

A Holistic Approach to *Quantum Physics*

Lucian Miti Ionescu

Mathematics Department, Illinois State University, Normal IL, USA

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Quantum phenomena reflect *resonance* at the very fundamental level of a network processing Quantum Information. The term “particle” is a hindrance preventing the shift to a new paradigm. A top-level mental-visual model for “elementary particles” and some other quantum experiments is needed, and provided: a foamy Riemann Surface is interpreted as a Quantum Chip, having both fermionic parameters (sources as integrals), and bosonic character (propagating energy-momentum).

Introduction

There are quite good and diverse mathematical models for Quantum Physics, computationally speaking; yet the article proposes a different level of understanding by considering a top-level, macroscopic “picture” of quantum phenomena: consider a “foamy” (wave function like) “ethereal” (quantum orbital-like) Riemann Surface shaped structure as a Geometric Model for *any* quantum interaction. Flows on Networks represent by now a universal paradigm, in place of the traditional paradigm of Mechanics, which requires an underlying “space” and “time”. *Think* first, about such a structure, usually represented in physics theories as a Feynman Diagram or String world sheet, as a “Quantum Chip” processing quantum information (qubits \sim spinors/twistors [1]: all based on the unitary group SU_2). *Then* “implement” this image in your favorite modern mathematical language: Topological Quantum Field Theory (cobordisms modeling change, which do *not* require “time” as a “real parameter”), or QFT with “quark lines” tracing the x, y, z frame of a qubit as quarks, subject to the Gauge Theory paradigm of Standard Model; and do use Feynman Diagrams as a good, well tested tool for computing scattering amplitudes of probabilities in the current strong measurements (in the sense of Quantum Computing) in high energy physics experiments, to get the numbers to compare with the experiments (e.g. [2]). These are more than “combinatorial devices” to keep track of a perturbation series, and its internal lines are much more than just “virtual” [3].

Let us marvel at the precision achieved by both experiment and theory; and then get stumped by the coincidence: Feynman amplitudes are linear combinations of Multiple Zeta Function values (up to 10 loops!?). It looks like Mathematics is “unexpectedly effective” because, maybe, reality *is* “just” Number Theory [4-9].

In what follows we aim to get a better “feeling” of a holistic approach to understanding quantum phenomena, through analogy and pictures. The way we think determines what to expect, beyond the traditional experiments targeting more decimal places, in both experiments and theoretical calculations.

Similar considerations, stepping back and overseeing the broader landscape of science, is done periodically, with some older insights [10], as well as new ones, notably by [11], who is advocating similar ideas, at a more philosophical level, and [12], considering a holistic approach from the point of view of Quantum Computing, in the vein of the present author’s holistic approach, towards a theory of Infotonics and Quantum Information Dynamics [13-15].

In this article the author extends the somewhat philosophical considerations mentioned above, to the mathematical-physics realm, and tie them to concrete mathematical structures at hand, but not so much used by modern physicists.

A brief list of modern mathematical frameworks is provided, beyond the traditional framework of Newton of Differential Equations in continuum space (or space-time, regardless of the number of dimensions).

From Electronics to Infotonics

The author's "big-picture" presented in this article, is composed of a few new key *ideas* which emerged from the scientific literature, articles and books of mathematics, physics and computer science mainly, but also taking into account other areas, like biology, as well as not yet understood phenomena. Some topics were the author's subject of research in several published articles, concerned for example with mathematical models in Quantum Physics: TQFTs and deformation quantization; Cohomology of Feynman Graphs, on a theme by Kontsevich and Kreimer-Connes; Riemann Hypothesis and Primes – Zeros duality etc. More details can be found in the author's physics essays: In Search of a New Equivalence Principle (Should information be part of the action?), Quantum Relativity (Is Gravity a residual "force"?), "On the arrow of Time" (Time is just a parameter, and there is no "real" arrow of time; conf. Feynman, 2D-time of Cramer, Wheeler-Feynman retarded-advanced solutions etc.).

The mathematical-physics foundations being well covered and understood, let us ponder on the implications and meaning, in a much more concrete way than what David Bohm did in the "implicate order" [16].

QED – A Jewel of Physics

The Feynman Diagrams are still considered a mere tool for expanding a Gaussian integral into a perturbation series. The natural conclusion from Wilson camera pictures and HEP experiments, together with other aspects, is: they actually represent schematically the "quantum circuits" formed in our modern high-energy alchemy!

One who knows about Weil Conjectures might reinterpret as follows: the partition function of QED as a generating function of the amplitudes of probability for the possible actual reaction processes (including the elastic collisions) represented by the Feynman Diagrams, miraculously has a rational closed form as an exponential period (Feynman Path Integral of $\exp(iS(\gamma))$, via Wick Theorem.

In view of the other important experiments (*Double Slit Interference*, with light or electrons [17], *Quantum Optics Which-Way Experiments* [18], etc.; see [13] and references within), the conviction that there is a quantum network formed "guiding" the quantum process, builds gradually. This is, of course, reminiscent of de Broglie pilot waves, yet generalized to a global concept, from its original local role. A good graphical model for this, also because of the mathematical connections, is that of an Etheric Riemann Surface (RS), in what concerns the topology and (conformal) flow ("etheric" should suggest a wave-function-like 3D-density).

One of the main conceptual benefits is that it implies a unified view of "particles and waves", avoiding the historically understandable split/separation between the concepts of fermion, as a carrier of the source of an interaction, and boson as the carrier of the "force field" (and "space", as a necessary background).

Secondly, in combination with the fermion-boson unification idea, the Quantum Channel concept (branches of RS if having a higher genus, e.g. $g=1$ in a double slit experiment), not unlike a "wave guide" in EM / radio electronics, *guides the "particle" as envisioned by de Broglie*; except now it is a concrete, consistent model! The network's cycles also yield charges, as envisioned by Wheeler and Misner in their article "Charges without charges ..." [19] (developed further as *Geometrodynamics* [20]), playing the role of fermions (portrayed by Feynman Diagrams as arrows/links; or quark lines accordingly for bosons and hadrons; except that the topological charge/source is "distributed", a global concept!), while the *pulses of energy*, playing the role of bosons, propagates on these channels, having the properties of the associated de Broglie "pilot wave".

The Role of Charges in Physics

Wheeler and Misner, in their Geometrodynamics [19-20], showed how charges, as sources of the field and hence of the dynamics, can be implemented topologically, as a network of wormholes, in Einstein's General Relativity, with an empty space-time (i.e. no local sources, i.e. pointwise, leading to singularities in the theory, are needed anymore), yet with non-trivial topology instead.

If we accept the RS picture of interactions, in place of the expected big wormholes, e.g. due to black holes, i.e. if the whole network consists only of "wormholes", but at our scale, then cross-sections are closed curves consistent with the idea of "strings", qualitatively speaking.

It is important to note that this idea applies to micro and macro-level quantum processes alike. This is why the author further advocates the Mathematic Models of the type used in Quantum Invariants of Knots, TQFTs, or Quantum Computing [21-23]; and also insists, for instance, that "fat Feynman Graphs" (or ribbon graphs, framed knots etc. as in Turaev's Calculus [21]), are a good model for this interpretation of interactions as RS-like. But the more flexible model than a mere network of coupons and arrows

[21], which would definitely serve the quantum computing logical framework, is of course, a Riemann surface, also because of the tradition, and connections with String Theory; maybe a better model is that of a “fat” Riemann surfaces (“Ethereal” RS), as if they model a Schrodinger wave function distribution (with some density and thickness), with non-trivial topology.

From Axiomatic Systems to Expert Systems: An Intermezzo

At this stage the reader may have the feeling that the author denies the purpose of fundamental theories of starting from primary, structure-less concepts, like point-particles (String Theory already departures, courageously, away from this), in order to derive the observed phenomena, and we start from a complex structure, the Ethereal RS, instead ... The reader is right!

To give a brief justification, we recall the evolution of Axiomatic Systems, from “glory to dust”: Euclid’s masterpiece (Euclidean Geometry), Hilbert’s Program to axiomatize mathematics, work on foundations of mathematics (constructivism, intuitionism etc.), Goedel’s assessment (limitations of Axiomatic Systems); and compare with Physics conceptual frameworks (some “hand-waving” as parts of the “proofs”), and the modern paradigm in Computer Science Applications: *The Expert System* (Based on a Procedural Data Base which *empowers the User*; yet a Labyrinth where going in circles is a threat! ... Quite deadly for computers!)

So, what we need is a 1st top-level of description, which is conceptual, intuitive and graphically representable, as a Model (compare with the Finite Element rendering of a CAD product, to be executed by a CAE process/programmable machine), a Physics Expert System as a 2nd level of “precision” description; 3rd a precise Mathematical Model; 4th a computationally tractable implementation of the Mathematical Model.

This approach to R&D is the norm in institutions designing software. In order to design and build such a “Model of the Operating System of the Universe”, there is the need not just for team work, or a “lab consisting on several teams”; rather of an institute, or of an infrastructure similar to that existing at the large research centers like Fermi Lab, SLAC or CERN etc.

This departs from the traditional goals, like the one of Prof. Michio Kaku, to know what equation governs the Universe, or “The Ultimate Physics Theory”, as an axiomatic system.

The Mother of All Quantum Experiments

This is how Richard Feynman called the double slit experiment; if we understand one such quantum experiment, we understand them all.

The Riemann Surface model (or tube-like, maybe with some density: “fat” RS), would explain easily the experiment: now we don’t “trace” a discrete particle (photon or electron), but we have a network “medium” (reminiscent of the ether, of course, and this is *not* a coincidence!), carrying the energy pulses as quanta, exhibiting a topological charge as a loop integral, distributed in “various places” (localized over string-like cycles; again, maybe with density: tori).

The “wave properties”, including interference, which is in fact due to resonant modes of vibration of the RS-network, like a 3D-cymatics picture (Figure 1), are easy to understand as emerging from this model.

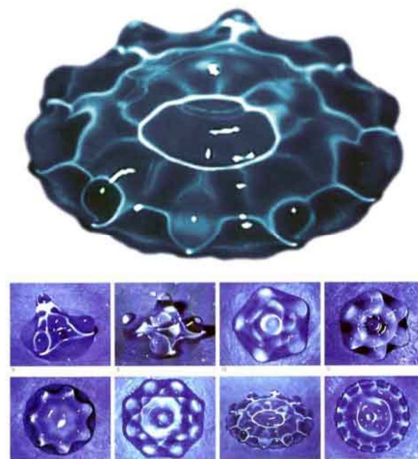


Figure 1. Resonant modes of a water droplet.

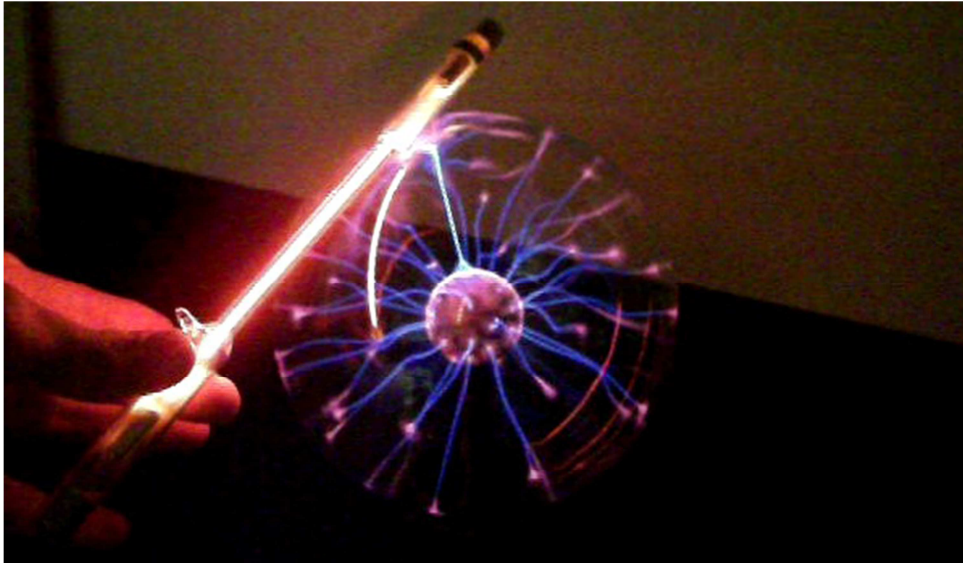


Figure 2. Neon plasma globe.

The localization as a “particle” makes perfect sense in the spiky tubes originating in the source, with one or several, ending in the target, in a slightly variable location, similar to what a plasma/neon globe toy shows (Figure 2), and producing the usual interference pattern.

In the case of “shooting” electrons, one at a time, imagine this RS structure (orbital-like wave function), forming as an “electric arc” between source and target, like a lightning, one at a time, with variations geometry (like when several lightning striking in a row), but with a geometry “quantized” by *resonance constraints*, roughly like in the original Bohr’s model (as type of intuitive conditions), where an electron’s orbit in an atom, had to accommodate an integer number of wavelengths. Here one may use one’s favorite mathematical formalism for computations (Schrodinger, Feynman Path Integral etc.), yet the understanding needs a “high-level language” description and “picture”. What we lack now is a mental model, an image for the quantum phenomena, which otherwise we can calculate quite well in several ways. This “big picture”/intuitive geometric model allows to “see” what is going on, without your mind “going down the drain”, as Feynman used to say.

Now, how close is this from the original idea of String Theory!? Very close; except one may choose to attach internal symmetry to the pulses, like qubits/quantum registers as in a Quantum Computation diagram (Computer Science oriented model), or quarks as in High Energy Physics model, to benefit from a connection with the Standard Model etc.), or just “generic spinors”; but we should avoid embedding the RS into a Calabi-Yau manifold and have extra, external, “manifold dimensions” (for a total of 11 or 21), which do not correspond to actual “mechanical positions”, where “something” may move, as just an old habit to remain in the framework of “motion dynamics”, instead of upgrading towards “Quantum Information Processing”.

Aharonov-Bohm Effect: Bosons and Fermions ... What’s the Difference Anyways

The Aharonov-Bohm experiment really shows how close QM is of Maxwell’s Electromagnetism, and the crucial role of *magnetic vector potential* A , which was later dismissed as “unphysical” by Heaviside’s version of EM, the same person who truncated Hamilton’s quaternions, separating time from space, and the product into scalar and vector product ...

In the RS model, generalized momentum $P=p+e/c A$ should somehow be associated with the flow of energy-momentum (spin, via SU_2 2:1 cover of SO_3) on it (no details for now; which are not “clear”, and not the goal of this article). Yet, it is interesting to note how this would allow a connection with the *Helical Model of the Electron*, which resurfaced quite periodically in physics articles (because of some tempting “coincidences” in the corresponding formulas; compatible with de Broglie, formulation, and with special relativity).

High Energy Physics – Elementary Particle Accelerator Experiments

Looking at the images in Wilson chamber, bubble chambers etc., a growing feeling emerges: Feynman Diagrams are for real; WYS-I-WYG (Bubble diagrams are pictures of quantum circuits!).

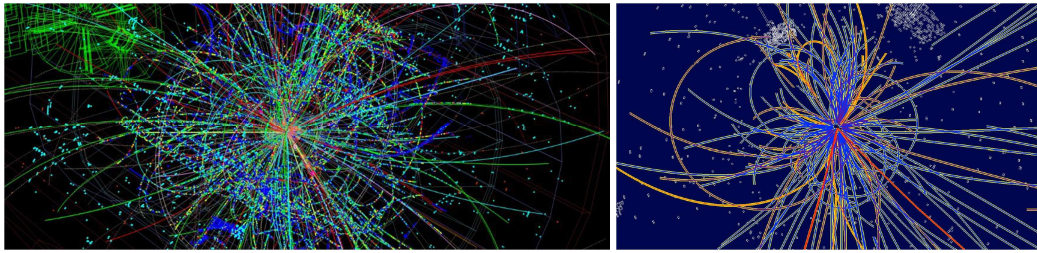


Figure 3. Buble diagrams in particle accelerator scattering experiments.

There is a wide misconception that quantum phenomena happen only at tinny distances; yet entanglement implies macro-separation, contradicting this [24]. There are many other phenomena where coherence happens over large structures (laser etc.). Quantum tunneling can be interpreted as a “proof” that there is “no space” in between, and a “short-cut” through the network can be built, “spontaneously” (given enough time etc.).

Quantum Optics Experiments

Low Energy Quantum Circuits are studied in Quantum Optics for instance [18], in steady-state regime, and are not “blown away” as in particle accelerator high energy experiments! Here we can measure the (relative) quantum state, not just the probability! (i.e. performing *weak measurements* instead of strong measurements which blow-up the ephemeral “ethereal” network forming the quantum circuit; i.e. the so called collapse of the wave function just reflects the collapsing of the quantum circuit, initially prepared by the experiment, between source and target!).

The main “novelty” of Quantum Computing in general, and Quantum Optics in particular, is the discovery (introduction?) of weak measurements, which do NOT collapse the “wave function”, as in a Schrodinger Wave Mechanics interpretation.

Our RS picture of QC and quantum interactions, supports the distinction between strong and weak interactions; the former “burst the ethereal bubble”, collapsing the network, or at least implementing a “cut” of one of the loops, containing the branch containing the measured “spot” (position of interaction node with the exterior). When observing a double slit experiment with “a photon”, in the RS model of interactions, there is “puncture” due to the external, secondary source of light used for observation, which may “destabilize” the RS and bursting one branch, reducing the number of holes (the genus).

Compare with Electronic Circuits

Quantum interactions, modeled by Feynman Diagrams or better RS, are Quantum Communications (duplex mode; corresponds to the establishing of a resonant structure). Modeled as a Quantum Computation, it can be decomposed into universal gates (similar to Boolean representation for classical computing and logic). An analysis of Feynman Graph from this point of view is worth doing. Recently, the twistor model came with an alternative involving the so called positive Grassmanian; this is probably another way to build a resolution of the quantum process (close to what happens in Homological Algebra; deeper even, reflecting the role of motives – Tate, Grothendieck, Hodge etc. – in the understanding of periods, as recently introduced by Kontsevich (1997) and Zagier (2001), explaining in principle why Multiple Zeta Values appear in Feynman Integrals; motives are universal summands of cohomologies).

Then, viewing such quantum networks (represented by Feynman Graphs, RS or QC models) as “Quantum Chips”, allows a parallel with Electronics. We would expect resonant modes, spectra and an analog of R, L, C elements of circuit; or parameters. “R” would correspond to lose of coherence; L and C to electric and magnetic aspects ...

The Human Being as a Quantum Circuit

This brings us to the ultimate question: What is Life!? 42 is not the answer, but Mathematics, Physics and Computer Science, together can by now explain it! The author pondered on a formal similarity between the 5-Elements Chinese Model of acupuncture treatment of diseases, for control of chi flow in meridians and the 3-Element Transistor: there are too many qualitative features not to think that it looks like Cybernetics (some theory of control for complex informational systems), but in different era. The analogy is much more technical then for instance comparing Greek atomism and Modern Atomism, or even with Quantum Mechanics. But this is really another story!

Conclusion

The evolution of physics theories (Appendix A: some main stages of development in Physics), was marked by the “unreasonable effectiveness of mathematics” [6-8].

With Feynman as apogee of conceptual (mindful) physics going to extremes of saying “If mathematics would disappear over night Physics would be set back one week!”, followed the frustration of not being able to “feel” like we understand physics, leading to denials like “Don’t think; just compute!”, and what an MIT professor call “the dark ages”: a *mathematization of physics*, trying to justify more and more decimals (see [2]), forgetting the power of the mind (Greeks deduced without any experiments: matter is indivisible at some level; Zeno: action, space and time can’t be divided infinitely many times – we just force this in our mathematical imagination, using limits etc.).

What is now needed is a holistic approach to modeling observed systems, totally similar with the “problem” of western medicine, except “they” are beginning to understand the main points of eastern medicine, and holistic approach, with its germs in the Tao (with others understand this too, in physics, biology etc., e.g. “Tao of Physics” [10] etc.).

The main unifying step is to acknowledge the past: 1) separation of experimental investigations, with the emergence of “different interactions”, together with 2) the imprint of “locality” and reductionism, i.e. there are “particles”, which should be “elementary” (independent), carrying all the ensuing interactions as “effects” and consequences of what “they” are!

The present is that of: entanglement, i.e. distributed properties, and, here the author propose, “collective properties” which emerge because of the way the parts of the system are structured, *not* because of what these parts are.

Plainly, an elementary particle experiment in a particle accelerator creates a resonant structure, which like a Quantum Chip, has resonant modes, and can be modeled as being made of “elements of circuit”, in a similar way an electronic circuit is composed of “passive elements” RLC and E (sources), and control elements (transistor). The measurements in such a particle accelerator are strong measurements, in the sense of Quantum Computing, amenable to computing probability amplitudes of the various possible Q-Chips being “made” at a given level of “currency” to “buy them from Nature”: energy and momenta; but these strong measurements destroy the circuit! This corresponds to the collapse of the wave function in a Schrodinger formulation of QM, and give the false impression that the measurement is local. Notably in high energy experiments, they lead to the conclusion, a mere interpretation in fact, that the electron is a pointwise particle.

Nature does not play dice, we just don’t now (yet) how to control the process (“how to ask politely”). In Quantum Optics (“Low Energy Physics”), we learned how to control light (and slow it down), because we build a resonant laser array (the Q-chip) in steady-state-mode; in particle accelerators, after a transient stage of building the circuit, we “blow it” apart, in the final transient phase, followed by a strong measurement; we obviously can’t control the final “commodity” (Q-chip) produced.

The solution is to redesign the theory top-down, starting from an *intuitive model*, inspired by the beautiful ideas of String Theory; that of a “foamy” Riemann Surface playing the role of Schrodinger’s wave function (governing its density) AND Heisenberg’s S-matrix, as a Jacobian matrix with entries (integral of 1-forms over paths) as amplitudes (think Markov Systems / Flows on Graphs; matrix: complex numbers labeling a total graph, as amplitudes of transition probabilities).

A recent indication that such a global structure exists, allowing the theory to compute an *exact* solution to the S-matrix scattering amplitude problem, and not as a perturbation, are the recent BCFW recursion method for computing the amplitude [25]. The new methods centered on the so called *amplituhedron* [26-27], as an alternative to Feynman diagrams and Riemann Surfaces, is currently under investigation by the author in a joint work with an expert in the corresponding physics [28].

The main consequences of such a “visual model”, which “forces” upon us a new way of thinking and new concepts, are:

- 1) Unification of fermions and bosons, without “super-symmetry” (which is still un-natural); why separate “field” from “source” in the first place! A holistic approach, like that using “real” networks with geometry and topology of Riemann Surfaces, should be able to derive the properties of fermions and photons, when choosing to separate them, in order to compare with traditional theories.
- 2) “Empty space-time” does not exist. “Space” and “time”, i.e. positions and momenta (Special Relativity “events”) are not physical (observable) natural primary aspects of the theory; yet introducing “coordinates” is a good approach, since Descartes and beyond! For example, in Mathematics is a current pedagogical practice to teach Manifold Theory embedded in \mathbb{R}^n , to avoid the heavier formalism of charts and atlases. In fact “empty space-time” was refuted by Einstein long time ago, with his Theory of General Relativity, following the teachings of Mach. If String Theory did not find yet the “right landscapes”, maybe it is time to understand: there is none!

- 3) The foundational language becomes that of Quantum Computing, which is the logical foundation of Quantum Phenomena and that of “reality”: qubits represent the prototypical “union” of possibilities (Yin-Yang polarity/duality; including spin / spinors), and their “manipulation” / transformation is performed through quantum gates. The transfer of properties through the “unseen” Quantum Network (like the hard to detect magnetic vector potential), appears as dynamics of “particles”. It is quite similar to how images on a TV screen change, and we see “colored areas” travel; we call them “particles”.

Now the correspondence between the traditional approaches (QFT/CFT/ Twistor Models, Standard Model: Gauge Theory +QFT, ST, TQFTs) are quite well developed: from Feynman Diagrams to Riemann Surfaces embedded in Calabi-Yau manifolds, via “fat graphs”; and from TQFTs, foam models and spin networks, with the basic mathematics of knot invariants, Turaev’s Calculus of ribbon graphs with “coefficients” in a braided category, to plain Quantum Computing.

The main “addition”, on top of the mathematics (with its corresponding physics interpretation), is “just” a replacement of the Copenhagen Interpretation; namely, in this unified interpretation there is no enforced dichotomic particle-wave duality, because there is *no separation between sources and field* in the first place! Every physical interaction is discrete (quantum), at the level of *energy-momentum transferred* (quantum information exchanged), through a network formed in the steady-state phase, with resonant properties and proper frequencies. The energy-momentum transfer is via “pulses”, which can be interpreted as bosons, and portions of the network having the properties of sources we call “electric charge” and, yes “magnetic charges” do exist! (see fluxons and quantum Hall effect, Abrikosov’s vortex etc.; they are natural as topological invariants, as opposed to singularities in a local continuum space modeled by \mathbb{R}^n). Then, even the “mother of all quantum experiments” can also be explained, provided that we don’t restrict ourselves by making the “hypothesis” that material “particles” are emitted by the sources, and then trying to trace them one way or another! If we do, “which-way-did-it go” makes our mind go “down the drain”, as Richard Feynman used to say. ■



Lucian Miti Ionescu

The author is a Professor of Mathematics at Illinois State University and a researcher in Mathematical-Physics. With prestigious recommendations from the late Prof. Emeritus Saunders MacLane and encouragements from other current active researchers, he is also one of the advocates of a Computer Science perspective on modern physics, in consonance with the new revolutionary approaches in biology, and in the line of thought of ancient eastern philosophy. Technical results on Quantum Field Theory and Deformation Theory, available on the electronic arXiv, provided him the opportunity of several visiting positions at the prestigious l’Institut des Hautes Etude Scientifique, France.

In his personal life, he enjoys music and sports, in the spare-time made possible with support from his wife and two children. Email: lmiones@ilstu.edu

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