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Adiabatic Compressibility of the Variable Chaplygin Gas

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Keywords

Adiabatic Compressibility, Chaplygin Gas, Cosmological Theories, Equation of State, Exotic Matter

In this paper, we have deduced an analytic expression for the adiabatic compressibility β_S for a Chaplygin gas which is a function of the pressure P and temperature T. We have considered the work of Panigrahi (2015) for variable Chaplygin gas, exotic matter used in some cosmological theories whose equation of state is of the form $P=-\frac{B}{\rho}$ with $B=B_0V^{-\frac{n}{3}}$ and n is a constant. The expression obtained for β_S was used for the determination of the value of the thermal capacity at constant pressure C_P for variable Chaplygin gas. It is predicted in this research the behaviour of the adiabatic compressibility β_S of Chaplygin gas in the limit of high and low pressure. We have found that the adiabatic compressibility β_S is function of the pressure and $\beta_S \to \infty$ when $P \to 0$ and $\beta_S \to 0$ if $P \to \infty$ as in the ideal gas and C_P is always positive with n<0.

Introduction

Recent observational evidences suggest that the present universe is accelerating [1, 2, 3] and the cosmological model of Chaplygin gas (CG) is one of the most reasonable explanations of recent phenomena [4, 5, 6, 7]. The form of the CG equation of state is the following $P=-\frac{B}{\rho}$ where P is the pressure of the fluid, ρ is the energy density of the fluid and B is a constant. The thermodynamic behaviour of the CG model was studied by Santos et al. [8],Guo and Zhang [9], Myung [10] and Panigrahi [11]. Santos et. al [8] have studied the thermodynamical stability in generalized Chaplygin gas model and determined that for an equation of state that depends only on the temperature the fluid presents thermodynamic stability during any expansion process. Guo and Zhang [9] proposed a new generalized Chaplygin gas model that includes the original Chaplygin gas model as a special case and found that the background evolution for the model is equivalent to that for a coupled dark energy model with dark matter. Myung [10] found a new general equation of state that describes the Chaplygin gas completely and confirms that the Chaplygin gas could shows a unified picture of dark energy and energy which cools down through the universe expansion without any critical point. Panigrahi [11] analyzed the thermodynamical behaviour of the Variable Chaplygin gas and found that the third law of thermodynamics is satisfied in this model and that the volume increases when temperature falls during adiabatic expansions, which also is observed in an gas ideal [12].

Considering the cosmological constant as a pressure, several authors [13, 14, 15, 16] have researched the thermodynamic behaviour of the black holes. Wang et. al [13] studied the first law of thermodynamics in black holes and Kerr-de Sitter spacetimes and found that by assuming the cosmological constant as a variable state parameter the mass formulas of the first law of black hole thermodynamics in the asymptotic flat spacetimes can be directly extended to rotating black holes. Sekiwa [14] analyzed the thermodynamic properties and the cosmological horizon for black holes solutions and showed that cosmological constant must decrease when takes into account the quantum effect. Larrañaga [15] found that using the cosmological constant as a new thermodynamic state variable, the differential and integral mass formulas of the first law of thermodynamics for asymptotic flat spacetimes can be extended to be used in black holes. Dolan [16] has investigated the behaviour of the adiabatic compressibility and the speed of sound v_s of the black hole and obtain that β_S vanishes for non-rotating black holes but it increases when the angular momentum reaches a maximum value. Dolan [16] also found that v_s is equal to c for a non-rotating black hole and decreases when the angular momentum is increased.

An ideal gas is a gas composed of a group of randomly moving, non-interacting point particles. The ideal gas approximation is useful because it obeys the gases laws and represent the vapor phases of fluids at high temperatures for which the heat engines is constructed [17]. A heat engine that can work with an ideal gas as working substance is the Carnot cycle. Malaver [18] found that the thermodynamic efficiency of Carnot cycle for CG model only depend on the limits of maximum and minimal temperature as in case of the ideal gas and the photons gas.

In this paper an expression is deduced for the adiabatic compressibility of the variable Chaplygin gas from the thermal equation of state for the pressure given for Panigrahi [11]. With the equation obtained for the adiabatic compressibility we have deduced an expression for the thermal capacity at constant pressure in a Chaplygin gas. We have found that the adiabatic compressibility for CG model only will depend on the pressure as in case of the ideal gas and C_P is always positive. The article is organized as follows: in Section 2, it presents the deduction of the compressibility for an ideal gas; in Section 3, we show the deduction for the adiabatic compressibility for a Chaplygin gas; in Section 4, we conclude.

Adiabatic Compressibility in an Ideal Gas

Following Dickerson [19] and Nash [20] an adiabatic process is one in which is no heat flow is or out of the system. In this case Q=0 and in agreement with the first law ΔU=W. This implies that

$$dU = C_V dT = -PdV \tag{1}$$

and for an ideal gas,

$$PdV + C_V dT = 0 \tag{2}$$

Where C_V is the thermal capacity at constant volume.

In a reversible expansion of an ideal gas $P_{ext} = P_{\text{int}} = RT/V$. Substituting this equality in (2) and dividing by T, we obtain

$$R\frac{dV}{V} + C_V\frac{dT}{T} = 0 \tag{3}$$

Integrating (3)

$$R\ln\frac{V_2}{V_1} + C_V\ln\frac{T_2}{T_1} = 0 \tag{4}$$

Converting to the exponential form

$$\left(\frac{V_2}{V_1}\right)^R = \left(\frac{T_1}{T_2}\right)^{C_V} \tag{5}$$

Substituting $\frac{T_1}{T_2} = \frac{P_1}{P_2}\frac{V_1}{V_2}$ in (5)

$$\left(\frac{P_2}{P_1}\right)^{C_V} = \left(\frac{V_1}{V_2}\right)^{C_P} \tag{6}$$

and for a reversible adiabatic process with a ideal gas we have

$$PV^{\gamma} = const \tag{7}$$

where $\gamma = \frac{C_P}{C_V}$

C_P is the thermal capacity at constant pressure. The adiabatic compressibility is defined as [20, 21]

$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S = -\frac{1}{V\left(\frac{\partial P}{\partial V}\right)_S} \tag{8}$$

According Zemansky and Dittman [21] of the eq.(7) we obtain

$$\left(\frac{\partial P}{\partial V}\right)_S = -\gamma const V^{-\gamma-1} = -\gamma \frac{P}{V} \tag{9}$$

Replacing (9) in (8) we have that for an ideal gas

$$\beta_S = \frac{1}{\gamma P} \tag{10}$$

Adiabatic Compressibility in an Chaplygin Gas

For a Chaplygin gas, the thermal equation of state for the pressure [11] is given by

$$P = -\left(\frac{B_0 N}{2}\right)^{1/2} V^{-n/6}\left(1-\frac{T^2}{\tau^2}\right)^{1/2} \tag{11}$$

where B_0 is a positive universal constant, n is a constant, $N = \dfrac{6-n}{3}$ and τ is a universal constant with dimension of temperature.

Following Zemansky and Dittman [21], the adiabatic compressibility (8) can be written as

$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S = -\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S\left(\frac{\partial T}{\partial P}\right)_S = -\frac{\left(\partial V/\partial T\right)_S}{V\left(\partial P/\partial T\right)_S} \tag{12}$$

For a reversible adiabatic process in a Chaplygin gas [18]

$$\frac{V}{\left(1-\frac{\tau^2}{T^2}\right)^{1/N}} = const \tag{13}$$

The eq.(13), we obtain

$$\left(\frac{\partial V}{\partial T}\right)_S = \frac{const}{N}\left(1-\frac{\tau^2}{T^2}\right)^{\frac{1-N}{N}}\frac{2\tau^2}{T^3} \tag{14}$$

In this paper, we have deduced an expression for an adiabatic reversible process in terms of P and T (see Appendix):

$$\frac{P T^{\frac{N-2}{N}}}{\left(\frac{T^2}{\tau^2}-1\right)^{\frac{N-1}{N}}} = const \tag{15}$$

With the eq.(15), $\left(\dfrac{\partial P}{\partial T}\right)_S$ takes the form

$$\left(\frac{\partial P}{\partial T}\right)_S = const\left(\frac{T^2}{\tau^2}-1\right)^{-\frac{1}{N}} T^{\frac{2-N}{N}}\left[2\left(\frac{N-1}{N}\right)\frac{T}{\tau^2}-\left(\frac{N-2}{N}\right)\left(\frac{T^2}{\tau^2}-1\right)\frac{1}{T}\right] \tag{16}$$

Replacing (15) in (16) and rearranging terms

$$\left(\frac{\partial P}{\partial T}\right)_S = \frac{P\left[\frac{2(N-1)}{N}\frac{T^2}{\tau^2}+\frac{(N-2)}{N}\left(1-\frac{T^2}{\tau^2}\right)\right]}{T\left(\frac{T^2}{\tau^2}-1\right)} \tag{17}$$

Substituting (14) and (17) in eq.(12), we have

$$\beta_S = \frac{2\tau^2}{PT^2}\frac{\left(1-\frac{T^2}{\tau^2}\right)}{\left(1-\frac{\tau^2}{T^2}\right)\left[2(N-1)\frac{T^2}{\tau^2}+(N-2)\left(1-\frac{T^2}{\tau^2}\right)\right]} \tag{18}$$

The expression for adiabatic compressibility (18) is an explicit function of the temperature and the pressure.

The equation (18) can be used to calculate the thermal capacity at constant pressure in a Chaplygin gas from the following expression [21]

$$\frac{C_P}{C_V} = \frac{\beta_T}{\beta_S} \tag{19}$$

where β_T is the isothermal compressibility defined as [20, 21]

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{V\left(\partial P/\partial V\right)_T} \tag{20}$$

with (11), for a Chaplygin gas we have

$$\left(\frac{\partial P}{\partial V}\right)_T = -\left(\frac{N-2}{2}\right)\left(\frac{B_0 N}{2}\right)^{1/2} V^{\frac{N-2}{2}-1}\left(1-\frac{T^2}{\tau^2}\right)^{1/2} \tag{21}$$

Substituting (11) in (21), we obtain

$$\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{N-2}{2}\right)\frac{P}{V} \tag{22}$$

Replacing (22) in (20), β_T can be written as

$$\beta_T = -\frac{2}{(N-2)P} \tag{23}$$

and from the equations (18) and (23), we obtain

$$\frac{\beta_T}{\beta_S} = \frac{T^2}{(N-2)\tau^2}\frac{\left(\frac{\tau^2}{T^2}-1\right)\left[2(N-1)\frac{T^2}{\tau^2}+(N-2)\left(1-\frac{T^2}{\tau^2}\right)\right]}{\left(1-\frac{T^2}{\tau^2}\right)} \tag{24}$$

The thermal capacity at constant volume for a Chaplygin gas is given by [11]

$$C_V = \left(\frac{2B_0}{N}\right)^{1/2} V^{\frac{N}{2}}\frac{T}{\tau^2}\frac{1}{\left(1-\frac{T^2}{\tau^2}\right)^{\frac{3}{2}}} \tag{25}$$

with eq.(24) and eq.(25) C_P can be written as

$$C_P = \left(\frac{2B_0}{N}\right)^{1/2} V^{\frac{N}{2}} \frac{T^3\left(\frac{\tau^2}{T^2}-1\right)\left[2(N-1)\frac{T^2}{\tau^2}+(N-2)\left(1-\frac{T^2}{\tau^2}\right)\right]}{(N-2)\tau^4\left(1-\frac{T^2}{\tau^2}\right)^{\frac{5}{2}}} \tag{26}$$

$C_P > 0$, this implies that $n < 0$ and $\tau > T$. Form the thermodynamic stability considerations should n always have negative value [11].

Conclusions

We have deduced an expression for the adiabatic compressibility β_S of a Chaplygin gas, which is a function of the pressure and the temperature. Is predicted for β_S that $\beta_S \rightarrow \infty$ when $P \rightarrow 0$ and $\beta_S \rightarrow 0$ if $P \rightarrow \infty$ as in the ideal gas. Furthermore, with the equation for β_S we found a new formula for the thermal capacity at constant pressure C_P for CG model that depends only the temperature and that always is positive for $n < 0$ and $0 < T < \tau$.

The study of Chaplygin gas can enrich the courses of thermodynamics, which contributes to a better compression of the thermal phenomena. The thermodynamic equations that describe the behavior of the Chaplygin gas are tractable mathematically and offer a wide comprehension of the accelerated universe expansion and of the basic ideas of the modern cosmology.

Appendix

In this Appendix, we have deduced an expression for a reversible adiabatic process in terms of P and T in a Chaplygin gas. With the eq. (11) we obtain

$$P_1 = -\left(\frac{B_0 N}{2}\right)^{1/2} V_1^{-n/6}\left(1-\frac{T_1^2}{\tau^2}\right)^{1/2} \tag{A.1}$$

and

$$P_2 = -\left(\frac{B_0 N}{2}\right)^{1/2} V_2^{-n/6}\left(1-\frac{T_2^2}{\tau^2}\right)^{1/2} \tag{A.2}$$

Dividing A.1 with A.2, we have

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^{\frac{N-2}{2}} \sqrt{\frac{T_1^2-\tau^2}{T_2^2-\tau^2}} \tag{A.3}$$

Following Malaver [18], for a reversible adiabatic process in terms of V and T

$$\frac{V}{\left(1-\frac{\tau^2}{T^2}\right)^{1/N}} = const \tag{A.4}$$

This implies that

$$\frac{V_1}{V_2} = \left[\frac{T_2\sqrt{T_1^2-\tau^2}}{T_1\sqrt{T_2^2-\tau^2}}\right]^{\frac{2}{N}} \tag{A.5}$$

Substituting A.5 in A.3

$$\frac{P_1}{P_2} = \left(\frac{T_2}{T_1}\right)^{\frac{N-2}{N}} \left(\frac{T_1^2 - \tau^2}{T_2^2 - \tau^2}\right)^{\frac{N-1}{N}} \quad \text{A.6}$$

rearranging A.6 we have

$$\frac{P_1 T_1^{\frac{N-2}{N}}}{\left(\frac{T_1^2}{\tau^2} - 1\right)^{\frac{N-1}{N}}} = \frac{P_2 T_2^{\frac{N-2}{N}}}{\left(\frac{T_2^2}{\tau^2} - 1\right)^{\frac{N-1}{N}}} \quad \text{A.7}$$

$$\frac{P\, T^{\frac{N-2}{N}}}{\left(\frac{T^2}{\tau^2} - 1\right)^{\frac{N-1}{N}}} = const \quad \text{A.8}$$

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References

[1] Sushkov, S. (2005). Wormholes supported by a phantom energy. *Phys. Rev, D71*, 043520.

[2] Lobo, F.S.N. (2005). Stability of phantom wormholes. *Phys. Rev, D71*, 124022.

[3] Lobo, F.S.N. (2006). Stable dark energy stars. *Class. Quant.Grav, 23*, 1525-1541.

[4] Fabris, J.C., Goncalves S.V.B. and de Souza, P.E. (2002*). Gen. Relativ. Grav. 34*, 53.

[5] Lima, J.A.S., Cunha, J.V. and Alcaniz, J.S. (2008). A simplified approach for Chaplygin-type cosmologies. *Astropart. Phys. 30*, 196-199.

[6] Bento, M.C., Bertolami, O. and Sen, A.A. (2002). *Phys. Rev. D66*, 043507.

[7] Panigrahi, D. and Chaterjee, S. (2011). *JCAP 10* 002.

[8] Santos, F.C., Bedran, M.L. and Soares, V. (2006). *Phys. Lett. B636*, 86.

[9] Guo, Z.K. and Zhang, Y.Z. (2007). *Phys. Lett. B645*, 326.

[10] Myung, Y.S. (2011). Thermodynamics of Chaplygin gas. *Astrophys. Space Sci.335*, 561-564.

[11] Panigrahi, D. (2015). Thermodynamical behaviour of the variable Chaplygin gas. *Int. J. Mod. Phys. D24*, 1550030.

[12] Leff, H. S. (2002). Teaching the photon gas in introductory physics. *Am. J. Phys, 70*, 792-797.

[13] Wang, S., Wu, S-Q., Xie, F. and Dan, L. (2006). *Chin. Phys. Lett. 23*, 1096.

[14] Sekiwa, Y. (2006). *Phys. Rev. D73*, 084009.

[15] Larrañaga, E.A. (2007). http://arxiv.org/abs/0711.0012

[16] Dolan, B.P. (2011). Compressibility of rotating black holes.*Phys.Rev, D84*,127503.

[17] Lee, H. (2001). Carnot cycle for photon gas? Am. J. Phys, 69, 874-878.

[18] Malaver, M. (2015). Carnot engine model in a Chaplygin gas. *Research Journal ofModeling and Simulation, 2(2)*, 42-47.

[19] Dickerson, R. (1969). Molecular Thermodynamics, W. A. Benjamin, Inc, Menlo Park, California, ISBN: 0-8053-2363-5.

[20] Nash, L. (1970). Elements of Classical and Statistical Thermodynamics, Addison-Wesley Publishing Company, Inc, MenloPark, California.

[21] Zemansky, M.W. and Dittman, R.H. (1985). Termodinámica, McGraw-Hill Interamericana, Sexta Edición, ISBN: 0-07-72808-9.