ for $y = h_0$ and $y = -h_0$ respectively

$$\mathrm{w}_1^*(x;-h_0;t) = \mathrm{w}_1(x;-h_0;t)\,;\ \sigma_{yz}^{1*}(x;-h_0;t) = \sigma_{yz}^{(1)}(x;-h_0;t) \tag{10}$$

$$\mathrm{w}_1^*(x;h_0;t) = \mathrm{w}_2(x;h_0;t)\,;\ \sigma_{yz}^{1*}(x;h_0;t) = \sigma_{yz}^{(2)}(x;h_0;t) \tag{11}$$

Along the width of each protrusion $\Omega_2^*\{x;y\}$, the continuity surface conditions of mechanical fields will be satisfied on intervals $-(1+n)b+n(a+b) \le x \le n(a+b)$ for $y = h_0$ and $y = -h_0$ respectively

$$\mathrm{w}_2^*(x;-h_0;t) = \mathrm{w}_1(x;-h_0;t)\,;\ \sigma_{yz}^{2*}(x;-h_0;t) = \sigma_{yz}^{(1)}(x;-h_0;t) \tag{12}$$

$$\mathrm{w}_2^*(x;h_0;t) = \mathrm{w}_2(x;h_0;t)\,;\ \sigma_{yz}^{2*}(x;h_0;t) = \sigma_{yz}^{(2)}(x;h_0;t) \tag{13}$$

In surface equations (10)÷(13) shear mechanical stresses are presented by material relations

$$\sigma_{yz}^{(n)}(x;y;t) = G_n \cdot \partial \mathrm{w}_n(x;y;t)/\partial y\,; \tag{14}$$

$$\sigma_{yz}^{n*}(x;y;t) = G_n \cdot \partial \mathrm{w}_n^*(x;y;t)\big/\partial y \tag{15}$$

where the index $n = 1;2$ corresponds to the materials in homogeneous half-spaces and their surface protrusions.

In addition to the given boundary conditions on virtual selected sections $y = -h_0$ and $y = h_0$ (10)÷(13), surface conditions of full mechanical contact are satisfied on the all lateral surfaces of protrusions $x_{ab}^* = n(a+b)$ and $x_{ab}^{**} = (n-1)b + n(a+b)$, where $n \in \mathbb{N}^+$.

The conditions of full mechanical contact will be as follows in recurrently repeated these sections of the composite layer (6)

$$\mathrm{w}_1^*(x_{ab}^*;y;t) = \mathrm{w}_2^*(x_{ab}^*;y;t)\,;\ \sigma_{zx}^{1*}(x_{ab}^*;y;t) = \sigma_{zx}^{2*}(x_{ab}^*;y;t) \tag{16}$$

$$\mathrm{w}_2^*(x_{ab}^{**};y;t) = \mathrm{w}_1^*(x_{ab}^{**};y;t)\,;\ \sigma_{zx}^{2*}(x_{ab}^{**};y;t) = \sigma_{zx}^{1*}(x_{ab}^{**};y;t) \tag{17}$$

taking into account the periodic characteristics of the wave field in the composite layer (6)

$$w_1^*(x;y;t) = w_1^*(x+(a+b);y;t)\,;\ \mathrm{w}_2^*(x;y;t) = \mathrm{w}_2^*(x+(a+b);y;t) \tag{18}$$

$$\sigma_{zx}^{1*}(x;y;t) = \sigma_{zx}^{1*}((x+(a+b);y;t);y;t)\,;\ \sigma_{zx}^{2*}(x;y;t) = \sigma_{zx}^{2*}((x+(a+b);y;t);y;t) \tag{19}$$

The shear mechanical stresses in the boundary conditions (16) and (17), as well as in conditions of periodicity (19) are represented by the following material relations

$$\sigma_{zx}^{n*}(x;y;t) = G_n \cdot \partial \mathrm{w}_n^*(x;y;t)\big/\partial x \tag{20}$$

Here, the indexes $n = 1;2$ correspond to the materials of surface protrusions of half-spaces. Equation Section (Next)

The solutions of equations (7) and (8) in homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$, as well as in the respective projections $\Omega_{1n}^*\{x;y\}$ and $\Omega_{2n}^*\{x;y\}$, construct by the method of separation of variables respectively

$$\mathrm{w}_n(x;y;t) = X_{0n}(x)\cdot Y_{0n}(y); \qquad \mathrm{w}_n^*(x;y;t) = X_{0n}^*(x)\cdot Y_{0n}^*(y) \tag{21}$$

The system of equations of relatively unknown functions $\left\{X_{0n}(x);\ Y_{0n}(y);\ X_{0n}^*(x);\ Y_{0n}^*(y)\right\}$, that describes the directional propagation of the wave signal along the axis ox in homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$, as well as in their surface rectangular protrusions $\Omega_{1n}^*\{x;y\}$ and $\Omega_{2n}^*\{x;y\}$, will be presented in the following form

$$\begin{cases} X_{0n,xx}(x) + k_n^2 \cdot X_{0n}(x) = 0 \\ Y_{0n,yy}(y) - k_n^2\alpha_{nt}^2 \cdot Y_{0n}(y) = 0 \end{cases} \tag{22}$$

The following assignations are taken in the system of equations (22) for the corresponding homogeneous half-spaces: $\alpha_{nt} \triangleq \sqrt{1-\omega^2/k_n^2c_{nt}^2}$ are the wave formation coefficients (in the case of slow waves – coefficients of attenuation into the corresponding half-spaces), and k_n are the wave numbers of normal waves in corresponding environment.

Damping through the depth of homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$ normal waves are presented in the form, respectively for $y \to \pm\infty$

$$\mathrm{w}_1(x;y;t) = A_1 \cdot X_{01}(x)\cdot \exp(\alpha_{1t}k_1y)\cdot \exp(-i\omega t) \tag{23}$$

$$\mathrm{w}_2(x;y;t) = A_2 \cdot X_{02}(x)\cdot \exp(-\alpha_{2t}k_2y)\cdot \exp(-i\omega t) \tag{24}$$

The attenuation of waves into the half-spaces in the solutions (23) and (24) naturally is provided by the slowness of the waves $\omega/k_n < c_{nt}$ in each half-space.

It is obvious, that from the demand of dumped wave propagation in both homogeneous half-spaces, for the phase velocity of formed wave forms through the thickness of the composite waveguide have $V_\phi(k) < \min_{n=1;2}\left\{c_{nt} \triangleq \sqrt{G_n/\rho_n}\right\}$, where $V_\phi(k) \triangleq \omega/k$.

In the surface rectangular protrusions $\Omega^*_{1n}\{x;y\}$ and $\Omega^*_{2n}\{x;y\}$, where it is also possible propagation of fast shear waves, for which $V_\phi(k) \geq \max_{n=1;2}\left\{c_{nt} \triangleq \sqrt{G_n/\rho_n}\right\}$, the system of equations for the unknown functions $X^*_{0n}(x)$ and $Y^*_{0n}(y)$ can also be represented in the form

$$\begin{cases} X^*_{0n,xx}(x) + k_n^2 X^*_{0n}(x) = 0 \\ Y^*_{0n,yy}(y) + k_n^2 \beta^{*2}_{nt} \cdot Y^*_{0n}(y) = 0 \end{cases} \tag{25}$$

Here $\beta_{nt} \triangleq \sqrt{\omega^2/k_n^2 c_{nt}^2 - 1} = i\alpha_{nt}$ is the new assignation for the coefficient of wave formation.

As follows from the first equations of systems (22) and (25), the solutions providing synchronicity of propagation waves in homogeneous half-spaces and their rectangular protrusions, match and are written by harmonic functions in periodic, laterally inhomogeneous layer

$$X^*_{0n}(x) = X_{0n}(x) = C_n \sin(k_n x) + D_n \cos(k_n x) \text{ for } n = 1;2 \tag{26}$$

As analytic continuation of solutions (23) and (24), the non-damped solutions for convenience will be presented by hyperbolic functions in each surface protrusion of the corresponding half-space in cut $y \in [-h_0;\ h_0]$,

$$\mathrm{w}^*_1(x;y;t) = X_{01}(x) \cdot \left[A^*_1 \cdot sh(k_1\alpha_{1t}y) + B^*_1 \cdot ch(k_1\alpha_{1t}y)\right] \cdot \exp(-i\omega t) \tag{27}$$

$$\mathrm{w}^*_2(x;y;t) = X_{02}(x) \cdot \left[A^*_2 \cdot sh(k_2\alpha_{2t}y) + B^*_2 \cdot ch(k_2\alpha_{2t}y)\right] \cdot \exp(-i\omega t) \tag{28}$$

Taking into account the periodicity of the structure of internal virtual layer, let us use the theory of Fouquet-Lyapunov and the boundary value problem solve for the repeated cell with the number zero $\Omega^*_0\{x;y\} \triangleq \Omega^*_{10}\{x;y\} \cup \Omega^*_{20}\{x;y\}$.

Satisfying the conditions of full mechanical contact (16) on the lateral surface of the protrusions $x = 0$,

$$\mathrm{w}^*_1(0;y;t) = \mathrm{w}^*_2(0;y;t);\ G_1 \cdot \mathrm{w}^*_{1,x}(0;y;t) = G_2 \cdot \mathrm{w}^*_{2,x}(0;y;t) \tag{29}$$

as well as the conditions (17) on lateral surfaces of protrusions $x = -b$ and $x = a$, taking into account the periodicity of solutions (18) and (19) by x coordinate

$$\mathrm{w}^*_1(a;y;t) = \lambda^{-1}\mathrm{w}^*_2(-b;y;t);\ \lambda G_1 \cdot \mathrm{w}^*_{1,x}(a;y;t) = G_2 \cdot \mathrm{w}^*_{2,x}(-b;y;t) \tag{30}$$

receive the condition of propagation of elastic shear wave signal in periodically laterally inhomogeneous inner layer in the following form

$$\cos\left(Lk_F(\omega)\right) = \cos(k_1 a) \cdot \cos(k_2 b) - \frac{G_2^2 k_2^2 + G_1^2 k_1^2}{2G_2 k_2 G_1 k_1} \sin(k_1 a) \cdot \sin(k_2 b) \tag{31}$$

In the boundary conditions (30) and dispersion relation (31) $\lambda = \exp(Lk)$ is a multiplier, and $L = a + b$ is the period of $k_1(\omega)$ and $k_2(\omega)$ wave numbers in the surface protrusions $\Omega^*_{10}\{x;y\}$ and $\Omega^*_{20}\{x;y\}$, in homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$, respectively, $k_F(\omega) = 2\pi/\lambda(\omega)$ is the wave number of formed wave (Fouquet wave number) corresponding to allowed wave length $\lambda_F(\omega)$ (allowed frequency ω).

Considering that $\left|\cos\left(Lk_F(\omega)\right)\right| \leq 1$, from the dispersion relations (17) or (18) have the transmission condition of wave signal in periodic, laterally inhomogeneous layer

$$-1 \leq \frac{(G_2k_2 + G_1k_1)^2}{4G_2k_2G_1k_1}\cos(k_1a + k_2b)\left[1 - \frac{(G_2k_2 - G_1k_1)^2}{(G_2k_2 + G_1k_1)^2} \cdot \frac{\cos(k_1a - k_2b)}{\cos(k_1a + k_2b)}\right] \leq 1 \tag{32}$$

Synchronization of shear wave propagation in general assumes the same allowed wave number, determined from (31)

$$k(\omega) = \frac{1}{L} \cdot \arccos\left\{\frac{1}{4G_2k_2G_1k_1}\left[(G_2k_2 + G_1k_1)^2 \cos(k_1a + k_2b) - (G_2k_2 - G_1k_1)^2 \cos(k_1a - k_2b)\right]\right\} \tag{33}$$

Considering the received relations as a area of definition for the allowed lengths of the wave signal in the periodic structure, from (33) we get

$$\lambda(\omega) = 2\pi L \cdot \arccos^{-1}\left\{\frac{\left(G_2k_2+G_1k_1\right)^2}{4G_2k_2G_1k_1}\left[\cos(k_1a+k_2b)-\frac{\left(G_2k_2-G_1k_1\right)^2}{\left(G_2k_2+G_1k_1\right)^2}\cos(k_1a-k_2b)\right]\right\} \tag{34}$$

It should be noted, that the allowed wave lengths for the known inhomogeneity always are of order of the composite layer widths $\lambda(\omega) \sim \min\{a;\ b\}$. The wave numbers $k_1(\omega)$ and $k_2(\omega)$ in homogeneous half-spaces $\Omega_{10}\{x;y\}$ and $\Omega_{20}\{x;y\}$, and respectively, in the surface protrusions $\Omega_{10}^*\{x;y\}$ and $\Omega_{20}^*\{x;y\}$ are determined from the boundary value problem of wave formation through the thickness of the composite waveguide.

Satisfying the boundary conditions of continuity of mechanical fields (10)÷(11) along the width of protrusion $\Omega_{10}^*\{x;y\}$ (on cut $0 \le x \le a$), as well as the continuity conditions of mechanical fields (12)÷(13) along the width of the protrusion $\Omega_{20}^*\{x;y\}$ (on cut $-b \le x \le 0$), we obtain the dispersion relations taking into account the relations (14) and (15) as well as the solutions (23), (24), (27) and (28) on the virtually selected sections $y = h_0$ and $y = -h_0$

$$tg(2\beta_{1t}k_1h_0) = G_1\beta_{1t}\cdot\left(G_1\alpha_{1t}+G_2\alpha_{2t}\right)\Big/\left(G_1\alpha_{1t}G_2\alpha_{2t}-G_{01}^2\beta_{01}^2\right) \tag{35}$$

$$tg(2\beta_{2t}k_2h_0) = G_2\beta_{2t}\cdot\left(G_1\alpha_{1t}+G_2\alpha_{2t}\right)\Big/\left(G_1\alpha_{1t}G_2\alpha_{2t}-G_0^2\beta_{2t}^2\right) \tag{36}$$

The system of dispersion relations (35) and (36) itself represents the dispersion equation of wave formation through the thickness of composite waveguide. Their joint solution provides synchronized values of wave numbers $k_1(\omega)$ and $k_2(\omega)$ in components of waveguide.

The formation of the wave-forms in homogeneous half-spaces and corresponding protrusions naturally corresponds to the value of phase velocity $V_\phi\left(k\right) \triangleq \omega/k(\omega)$ in the phase region of determination. Without interrupting the generality of discussion, we assume that the speed of bulk shear wave in the first homogeneous half-space greater than the speed in the second half-space $c_{1t} = \sqrt{G_1/\rho_1} > c_{2t} = \sqrt{G_2/\rho_2}$. Then on the phase coordinate axis $V_\phi\left(k(\omega)\right)$ will have three intervals of the forming waves.

In the case of slow wave propagation $V_\phi(\omega/k) < \min\{c_{1t};c_{2t}\}$ in both environments, when $\beta_{nt} \triangleq \sqrt{\omega^2/k_n^2c_{nt}^2-1} = i\alpha_{nt}$, the system of dispersion relations (35) and (36) takes a rather simple form

$$\begin{cases} th(2h_0\alpha_{1t}(\omega)\cdot k_1(\omega)) = 1 \\ th(2h_0\alpha_{2t}(\omega)k_2(\omega)) = 1 \end{cases} \tag{37}$$

Since the hyperbolic tangents are quickly descending in equations (37) and allowed wave lengths for periodic inhomogeneity are always of order of the composite layer widths $\lambda(\omega) \sim \min\{a;\ b\}$ in the relation (35), then from (37) is received the approximation of the solution with great precision for arguments $2\alpha_{nt}k_nh_0 = 4\pi$

$$k_n(\omega) = \left(\omega/c_{nt}\right)\cdot\sqrt{1+\left(2\pi c_{nt}/\omega h_0\right)^2}\ ;\ n = 1;2 \tag{38}$$

Here the found wave number in both half-spaces provide lower phase velocity than bulk shear wave in each medium. Corresponding to the slow wave, phase velocity in each composite layer can be written as

$$V_{a\phi}(\omega/k) = c_{1t}\Big/\sqrt{1+\left(2\pi c_{1t}/\omega h_0\right)^2}\ ; \qquad V_{b\phi}(\omega/k) = c_{2t}\Big/\sqrt{1+\left(2\pi c_{2t}/\omega h_0\right)^2} \tag{39}$$

The obtained value of phase velocity $V_{a\phi}(\omega/k)$ in the first layer satisfies the condition of slow waves $V_{1\phi}(\omega/k) < c_{2t}$ in the frequency range

$$0 < \omega \le \left(2\pi/h_0\right)\cdot\left(c_{1t}c_{2t}\Big/\sqrt{c_{1t}^2-c_{2t}^2}\right) \tag{40}$$

Therefore, the connection of isotropic, elastic half-spaces with canonical surface protrusions, for some ratios of the linear dimensions of protrusions leads to the localization of wave energy of elastic shear signal with certain frequency ω, near the virtual surfaces of homogeneous half-spaces. The wave field distribution can be cast as

$$Y_{0a}(y) = \begin{cases} A_1\cdot\exp[k_a(\omega)\alpha_{1t}(\omega)y]; & -\infty < y \le -h_0 \\ A_1^*\cdot sh[k_a(\omega)\alpha_{1t}(\omega)y] + B_1^*\cdot ch[k_a(\omega)\alpha_{1t}(\omega)y]; & -h_0 \le y \le h_0 \\ A_2\cdot\exp[-k_a(\omega)\alpha_{2t}(\omega)y]; & h_0 \le y < \infty \end{cases} \tag{41}$$

$$Y_{0b}(y)=\begin{cases}A_1\cdot\exp[k_b(\omega)\alpha_{1t}(\omega)y]; & -\infty<y\le -h_0\\ A_2^*\cdot sh[k_b(\omega)\alpha_{2t}(\omega)y]+B_2^*\cdot ch[k_b(\omega)\alpha_{2t}(\omega)y]; & -h_0\le y\le h_0\\ A_2\cdot\exp[-k_b(\omega)\alpha_{2t}(\omega)y]; & h_0\le y<\infty\end{cases} \quad (42)$$

To find allowed frequencies (or lengths) for the obtained localized forms, the wave numbers $k_1(\omega)=k_a(\omega)$ and $k_2(\omega)=k_b(\omega)$ determined from (38) substitute in the relations (33) or (34).

In the case of propagation of fast shear waves in both environments $V_\phi(\omega/k)\ge\max\{c_{1t};c_{2t}\}$ we have $\alpha_{nt}=-i\beta_{nt}=\sqrt{1-\omega^2/k_n^2c_{nt}^2}$, and the dispersion relations (35) and (36) will already be written in the form

$$\begin{cases}tg(2h_0\beta_{1t}(\omega)k_1(\omega))=i\\ tg(2h_0\beta_{2t}(\omega)k_2(\omega))=i\end{cases} \quad (43)$$

It is obvious, that the obtained dispersion equation of fast waves (43), considering $\alpha_{nt}=-i\beta_{nt}$ gets converted to (37) and in the composite layers has real roots

$$k_1(\omega)=(\omega/c_{1t})\cdot\sqrt{1+(2\pi c_{1t}/\omega h_0)^2};\qquad k_2(\omega)=(\omega/c_{2t})\cdot\sqrt{1+(2\pi c_{2t}/\omega h_0)^2} \quad (44)$$

which correspond to phase velocities in composite layers

$$V_{1\phi}(\omega/k)=c_{1t}\Big/\sqrt{1+(2\pi c_{1t}/\omega h_0)^2};\qquad V_{2\phi}(\omega/k)=c_{2t}\Big/\sqrt{1+(2\pi c_{2t}/\omega h_0)^2} \quad (45)$$

It is obvious, that in this case the obtained phase velocities $V_{n\phi}(\omega/k)$ does not belong to the interval of fast shear waves. Therefore, the localization of the wave energy of the fast shear waves due to the presence of pin junction of the homogeneous half-space surface does not occur.

In the case of Love type wave propagation $\min\{c_{1t};c_{2t}\}\le V_\phi(\omega/k)<\max\{c_{1t};c_{2t}\}$ we get the following dispersion equation system

$$\begin{cases}th(2h_0\alpha_{1t}(\omega)k_1(\omega))=1\\ tg(2h_0\beta_{2t}(\omega)k_2(\omega))=-1\end{cases} \quad (46)$$

Hence, obtain the solutions for both layers of the periodic cell

$$k_1(\omega)=(\omega/c_{1t})\cdot\sqrt{1+(2\pi c_{1t}/\omega h_0)^2};\qquad k_2(\omega)=(\omega/c_{2t})\cdot\sqrt{1-\left[(4m+3)\pi c_{2t}/8\omega h_0\right]^2} \quad (47)$$

It is obvious, that at a certain frequency of wave signal, the wave number $k_2(\omega)$ is valid for limited number of harmonics $0\le m(\omega)\le(2h_0\omega/\pi c_{2t})-3/4$, where $m\in\mathbb{N}^+$.

To the wave number (47) corresponds the phase velocities of adjacent waves in the corresponding layers

$$V_{a\phi}(\omega/k)=c_{1t}\Big/\sqrt{1+(2\pi c_{1t}/\omega h_0)^2};\qquad V_{b\phi}(\omega/k)=c_{2t}\Big/\sqrt{1-\left[(4m+3)\pi c_{2t}/8\omega h_0\right]^2} \quad (48)$$

It is shown from relations (47) and (48), that the obtained values of phase velocities correspond to the shear waves of Love type $\min\{c_{1t};c_{2t}\}\le V_\phi(\omega/k)<\max\{c_{1t};c_{2t}\}$, out of the frequency range of the wave signal (40)

$$\omega\ge(2\pi/h_0)\cdot\left(c_{1t}c_{2t}\Big/\sqrt{c_{1t}^2-c_{2t}^2}\right) \quad (49)$$

Considering the definition area (49), we also receive the number of generated harmonics depending on the characteristics of adjacent materials

$$0\le m<\left(16c_{1t}-3\sqrt{c_{1t}^2-c_{2t}^2}\right)\Big/4\sqrt{c_{1t}^2-c_{2t}^2}$$

Taking into account the obtained values of wave numbers (47) and phase velocities (48) the amplitude functions through the thickness of the composite waveguide $\{Y_0(y);\ Y_0^*(y)\}$ will be presented in the following descriptions:

- in periodic virtual vertical layers corresponding to the widths of the surface protrusions $n(a+b)\le x\le(1+n)a+nb$

$$Y_{0a}(y)=\begin{cases} A_1\cdot\exp[k_1(\omega)\alpha_{1t}(\omega)y]; & -\infty<y\le -h_0\\ A_1^*\cdot sh[k_1(\omega)\alpha_{1t}(\omega)y]+B_1^*\cdot ch[k_1(\omega)\alpha_{1t}(\omega)y]; & -h_0\le y\le h_0\\ \sum_{m=0}^{m_0}\left\{A_{2m}\cdot\sin[k_1(\omega)\beta_{2t}^{(m)}(\omega)y]+B_{2m}\cdot\cos[k_1(\omega)\beta_{2t}^{(m)}(\omega)y]\right\}; & h_0\le y<\infty\end{cases} \tag{50}$$

- in periodic virtual vertical layers corresponding to the widths of the surface protrusions $(1+n)a+nb\le x\le(1+n)(a+b)$

$$Y_{0b}(y)=\begin{cases} A_1\cdot\exp[k_1(\omega)\alpha_{1t}(\omega)y]; & -\infty<y\le -h_0\\ \sum_{m=0}^{m_0}\left\{A_{2m}^*\cdot\sin[k_{2m}(\omega)\beta_{2t}^{(m)}(\omega)y]+B_{2m}^*\cdot\cos[k_{2m}(\omega)\beta_{2t}^{(m)}(\omega)y]\right\}; & -h_0\le y\le h_0\\ \sum_{m=0}^{m_0}\left\{A_{2m}\cdot\sin[k_{2m}(\omega)\beta_{2t}^{(m)}(\omega)y]+B_{2m}\cdot\cos[k_{2m}(\omega)\beta_{2t}^{(m)}(\omega)y]\right\}; & h_0\le y<\infty\end{cases} \tag{51}$$

Putting the obtained values of wave numbers (47) in the dispersion equation (31) (or (33)), we obtain dispersion dependency of the propagation of adjacent shear waves in the composite waveguide $k(\omega)$.

Putting the values of wave numbers (47) in inequality (32) (or (33)) will receive the zones of allowed (or forbidden) frequencies for the formed harmonics of adjacent shear waves in the composite waveguide. Equation Section (Next)

3. Numerical and Analytical Comparative Analysis

Let us consider some obvious cases, in which the obtained results are the limit for the above considered tasks:
the materials of boundary half-spaces are the same. Then, obviously, all the boundaries disappear between the half-spaces, and the equality of the shear modulus $G_1=G_2=G$ and density of materials $\rho_1=\rho_2=\rho$ leads to the automatic execution of the dispersion equation (31) and the propagation condition of shear waves (32) for all wave numbers $k(\omega)=k_1(\omega)=k_2(\omega)$ of limited shear wave of constant amplitude, with speed $V_0(\omega/k)=G/\rho$.

For the second medium taking the characteristics $G_2=0$ and $\rho_2=0$, from the above relations it follows that there are not elastic waves outside of half-space $\Omega_1\{x;y\}$.

Virtually modeling the half-space with surface protrusions as a longitudinally inhomogeneous waveguide from periodically repeated layers

$$\Omega_a\{x;y\}=\left\{n(a+b)\le x\le(1+n)a+nb;\ -\infty<y\le h_0;\ |z|<\infty\right\} \text{ и}$$

$$\Omega_b\{x;y\}=\left\{-(1+n)b+n(a+b)\le x\le n(a+b);\ -\infty<y\le -h_0;\ |z|<\infty\right\}$$

and taking into account the periodicity, satisfying the boundary conditions on mechanically free lateral borders of surface protrusions

$$\mathrm{w}_{a,x}^*(0;y;t)=0\,;\ \lambda\cdot\mathrm{w}_{a,x}^*(a;y;t)=\mathrm{w}_{a,x}^*(-b;y;t)=0\,;\ \lambda^{-1}\cdot\mathrm{w}_{a,x}^*(0;y;t)=\mathrm{w}_{a,x}^*(a+b;y;t)=0\,,$$

dumped into the depth of homogeneous half-space, the normal waves in the corresponding periodic vertical layers can be represented in the following forms

$$\begin{aligned}\mathrm{w}_{1a}(x;y;t)&=A_{1a}\cdot\exp[\alpha_{1a}k_a(y+h_0)]\cdot\cos(k_a x)\cdot\exp(-i\omega t)\\ \mathrm{w}_{1b}(x;y;t)&=A_{1b}\cdot\exp[\alpha_{1b}k_b(y+h_0)]\cdot\cos(k_b x)\cdot\exp(-i\omega t)\end{aligned} \tag{52}$$

The wave numbers of propagating wave in corresponding surface protrusions and cavities of periodically repeating vertical layers will be

$$k_a = 2\pi m_a / a\,;\; k_b = 2\pi m_b / b \text{ where } m_a;\, m_b \in \mathbb{N}^+ \tag{53}$$

The propagation condition (31) of shear elastic wave along the surface with periodically canonical protrusions in this case will be written in a simplified form

$$\cos[k(\omega)\cdot(a+b)] = \cos(k_a a)\cdot\cos(k_b b) \tag{54}$$

Representing non-dumped solutions in the surface protrusions of the half-space (on the interval $y \in [-h_0;\, h_0]$) by hyperbolic functions

$$\mathrm{w}_{1a}^*(x;y;t) = \left[A_{1a}^*\cdot sh(k_a\alpha_{1a}y) + B_1^*\cdot ch(k_a\alpha_{1a}y)\right]\cdot\cos(k_a x)\cdot\exp(-i\omega t) \tag{55}$$

from the condition of mechanically free surface on $y = -h_0$, for dumped normal waves in the layer $\Omega_b\{x;y\}$, from the condition of the mechanically free surface on $y = h_0$ and from the continuity conditions of mechanical fields in the layer $\Omega_a\{x;y\}$, we obtain the dispersion equations of wave formation in the layers, respectively

$$th(2h_0 k_a\alpha_{at}) = 1; \qquad \alpha_{1b}k_b = 0 \tag{56}$$

Which obviously allow the propagation of normal waves of the type (52) dumped into the depth of homogeneous half-space, in the case of short wavelengths

$$\lambda_a = a/m_a = 2\pi h_0 \Big/ \sqrt{4\pi^2 + \omega^2 h_0^2 / c_t^2}\,; \qquad \lambda_b = b/m_b < 2\pi c_t/\omega \tag{57}$$

This is possible for the following ratios of linear dimensions of surface protrusions

$$h_0(a;b;\omega) < \left(2\pi\, c_t/\omega\right)\cdot(4\cdot\left(bm_a/am_b\right)^2 - 1)^{-1/2} \tag{58}$$

Corresponding to adjacent waves (Love type waves), the wave number determined from the formation dispersion relation (56), given (53)

$$k_a(\omega) = \left(\omega/c_t\right)\cdot\sqrt{1+\left(c_t/\omega\right)^2\cdot\left(2\pi/h_0\right)^2}\,; \qquad k_b(\omega) = k_a(\omega)\big/\max\{m_b a; m_a b\} \tag{59}$$

The phase velocity in the layers will take the following values

$$V_{a\phi}(\omega) = c_t \Big/ \sqrt{1+\left(c_t/\omega\right)^2\cdot\left(2\pi/h_0\right)^2}\,; \qquad V_{b\phi}(\omega) = \left(a\, m_b/bm_a\right)\cdot V_{a\phi}(\omega) \tag{60}$$

For existing in the periodic vertical layers, the respective numbers of forms of the Love type waves we obtain

$$m_b/m_a < \left(b/a\right)\cdot\sqrt{1+\left(c_t/\omega\right)^2\cdot\left(2\pi/h_0\right)^2} \tag{61}$$

We get Floquet wave number by putting the values of wave numbers $k_a(\omega)$ and $k_b(\omega)$ from (59) in the dispersion equation (54)

$$k_F(\omega) = (a+b)^{-1}\cdot\arccos[\cos(k_a a)\cdot\cos(k_b b)] \tag{62}$$

The allowed (and/or forbidden) frequencies zones of existing adjacent Love type waves are received taking into account the conditions of attenuation of shear waves $V_\phi(\omega) = c_t$ in the half space.

Numerical calculations are carried out for cases when a conductor and a piezoelectric are bounded, without considering their electromagnetic properties (Table 1). A pair of materials (*PZT-4 with Ag (silver) and/or ZnO with AU (gold)*) is chosen so that the speed of bulk waves in them be different. If in the pair of *PZT-4 with Ag* the speed of bulk waves $c_{1t} = 1.848\times10^3$ (m/sec) and $c_{2t} = 1.67\times10^3$ (m/s) differ little, then in the pair of *ZnO with Au* $c_{1t} = 2.735\times10^3$ (m/sec) and $c_{2t} = 1.182\times10^3$ (m/s) they differ significantly.

Table 1. *Shear modulus, densities and speeds of shear waves in some conductors and piezoelectric crystals.*

	Gold Au	Copper Cu	Silver Ag	PZT-4	Zinc Oxide *ZnO*
Shear modulus G_i (n/m^2)	2.7×10^{10}	4.833×10^{10}	3.03×10^{10}	2.56×10^{10}	4.25×10^{10}
Material density ρ_i (kg/m^3)	19.32×10^3	8.93×10^3	10.49×10^3	7.5×10^3	5.68×10^3
Wave speed *(SH)* c_t (m/sec)	1.182×10^3	2.326×10^3	1.67×10^3	1.848×10^3	2.735×10^3

For the calculations of geometry of surface protrusions of both half-spaces it is characterized by the relative linear dimensions $2h_0 \sim \{a;\ b\}$, $a/2h_0 \sim 10^{-1}$ or $b = a/2$. The study of propagation of short (or super short) micrometer shear waves $\lambda = 2\pi/k \sim \{a;\ b\} \sim mkm$ implies numerical analysis for frequency of about $10^9 \div 10^{11}$ *Hertz*.

The formation of zones of forbidden and/or the allowed frequencies is shown on Figure 2. for slow high-frequency shear waves with phase speed less than the minimum bulk wave of adjacent materials $V_\phi(\omega/k) \le \min\{c_{1t}; c_{2t}\}$, in a composite waveguide of *ZnO and Au*. It is shown analytically, that slow waves are formed at relatively low frequencies (40), in this case up to $\approx 0.85\times10^{10}$ *Hertz*. The zones of allowed frequencies of these localized waves are already determined from the system (33) taking into account (38), in the definition range (40).

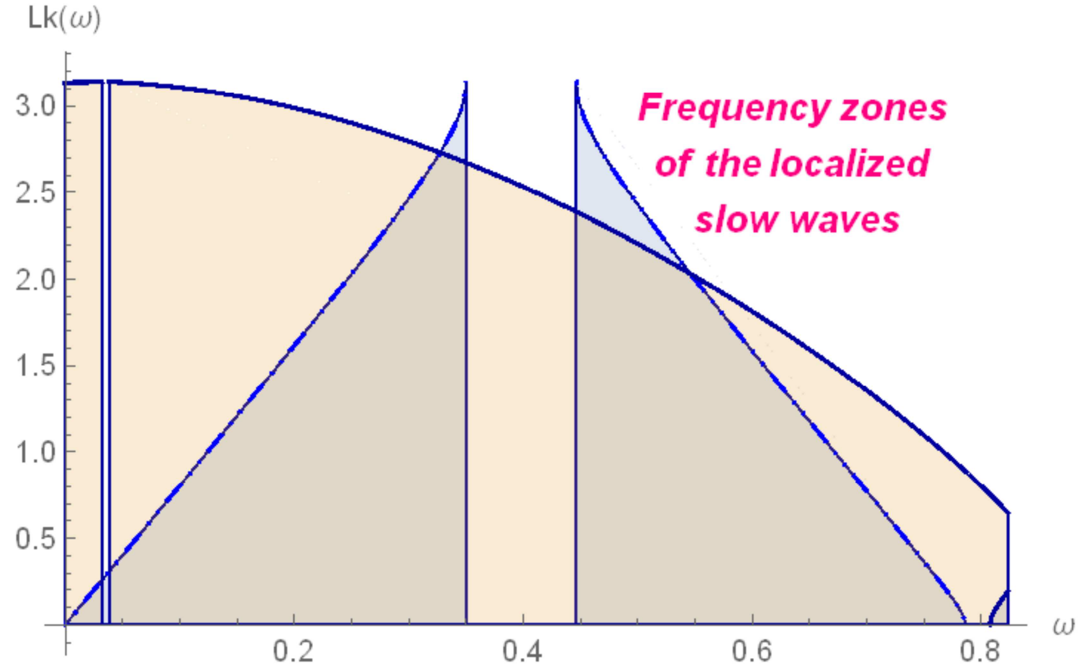

Figure 2. *Forbidden and/or allowed frequency zones for shear localized slow waves (Stoneley type waves) in composite waveguide of Zinc Oxide and Gold (or ZnO + Au).*

It follows from the calculations, that the formation of localized slow waves with the wave numbers (38) in the composite waveguide, in contrast to the case of propagation of shear bulk wave in periodically longitudinally inhomogeneous waveguide of homogeneous layers with wave numbers $k_n(\omega) = \omega/c_{nt}$ [10, 11, 20], have an almost continuous range of frequencies with one thin frequency slit. The nature of phase speed changes of localized slow waves in the virtual composite layers is shown in Figure 2. Up to a certain frequency, the phase speeds in both composite layers are less than the minimum speed of bulk shear waves in the adjacent environments.

It is shown from Figure 2, that for a pin contact of half-spaces, localization of Stoneley type wave occurs in the frequency band (40), which in this case is up to $\approx 0.85\times10^{10}$ *Hertz*. It is also important, that the total bandwidth of allowed frequencies in the case of slow localized waves is limited and its length is determined by the relation of physical characteristics of the adjacent materials. High-frequency shear Love type waves for which $\min\{c_{1t}; c_{2t}\} \le V_\phi(\omega/k) < \max\{c_{1t}; c_{2t}\}$ already will propagate in the composite waveguide of the same materials outside of frequency bands (40).

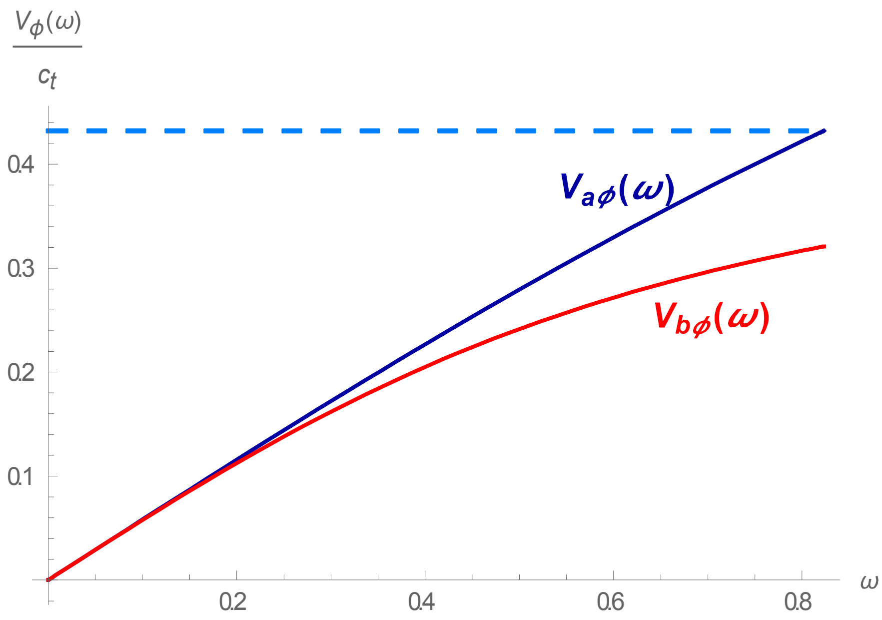

Figure 3. *The nature of phase speed changes of shear localized slow waves (Stoneley type waves) in composite waveguide of Zinc Oxide and Gold (or ZnO + Au).*

In this case, the forbidden (or allowed) frequency zones of localized Love type waves are also determined from the dispersion equation (31), already taking into account the wave numbers (48) in the definition area (49) (Figure 3).

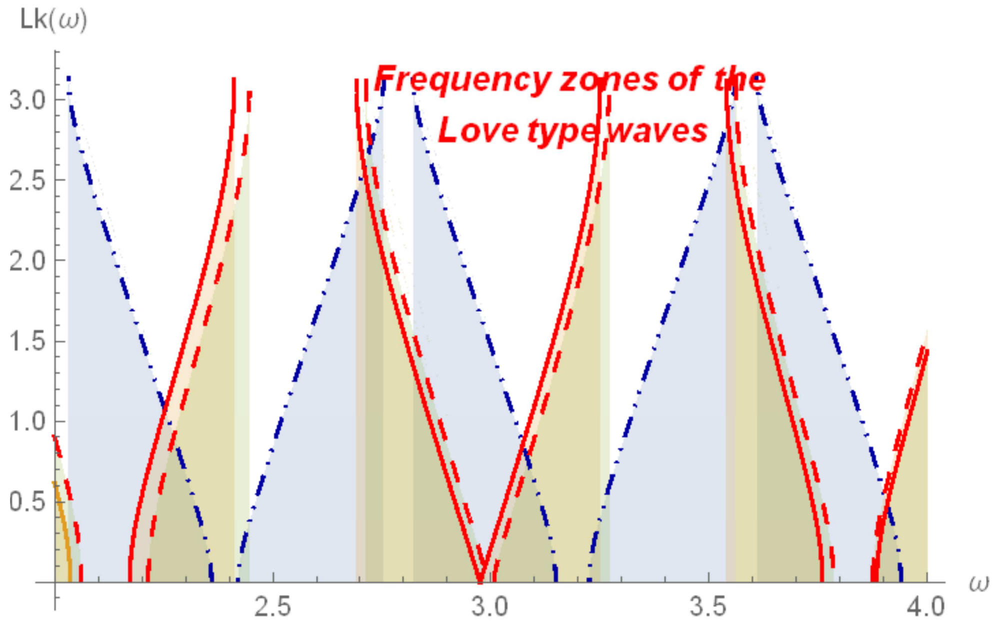

Figure 4. *Forbidden and/or allowed frequency zones for shear localized slow waves (Love type waves) in composite waveguide of Zinc Oxide and Gold (or ZnO + Au).*

It follows from Figure 4, that the allowed frequency zones of formed harmonic adjacent Love type waves do not differ much among themselves on the frequency bandwidth (49) and differ sharply from the allowed frequency zones in the problem of homogeneous layers. The zones of harmonic waves of Love type are already practically identical approximately at frequencies $\omega \approx 3.0\times10^{10} \div 3.0\times10^{11}$ Hertz. On the same frequency bandwidth, the forbidden frequency zones of the formed harmonics first expand, then shrink, but do not disappear.

The nature of changes of phase speeds of formed harmonics of high-frequency adjacent shear waves in the virtually selected vertical layers is shown in Figure 4. It follows from the given dependences of the normalized phase speeds on Figure 3, Figure 4, that the frequency value $\omega_{cr} = (2\pi/h_0)\cdot\left(c_{1t}c_{2t}/(c_{1t}^2 - c_{2t}^2)^{1/2}\right)$ is critical, at which the localization of shear waves of Stoneley type turns in an adjacent localization. Also, localization of Love type waves appears.

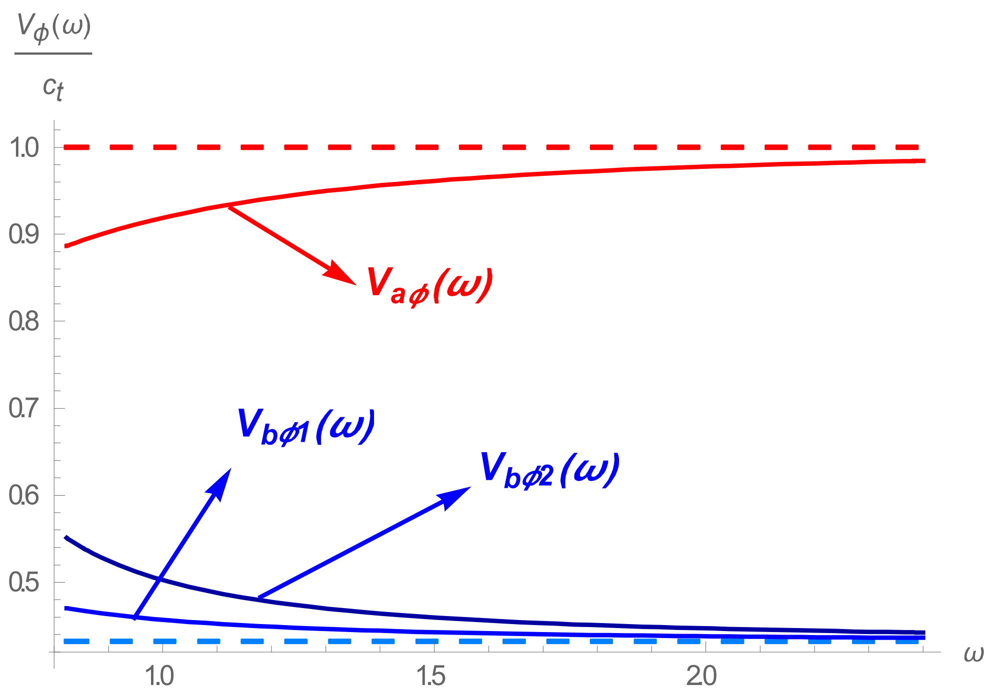

Figure 5. *The nature of phase speed changes of shear localized slow waves (Love type waves) in composite waveguide of Zinc Oxide and Gold (or ZnO + Au).*

The allowed (or forbidden) wave zones are determined by simplified Floquet equation (62) in the problem of propagation of shear wave signal in space with mechanically free surface with the canonical protrusions,.

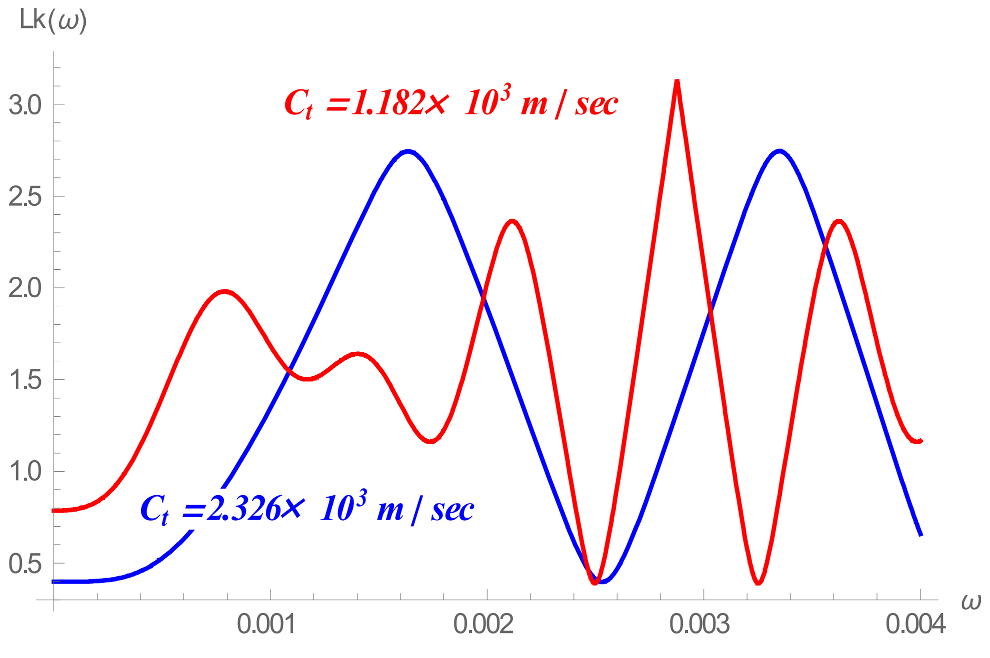

Figure 6. *The dispersion curves of localized shear millimeter waves $\lambda \sim 10^{-3} m$ at the mechanically free surface of half-space of piezoelectric crystal PZT - 4 and Gold (PZT - 4 or Au), for millimeter height $h_0 = 10^{-3}$ m and millimeter widths of protrusions and cavities of surface protrusions $a = 10^{-3}$ m and $b = 5\times10^{-4}$ m.*

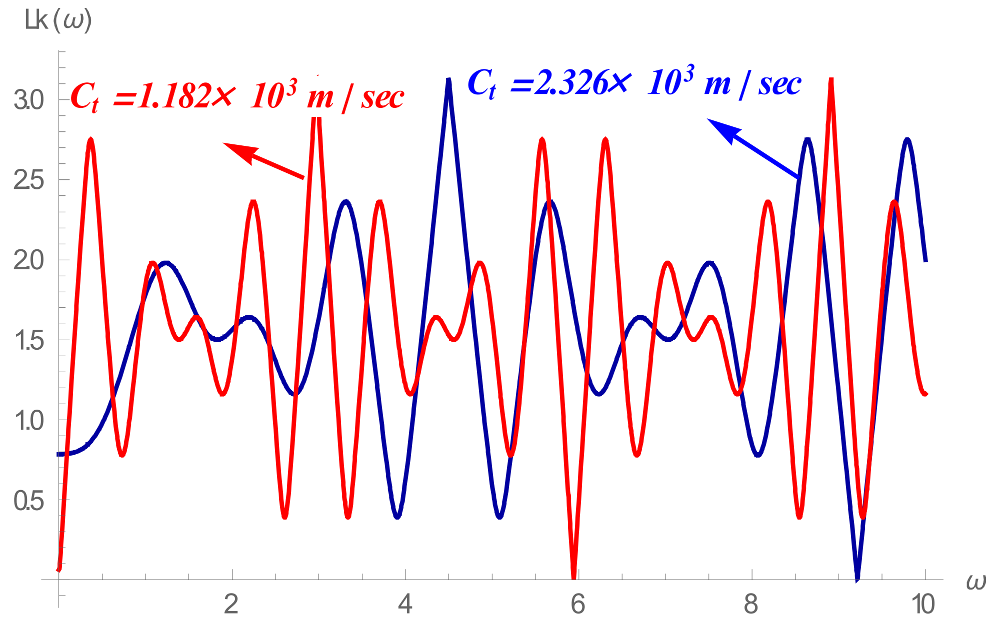

Figure 7. *The dispersion curves of localized shear micrometer waves $\lambda \sim 10^{-6} m$ at the mechanically free surface of half-space of piezoelectric crystal PZT - 4 and Gold (PZT - 4 or Au), for micrometer height $h_0 = 10^{-6}$ m and micrometer widths of protrusions and cavities of surface protrusions $a = 10^{-6}$ m and $b = 5\times10^{-7}$ m.*

Graphical images of dispersion curves, for different relative sizes of surface protrusions and different materials of half-space are shown on Figure 6, Figure 7. It follows from the calculations, that we have considerable dispersion changes for the propagation of relatively long (millimeter) waves (Figure 6) at quite low frequencies. The high frequencies of wave signal lead to frequency setting mode, as in the case of propagations of millimeter, as well as relatively short (micrometer) waves (Figure 7).

It also follows from Figure 7 that in the case of high-frequency signals, the dispersion lines $k(\omega)$ quite strong vary, but do not cross the frequency coordinate line.

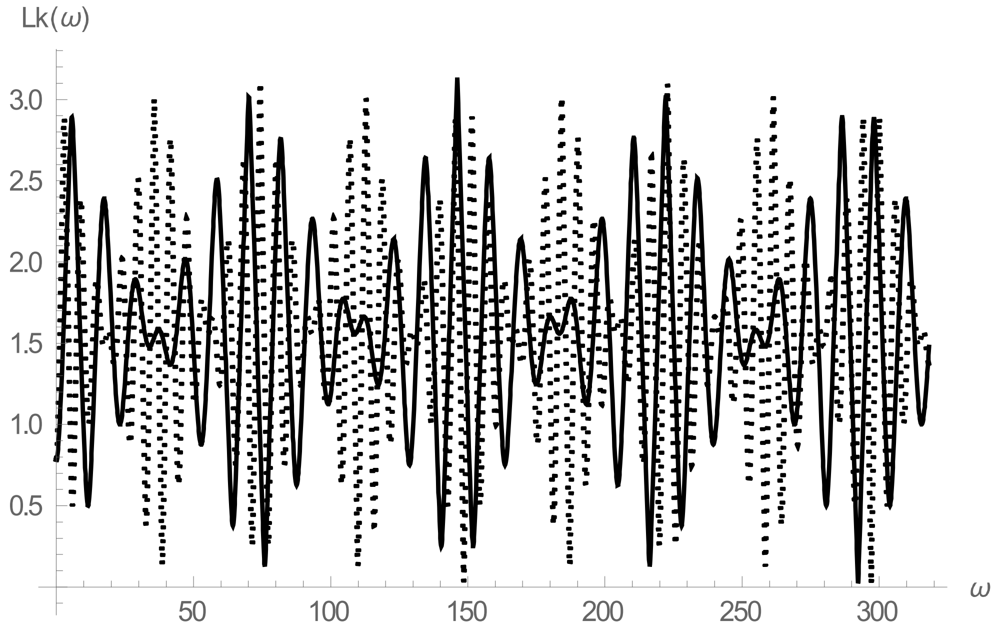

Figure 8. *The dispersion curves of localized shear nanometer waves $\lambda \sim 10^{-9} m$ at the mechanically free surface of half-space of piezoelectric crystal (PZT-4) or Gold (Au), for micrometer height $h_0 = 10^{-6}$ m and micrometer widths of protrusions and cavities of surface protrusions $a = 10^{-8}$ m and $b = 5\times10^{-7}$ m.*

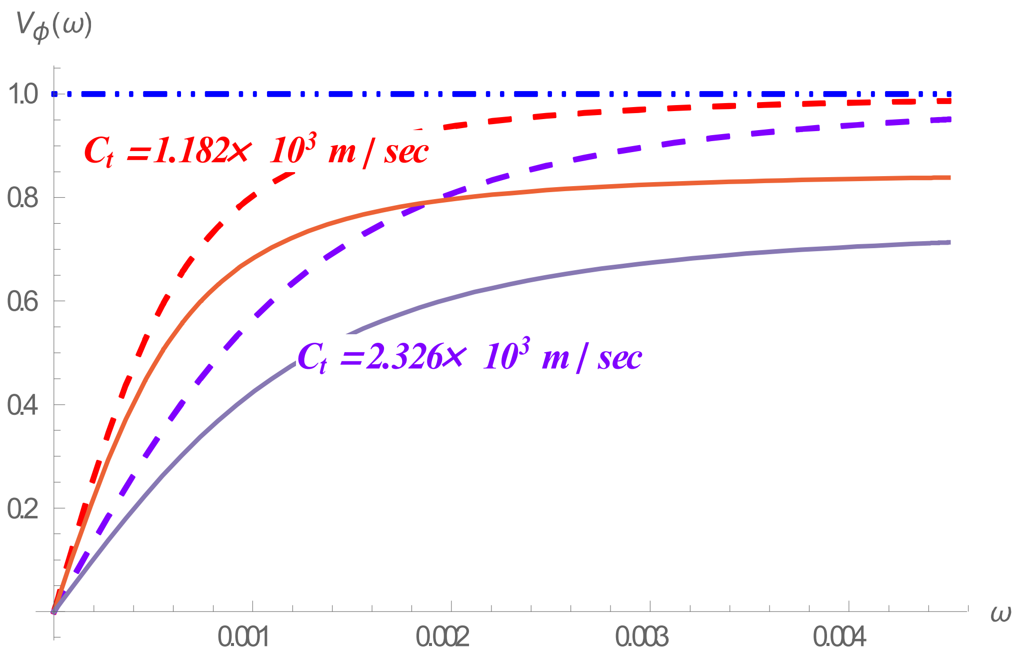

Figure 9. *The nature of phase speed changes of shear localized slow waves in half-space of piezoelectric crystal (PZT-4) or gold (Au) with mechanically free surface.*

It follows from more visual graphs of high-frequency propagation (Figure 8), that forbidden frequency zones do not form in this task, in which wave numbers $k(\omega)$ do not exist. In this case, the dispersion lines have clearly outlined envelopes at top and bottom. It is also obvious, that the different stiffness of the materials of half-spaces lead to frequency shear of the dispersion curves between each other.

It is interesting, that in all these cases the nature of changes of phase speeds are the same in the virtually selected layers (Figure 9), while the phase speed in the cavity layer ($a \le x \le a+b$) is less than the phase speed in the protrusion layer ($0 \le x \le a$).

4. Conclusions

The connection of two half-spaces with surface canonical protrusions is modeled as a composite waveguide consisting of periodically, laterally inhomogeneous embedded inner layer in two homogeneous half-spaces. The allowed and forbidden frequency zones of wave process are determined.

The conditions of wave propagation in periodically inhomogeneous layered waveguide are obtained for various physically possible cases of wave formation in lateral composite cells of composite waveguide. Depending on the relative linear dimensions of the surface protrusions, as well as on physico-mechanical characteristics of the waveguide materials, possible modes of localization of shear waves are analytically and numerically investigated along the uneven surface of the composite waveguide. The dispersion of waves, nature of change of the phase speeds and amplitude distribution through the thickness of the waveguide in the virtually selected layers are investigated.

As limiting cases, the propagation of shear wave signal in elastic half-space with mechanically free surface and periodic surface rectangular protrusions is studied. It is shown, that in this case forbidden frequency zones for the propagation of localized shear waves are not formed. Localization similar to localization of Love type waves occurs.

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