

Suitability of 3D Rock Failure Criteria for Wellbore Stability in the Niger Delta

Roland Ifeanyi Nwonodi^{1, *}, Solomon Ayuba², Titus Jibatswen Yusuf³, Adali Francis Eromosele⁴, Glory Chukwu¹, Tsokwa Tswenma¹

¹Department of Chemical Science, Federal University Wukari, Wukari, Nigeria

²Department of Civil Engineering, Bayero University Kano, Nigeria

³Department of Mechanical Engineering, University of Agriculture, Markudi, Nigeria

⁴Department of Petroleum and Gas Engineering, University of Port Harcourt, Choba, Nigerisa

Email address

ifeanyi@fuwukari.edu.ng (R. I. Nwonodi), rolandnwonodi@yahoo.com (R. I. Nwonodi) *Corresponding author

Citation

Roland Ifeanyi Nwonodi, Solomon Ayuba, Titus Jibatswen Yusuf, Adali Francis Eromosele, Glory Chukwu, Tsokwa Tswenma. Suitability of 3D Rock Failure Criteria for Wellbore Stability in the Niger Delta. *American Journal of Earth Science and Engineering*. Vol. 1, No. 4, 2018, 174-180.

Received: September 25, 2018; Accepted: November 4, 2018; Published: December 21, 2018

Abstract: In order to predict wellbore instability in the Niger Delta basin, there are several important rock failure criteria available in 3D, which produce different results as they were developed on the basis of different hypotheses. The geomechanical data used to develop these criteria were obtained from the Gulf Coast, the North Sea, and the Asia Pacific, but none from Africa, yet engineers apply these models in wellbore stability analysis in the region. Thus, this study was carried out to compare quantitatively the level of discrepancy among the commonly applied 3D rock failure criteria using data from the region. The justification for selecting 3D criteria is the evident significant influence of the intermediate principal stress on rock strength. The alternatives selected for this study are the commonly applied modified Lade, circumscribed Drucker-Prager, middle circumscribed Drucker-Prager, inscribed Drucker-Prager, and Mogi-Coulomb criteria. The authors computed the values of the safe mud weight required to prevent wellbore collapse for the various alternatives and considered their fitness with respect to the mean squared error. Bar charts and discrepancy table displayed the results in the study. From the results, the modified Lade criterion was the most accurate for a vertical wellbore orientation while the Mogi-Coulomb criterion gave the most accurate result for the horizontal wellbore orientation. In addition, the circumscribed Drucker-Prager criterion gave the least accurate result. The closest discrepancy existed between the Mogi-Coulomb criterion and the middle circumscribed Drucker-Prager criterion, while the widest discrepancy existed between the circumscribed Drucker -Prager and the inscribed Drucker-Prager. Thus, the modified Lade and Mogi-Coulomb criteria are more suitable for drilling vertical well and horizontal well respectively.

Keywords: Geomechanical Data, 3D Failure Criteria, Wellbore Instability, Discrepancy Table

1. Introduction

Rock failure criteria are mathematical expressions showing the condition of the principal stresses in the formation at the point of failure. They are usually 2D, like the conventional Mohr-Coulomb criterion, or 3D. In the literature, there are several 3D failure criteria for wellbore stability analysis, which include the Mogi criterion, the Drucker-Prager criterion, the Von Mises criterion and the modified Lade criterion [1-4]. These criteria capture the essential features of the three principal stresses in the subsurface. Their determination is important in the field of rock mechanics and in drilling engineering in general. One of the reasons for this is that accurate rock failure criterion is an integral component necessary for the management of wellbore instability, which cost the drilling industry a huge amount of dollars every year [5]. The poor prediction of wellbore instability leads to nonproductive time usually in the form of wellbore collapse and stuck pipe. In Nigeria, this non-productive time is about 15% of the well cost [6]. Subsequent to the development of the Mohr-Coulomb failure criterion, several researchers have shown the importance of the inclusion of the intermediate principal stress into rock failure analysis as evident in the works of Wiebols, Mogi and Ewy [7, 1, 4].

The most commonly applied 3D rock failure criteria for wellbore stability analysis are the Mogi-Columb criterion and the Drucker-Prager criterion, which is divided into the circumscribed, middle circumscribed and inscribed Drucker-Prager criteria. There is also the modified Lade criterion, which yields practical results [8]. The development of the Mogi-Coulomb failure criterion has helped to overcome the difficulty of using the power law form of the Mogi failure criterion for geomechanical studies of this nature. The results obtainable from the use of the several 3D rock failure criteria do not reflect results that agree numerically, but this should be the case. One good reason for the discrepancies among the results is the different hypotheses used in their developments. It implies that some of these criteria require refinement in order to produce consistent results. This does not concern the current study.

The data used to develop most of the 3D rock failure criteria, are from the Gulf Coast, North Sea, and the Asia Pacific. For example, Mogi used the Solenhofen limestone from Germany and the Shirahama Sandstone from Japan in his study [1]. Colmenares and Zoback used the KBT Amphibolite from Germany in their study [9], and Minaeian used the Pierre shale from the USA and the Yuubari shale from China in her study [10]. None is from the West African sub-region, yet these models are applicable in wellbore stability analysis in the region. It is therefore important to consider which of them is more suitable when applied to data from West Africa, but the way to go about this may not be easy.

The aim of this study is to compare quantitatively the Mogi-Coulomb, Modified Lade, circumscribed Drucker-Prager, middle circumscribed Drucker-Prager and inscribed Drucker-Prager criteria using data from the Niger Delta subregion. The objectives are to determine the safe mud weights of the well using the various alternative, to consider if there are significant similarities or differences in the models when applied to the Niger Delta wellbore, to compare the discrepancies in the models and to ascertain the most reliable criterion based on the analysis.

One cannot overemphasize the importance of this study when one considers the huge amount of dollars spent in the management of wellbore instability in the world at large, and in the Niger Delta in particular. The study is important, as it will provide the basis for carrying out better wellbore stability analysis in the complex terrains in the region. In order to minimize drilling costs and challenges that arise from drilling through complex formations, it is critical to use, as accurate as possible, models that are well suited to the particular zone.

2. Literature

Mogi conducted the first extensive compression tests on rocks using true triaxial tests. After concluding the experiments, he observed that the intermediate principal stress has an effect on rock strength. He also noted that brittle fracture of the rock samples occurred along a plane in the intermediate stress direction. The experiments by Takahashi and Chang also confirmed this observation [11, 12]. The Mogi criterion assumes that the mean normal stress opposes the fracture of the failure plane. He gave an expression between these stresses, which has a limitation that the parameters are difficult to relate to that in the Coulomb failure. For this reason, Al-Ajmi developed a linear expression known as the Mogi-Coulomb criterion now used in wellbore stability analysis [13]. The following is the mathematical expression for the Mogi-Coulomb criterion:

$$\tau_{oct} = a + b\sigma_m \tag{1}$$

Where τ_{oct} = octahedral shear stress, σ_m = average normal stress on the plane of failure, a = parameter related to cohesion, and b = parameter relation friction.

They gave an expression for the average normal stress acting on the plane of failure in terms of the maximum and minimum principal stresses:

$$\sigma_m = \frac{\sigma_1 + \sigma_3 - 2P}{2} \tag{2}$$

Where σ_1 = maximum principal stress, σ_3 = minimum principal stress and *P* = pore pressure.

The octahedral shear stress is a formulation written in terms of the three principal stresses. These stress formulations enable the principal stresses acting at the wellbore walls to simplify the analysis. The mathematical expression for this shear stress is the following:

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$
(3)

When compared to the conventional Mohr-Coulomb criterion, the parameters in eq. (1) are the following:

$$a = \frac{2cCos\varphi}{\sqrt{3}} \tag{4}$$

$$b = \frac{2Sin\varphi}{\sqrt{3}} \tag{5}$$

Where c = rock cohesion, $\varphi = \text{friction}$ angle between failure plane and the normal.

The Drucker-Prager failure criterion is one of the commonly applied rock failure criterion originally formulated for soil mass [2]. It resembles the Mohr-Coulomb failure criterion but does not make use of the shear stress and normal stress. Instead, it uses the octahedral shear stress and the octahedral normal stress. The mathematical expression for the criterion is the following [6, 9]:

$$\sqrt{J_2} = k + \alpha I_1 \tag{6}$$

Where J_2 = the second deviator stress invariant, I_1 = the

first principal stress invariant of Cauchy stress, k = parameter related to the rock strength, and $\alpha =$ parameter related to friction. The parameters used for the circumscribed (c), middle circumscribed (m), and inscribed (i) Drucker-Prager failure criterion are the following expressions:

$$k_c = \frac{6c\cos\varphi}{\sqrt{3}\left(3 - \sin\varphi\right)} \tag{7}$$

$$\alpha_c = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)} \tag{8}$$

$$k_m = \frac{6c\cos\varphi}{\sqrt{3}\left(3+\sin\varphi\right)} \tag{9}$$

$$\alpha_m = \frac{2\sin\varphi}{\sqrt{3}(3+\sin\varphi)} \tag{10}$$

$$k_i = \frac{3cCos\varphi}{\sqrt{9+3Sin^2\varphi}} \tag{11}$$

$$\alpha_i = \frac{Sin\varphi}{\sqrt{9 + 3Sin^2\varphi}} \tag{12}$$

$$\sqrt{J_2} = \frac{1}{6}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$
(13)

$$I_1 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{14}$$

Where σ_1 = maximum principal stress, σ_2 = intermediate principal stress, σ_3 = minimum principal stress, c = cohesion, and φ = friction angle.

Although these expressions are 3D formulations, they may yield different results when compared with one another and with other 3D models.

The modified Lade criterion is an important criterion applicable to any region with petroleum exploration activity. It is an empirical model obtained from observation of the behavior of samples to true triaxial stresses. This model accounts for the influence of the intermediate stress in a realistic way [8] and the mathematical expression for the criterion are the following expressions [4]:

$$\frac{I_1^3}{I_3} = 27 + \eta_L \tag{15}$$

$$I_1 = (\sigma_1 + s_L) + (\sigma_2 + s_L) + (\sigma_3 + s_L)$$
(16)

$$I'_{3} = (\sigma_{1} + s_{L})(\sigma_{2} + s_{L})(\sigma_{3} + s_{L})$$
 (17)

$$s_L = \frac{S_o}{\tan \varphi} \tag{18}$$

$$\eta_L = 4\tan^2 \varphi \frac{(9-7\sin\varphi)}{(1-\sin\varphi)} \tag{19}$$

Where σ_1 = maximum principal stress, σ_2 = intermediate principal stress, σ_3 = minimum principal stress, I_1 = principal stress invariant, I_3 = modified principal stress invariant, η_L = parameter related to friction angle, S_o = cohesion, φ = friction angle, and s_L = parameter related to the principal stresses.

The combination of a constitutive model with an accurate rock failure criterion yields the safe mud weight to prevent the collapse of the wellbore. Westergaard derived expressions for the distribution of in-situ stresses at the face of the wellbore in terms of radial, axial and tangential stresses [15]. When only the mechanical stresses are added into the stress concentration, the following expressions, in terms of wellbore coordinate system, are generated:

$$\sigma_r = P_w \tag{20}$$

$$\sigma_{\theta} = \sigma_x^o + \sigma_y^o - 2\left(\sigma_x^o - \sigma_y^o\right)\cos 2\theta - P_w$$
(21)

$$\sigma_z = \sigma_z^o - 2\mu \left(\sigma_x^o - \sigma_y^o\right) \cos 2\theta \tag{22}$$

Where σ_r = radial stress, P_w = safe wellbore pressure, σ_{θ} = tangential stress, σ_z = axial stress, μ = Poisson's ratio, σ_x^o = maximum horizontal stress in the wellbore coordinate system, σ_y^o = minimum horizontal stress in the wellbore coordinate system, and σ_z^o = vertical stress in the wellbore coordinate system.

This stress varies from the value of the wellbore pressure at the face of the well to reservoir pressure and in-situ stresses further away from the wellbore. The mud weight bears a part of the stress re-distribution created because of the excavation of earth materials. It is this mud weight that must be calculated if a stable and safe wellbore is targeted.

The location of Well X, used in this research, is in the swampy region of the Nembe community of the Niger Delta province. The Akata formation, which is the oldest lithological unit, the Agbada formation, which is typically an intercalation of sandstone and shale sequence, and the Benin formation comprising the continental sandstone aquifer make up the Niger Delta province. The impermeable shale of the region does not deform in an elastic manner, and together with compaction disequilibrium, there is the generation of overpressure in the Niger Delta. There is a constant reduction in the porosity in the zone since the age of the formation is from tertiary to Recent.

A recent comparative study of several rock failure criteria showed that the modified Lade, modified Wiebols-Cook, and Mogi-Coulomb criteria gave similar results for the lithology used. The results from these criteria predicted safe mud weight that was close to that used in drilling the field successfully. The Circumscribed Drucker-Prager criterion gave the least accurate result in the analysis done, and the result was always less than that of the Inscribed Drucker-Prager criterion. The author used a statistical analysis to determine the similarities and difference in results predicted. He used the percent difference and table of contradiction methods to display results. Colmenare and Zoback made a comparison of the Modified Lade criterion with other criteria using the fitting of polyaxial test data of several types of rocks. The Modified Lade criterion had one of the best fit for the data used. From their study, the inscribed and circumscribed Drucker-Prager criteria gave very poor results. Recent studies have recommended the use of modified Lade criterion for wellbore stability analysis. Yoshida et al. carried out a comparative study of several methods of pore pressure and fracture gradient prediction, detection and evaluation using both qualitative and quantitative methods [23]. The authors carried out several interviews and made comparative computations using actual formation and well data. Here in this study, the authors use the quantitative approach to compare the results.

3. Materials and Methods

This section focuses on the approach adopted in this study. The authors computed the values of the safe mud weight required to prevent wellbore collapse using the various alternatives and then computed the discrepancies among the results. Following the works of Gholami and Fjær et al. [14, 8], the authors substituted the constitutive stress equations of Westergaard into the various alternative rock failure criteria [15]. The minimum values of the polynomial expression obtained gave the values of the critical mud weight required to prevent wellbore collapse. The authors used a stress concentration in terms of the wellbore coordinate system to enable the computation of critical mud weight for deviated and non-deviated wellbores. This system can include the wellbore azimuth and inclination. The applicable stress regime for this study is in a normal stress system. Bar charts and discrepancy table, which shows the absolute difference in values between pairs of alternatives, displayed the results. The study field is a well in the Nembe oil field of the Niger Delta, which is Well X for a proprietary reason. In order to aid the analysis, a statistical method was applied using the arithmetic mean of the alternatives as the base value. Then the authors used the mean squared error (MSE) to obtain the group variance using the Microsoft Excel programme. The mathematical expression provided by Richard and Govindasami was applied in order to get the MSE [24].

The alternative having the smallest net error was taken as the most accurate. The discrepancy table, Table 1, is a display of the absolute value of the discrepancies between pairs of alternatives. This table is an nXn arrangement of cells, where n is the number of pairs considered in the study. The values of the non-diagonal elements show the absolute difference between pairs of alternatives, while the diagonal elements are typically zero, indicating no discrepancy with self; the cell with the lowest numeric value contains the most reliable discrepancy between pairs of alternatives, while the cell with the highest numeric value contains the most significant discrepancy between pairs of alternatives.

 Table 1. Discrepancy table system for study.

	Discrepancy	Table		-
$P_{w1} - P_{w1}$	$P_{w1} - P_{w2}$	$P_{w1} - P_{w3}$	$P_{w1} - P_{w4}$	$P_{w1} - P_{w5}$
$P_{w2} - P_{w1}$	$P_{w2} - P_{w2}$	$P_{w2} - P_{w3}$	$P_{w2} - P_{w4}$	$P_{w2} - P_{w5}$
$P_{w3} - P_{w1}$	$P_{w3} - P_{w2}$	$P_{w3} - P_{w3}$	$P_{\scriptscriptstyle W3} - P_{\scriptscriptstyle W4}$	$P_{_{W3}} - P_{_{W5}}$
$P_{\scriptscriptstyle w4} - P_{\scriptscriptstyle w1}$	$P_{\scriptscriptstyle W4} - P_{\scriptscriptstyle W2}$	$P_{w4} - P_{w3}$	$P_{w4} - P_{w4}$	$P_{\scriptscriptstyle w4} - P_{\scriptscriptstyle w5}$
$P_{w5} - P_{w1}$	$P_{w5} - P_{w2}$	$P_{w5} - P_{w3}$	$P_{w5} - P_{w4}$	$P_{w5} - P_{w5}$

4. Results and Discussions

The convention adopted is that the numerical value of the wellbore collapse obtained using the various alternatives are as follows: Mogi-Coulomb criterion is designated P_{w1} , Modified Lade criterion is designated P_{w2} , Circumscribed Drucker-Prager criterion is designated P_{w3} , Middle Circumscribed Drucker-Prager criterion is designated P_{w3} , and Inscribed Drucker-Prager criterion is designated P_{w5} . The most reliable discrepancy is for pairs of alternatives that are the best substitutes while the most significant discrepancy is for pairs of alternatives.

4.1. Case Study: $\sigma_{\theta} \ge \sigma_z \ge \sigma_r$

The incorporation of the constitutive stress equation of Westergaard into the various rock failure criteria yields the values of the wellbore collapse gradient [15]. Using the condition that the tangential stress is maximum and the radial stress is minimum, then $\sigma_1 = \sigma_{\theta}$, $\sigma_2 = \sigma_z$ and $\sigma_3 = \sigma_r$. In this case, the expression for the mean normal stress, σ_{mean} , becomes the following expression:

$$\sigma_{mean} = 0.5A - P \tag{23}$$

We can present the following substitutions from eq. (20) to eq. (22):

$$A = \sigma_x^o + \sigma_y^o - 2\left(\sigma_x^o - \sigma_y^o\right)(-1)$$
(24)

$$B = \sigma_z^o - \mu [2(\sigma_x^o - \sigma_y^o)(-1)]$$
⁽²⁵⁾

The substitution of the constants in eq. (24) and eq. (25) into the Mogi-Coulomb failure criterion yields the following expressions:

$$P_{w1} = \frac{6A + \sqrt{36A^2 - 24S_1}}{12} \tag{26}$$

Where

$$S_1 = 2(A^2 + B^2 - AB) - a - 0.5bA + bP - 0.25A^2C_1 + APC_1 - P^2C_1 + 6\tau_{\theta Z}^2$$
(27)

The modified Lade criterion yields the following expression for the critical wellbore pressure by the substitution of eq. (24) and eq. (25) into the failure criterion of eq. (15):

$$P_{w2} = \frac{(L_3 - L_5) + \sqrt{(L_3 - L_5)^2 + 4\left(\frac{L_2 L_3 L_4 L_5}{L_2 L_4}\right)}}{2}$$
(28)

$$L_1 = A + B - 3P + 3s_L , \ L_2 = 27 + n_L , \ L_3 = A - P + s_L , \ L_4 = B - P + s_L \& \ L_5 = -3P + 3s_L$$

The expression for the critical wellbore pressure using the Drucker-Prager criteria is as follow:

$$P_{w3} = \frac{6A + \sqrt{36A^2 - 24S_3}}{12} \tag{29}$$

$$S_3 = 2(A^2 + B^2 - AB) - 6k^2 - 12k\alpha(A + B - 3p) - 6\alpha^2(A + B - 3p)^2 + 6\tau_{\theta Z}^2$$
(30)

In order to use eq. (29), the accurate substitution for the parameters in the various types of the Drucker-Prager criterion should be made.

4.2. Data Analysis

In order to facilitate this study, the authors use Well X with the properties in Table 2.

Table 2. Well X data extracted at 9,000 ft.				
$\sigma_v = 1.040 \text{ Psi/ft}$	$\mu = 0.203$			
$\sigma_{_H}$ =0.790 Psi/ft	φ=35.3233deg			
$\sigma_h = 0.764 \text{ Psi/ft}$	<i>z</i> =9,000 ft			
$p_o = 0.56 \text{ Psi/ft}$	$azimuth = 0 \deg$			
<i>c</i> =998.48 Psi	<i>inclination</i> $= 0 \deg$			





Figure 1. Display of results for collapse gradient using the alternatives.

Figure 1 is a display of the results obtained using the various alternatives. The Mogi-Coulomb criterion produced a reasonable result when compared to the pore pressure. Although the modified Lade criteria yielded the result with the least error, its value can result in the incidence of wellbore kick, which can be catastrophic, if appropriate measures are not taken. The reason for this may be due to the nature of the geomechanical data employed. The middle circumscribed and inscribed Drucker-Prager criteria also yielded feasible results, but the circumscribed Drucker-Prager criterion yielded

unfeasible result having a larger discrepancy. The major reason why it differs from the other Drucker-Prager criterion is that the circumscribed Drucker-Prager criterion touches the apex of the projection of the Mohr-Coulomb criterion on the Π plane, and so does not correlates properly with the parameters in the Mohr-Coulomb criterion. Since this criterion circumscribes the Mohr-Coulomb criterion, it tends to produce large values of the critical mud weight while the inscribed Drucker-Prager yields the least values.



Figure 2. Net error in the alternatives for a vertical wellbore orientation.

Figure 2 shows the result of the net error in the various alternatives. The Modified Lade criterion gave a more accurate result for the safe mud weight followed by the Mogi-Coulomb criterion. The argument is that the modified Lade criterion yields realistic results than others in wellbore stability analysis. Here, this is not the case as the result is more unrealistic than the ones from the middle and inscribed Drucker-Prager criteria. Increasing or decreasing the friction angle changes the statistics. The major reason is that a change in friction angle will cause a change in the failure plane, which leads to a change in the failure stress. When the wellbore inclination increases, there are also variations in the statistics. An inclined wellbore tends to be more unstable than a vertical one, thus requiring more mud weight to ensure stability. The Mogi-Coulomb criterion yields better statistics for all inclination, especially at an angle of ninety degrees



 (90°) , in the wellbore used in this research.

Figure 3. Net error in the alternatives for a horizontal wellbore orientation.

Figure 3 shows the result, using a horizontal wellbore orientation. The modified Lade criterion also yields a good result for the inclined wellbore, whereas, the circumscribed Drucker-Prager criterion does not produce good results.



Figure 4. Log data for well X used for study.

Table 3 shows the level of discrepancies among pairs of alternatives. From the results, the discrepancy between the Mogi-Coulomb and the middle circumscribed Drucker-Prager criteria is the lowest while that between the inscribed and circumscribed Drucker-Prager criteria is the highest. The implication is that the middle circumscribed Drucker-Prager criterion is an improvement of the inscribed and circumscribed Drucker-Prager criteria. Figure 4 shows the log data for well X used in this study.

Table 3. Discrepancy betw	een pairs of alternatives.
---------------------------	----------------------------

Discrepancy Table						
0	0.03546	0.145526	0.005726	0.000609		
0.03546	0	0.110066	0.029734	0.036069		
0.145526	0.110066	0	0.1398	0.146135		
0.005726	0.029734	0.1398	0	0.006335		
0.000609	0.036069	0.146135	0.006335	0		

5. Conclusions

In this study, the authors considered how suitable the most commonly used 3D rock failure criteria are when applied to wellbore stability analysis with respect to data from West Africa. The estimation of the mean sum of error allowed the computation of the invariance in the alternatives. From the study, the following points are worthy of note:

- 1. The Modified Lade criterion yields a more accurate result for the critical mud weight for vertical wells, while the Mogi-Coulomb criterion yields much better results for deviated and horizontal wells.
- 2. No single failure criterion yields accurate result for all wellbore orientation.
- The most reliable (closest) discrepancy exists between the Mogi-Coulomb and middle circumscribed Drucker-Prager criterion.
- 4. The most significant discrepancy exists between the inscribed and circumscribed Drucker-Prager criterion.

Conflict of Interest

There is no conflict of interest in this study.

Acknowledgements

The authors appreciate Dr. Abrakasha of the Department of Geology, University of Port Harcourt, Nigeria, for providing the platform from which data extraction for this study was possible. In addition the authors wish to thank Mr. Breshnahan of RockWare Inc. for helping to make the plot for well X in Figure 4.

References

- [1] Mogi, K. (1971). Fracture and flow of rocks under high triaxial compression. *J Geophys Res*, 76, 1255-1269.
- [2] Drucker, D. C., Prager, W. (1952). Soil mechanics and plastic analysis or limit design. *Quart Appl Math*, 10, 157-165.
- [3] Von Mises, R. (1913). Mechanik der festen Körper im plastisch deformablen Zustand. Göttin. Nachr. Math. Phys., vol. 1, pp. 582–592.

- [4] Ewy, R. T. (1999). Wellbore-stability predictions by use of a Modified Lade Criterion. SPE Drilling Comp, 14, 85-91.
- [5] Al-Ajmi, A. M. (2006). Wellbore stability analysis based on a new true-triaxial failure criterion, TRITA-LWR Ph.D. Thesis 1026.
- [6] Adewale Dosunmu. (2013). Fundamentals of Petroleum Geomechanics and Wellbore Stability in Well Design and Construction, Modelling and Predicting Well instability, SPE/NAICE conference, Lagos.
- [7] Wiebols, G. A., Cook, N. G. W. (1968). An energy criterion for the strength of rock in polyaxial compression. *Int J Rock Mech Min Sci Geomech Abstr*, 5, 529-549.
- [8] Fjær, E., Holt, R. M., Horsrud, P., Raaen, A. M., Risnes, R. (2008). *Petroleum Related Rock Mechanics*, Second Edition, Elsevier, Amsterdam, pp 72.
- [9] Colmenares, L. B. and Zoback, M. D. (2002). A Statistical Evaluation of Intact Rock Failure Criteria Constrained by Polyaxial test data for Five Different Rocks. *Int J Rock Mech Min Sci*, 39, 695-729.
- [10] Minaeian, Vida. (2014). True triaxial testing of sandstones and shales. Ph.D. Curtin University, Faculty of Science and Engineering, Department of Petroleum Engineering.
- [11] Takahashi, M., and H. Koide. (1989). Effect of the intermediate principal stress on strength And Deformation behavior of sedimentary rocks at the depth shallower than 2000 m. In *ISRM International Symposium*. Pau, France.
- [12] Chang, C., Haimson, B. (2000). True triaxial strength and deformability of the German Continental deep drilling program (KTB) deep hole amphibolite. *J Geophys Res*, 105, 18999-19013.
- [13] Al-Ajmi, A. M., Zimmerman, R. W. (2005). Relationship between the Mogi and the Coulomb failure criteria. *International Journal of Rock Mechanics and Mining Science*, 42, 431-439.
- [14] Gholami, R., Moradzadeh, A., Rasouli, V., Hanachi, J. (2013). Practical application of Failure criteria in determining safe mud weight windows in drilling operations, *J Rock Mech Geotech Eng.* 55.

- [15] Westergaard, H. M. (1940). Plastic state of stress around a deep well. J Boston Soc Civil Eng, 27, 1-5.
- [16] Opara, A. I. (2011). Estimation of multiple sources of overpressures using vertical effective stress approach: a case study of Niger Delta. Petroleum and Coal, 53 (4), 302 – 314.
- [17] Banerjee, S., Muhuri, S. (2013). Applications of geomechanics – based restoration in structural analysis along Passive Margin Settings – Deep Water Niger Delta example. In New Understanding of the Petroleum System of Continental Margins of the World.
- [18] Short, K. C., Stäuble, A. J. (1967). Outline of geology of Niger Delta: American Association of Petroleum Geologists Bulletin, (51), 761-779.
- [19] Emujapkorue, G. O. (2015). Growth Fault History Analysis of an Oil Field, Niger Delta, Nigeria, *International Journal of Geophysics and Geochemistry*. Vol. 2, No. 5, pp. 105-112.
- [20] Rahimi, Reza. (2014). Effect of Using Different Rock Failure Criteria in Wellbore Stability Analysis, Scholars Mine Master's Thesis, Paper 7270.
- [21] Nawrocki, P. A. (2010). Critical wellbore pressures using different rock failure criteria. ARMA/USRMS 05-794. In proceeding of ISRM International Symposium and 6th Asian Rock Mechanics Symposium, New Delhi, India, 23-27.
- [22] Yi, X., Ong, S. H., Russel, J. E. (2005). Improving borehole stability analysis by quantifying the effects of intermediate principal stress using polyaxial rock strength test data. ARMA/USRMS 05-794, Alaska, 25-29 June, 2005.
- [23] Yoshida, C., Ikeda, S., Eaton, B. A. (1996). An Investigative Study of Recent Technologies Used for Prediction, Detection, and Evaluation of Abnormal Formation Pressure and Fracture Pressure in North and South America, IADC/SPE Asia Pacific Drilling Technology Conference, Kuala Lumpur, 9-11 Sept. pp 131-151, IADC/SPE 36381.
- [24] Richard Bronson, Govindasami Naadimuthu. (1997). OPERATIONS RESEARCH, Second Edition, SCHAUM'S outlines, McGraw-Hill Company, ISBN: 978-0-07-178350-7, pp 281-289.