Structural-Parametric Model of Piezoactuator Nano- and Microdisplacement for Nanoscience

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Citation

Abstract
The structural-parametric model, the parametric structural schematic diagram, the decision of the wave equation, the transfer functions of the piezoactuator for the nanoscience are obtained. The effects of geometric and physical parameters of the piezoactuator and the external load on its static and dynamic characteristics are determined. The parametric structural schematic diagram and the transfer functions of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are obtained from the structural-parametric model of the piezoactuator. For calculation of the control systems for nanotechnology with the piezoactuator it’s the parametric structural schematic diagram and the transfer functions are determined. The generalized parametric structural schematic diagram of the piezoactuator is constructed.

1. Introduction
For the nanoscience, the nanobiology, the microelectronics, the nanotechnology is promising for use the actuator based on the piezoeffect. The piezoactuator is the piezomechanical device intended for the actuation of the mechanisms, the systems or the management based on the piezoelectric effect, converts the electrical signals into the mechanical movement or the force [1 – 5]. In the present work is solving the problem of building the structural parametric model of the piezoactuator in contrast the electrical equivalent circuits Cady and Mason for calculation of the piezotransmitter and the piezoreceiver [6 – 9]. Consider building the structural-parametric model of the piezoactuator, representing the system of equations, which, given the electromechanical parameters of the piezoactuator describes the structure and conversion the energy electric field into mechanical energy and the corresponding displacements and forces at its the ends. By Laplace transform of the wave equation and the equation of piezoelectric effect, the boundary conditions on working surfaces of the piezoactuator, the strains along the coordinate axes it is possible to construct the structural-parametric model of the piezoactuator. The transfer functions and the parametric structural schematic diagrams of the piezoactuator are obtained from its structural-parametric model [10 – 14].

Piezoactuator is used in the nanoscience for scanning tunneling microscopes, scanning force microscopes, atomic force microscopes. The use of piezoactuator solves the problems of precise adjustment and compensation of temperature and gravitational deformations in the nanoscience [15 – 19].
2. Structural-Parametric Model and Parametric Structural Schematic Diagram of Piezoactuator

Since the deformation of the electroelastic actuator corresponds to its stressed state, therefore in the piezoactuator there are six stress components $T_1, T_2, T_3, T_4, T_5, T_6$, where the components $T_1 - T_3$ are related to extension-compression stresses and $T_4 - T_6$ to shear stresses.

The matrix state equations [8, 11] connecting the electric and elastic variables for polarized piezoceramics have the following form:

$$ [D] = [d][T] + [e]^T[E], \quad (1) $$

$$ [S] = [s^E][T] + [d]^T[E], \quad (2) $$

where the first equation describes the direct piezoelectric effect, and the second - the inverse piezoelectric effect; $[D]$ is the column matrix of electric induction along the coordinate axes; $[S]$ is the column matrix of mechanical stresses; $[E]$ is the column matrix of electric field strength along the coordinate axes; $[s^E]$ is the elastic compliance matrix for $E = \text{const}$; $[e]$ is the matrix of dielectric constants for $T = \text{const}$; $[d]^T$ is the the transposed matrix of the piezoelectric modules.

In polarized piezoceramics from lead zirconate titanate PZT for a piezoactuator there are five independent components $s_{11}^E, s_{12}^E, s_{13}^E, s_{33}^E, s_{55}^E$ in the elastic compliance matrix, three independent components $d_{33}, d_{31}, d_{35}$ in the transposed matrix of the piezoelectric modules and three independent components $e_{11}^T, e_{22}^T, e_{33}^T$ in the matrix of dielectric constants.

The elastic compliance matrix has the form:

$$ [s^E] = \begin{bmatrix}
    s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\
    s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 \\
    s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 \\
    0 & 0 & 0 & s_{55}^E & 0 & 0 \\
    0 & 0 & 0 & 0 & s_{55}^E & 0 \\
    0 & 0 & 0 & 0 & 0 & 2(s_{11}^E - s_{12}^E)
\end{bmatrix}. \quad (3) $$

We can write the transposed matrix of the piezomodules as

$$ [d]^T = \begin{bmatrix}
    0 & 0 & d_{11} \\
    0 & 0 & d_{31} \\
    0 & d_{33} & 0 \\
    d_{15} & 0 & 0 \\
    0 & 0 & 0 \\
\end{bmatrix}. \quad (4) $$

The matrix of dielectric constants has the form:

$$ [e]^T = \begin{bmatrix}
    e_{11}^T & 0 & 0 \\
    0 & e_{22}^T & 0 \\
    0 & 0 & e_{33}^T
\end{bmatrix} \quad (5) $$

Let us consider the simplest piezoactuator in Figure 1 for longitudinal, transverse and shift deformations.

Let us consider the piezoactuator for the longitudinal piezoelectric effect, where $\delta$ is thickness and the electrodes deposited on its faces perpendicular to axis 3, the area of which is equal to $S_0$. The direction of the polarization axis $P$, i.e., the direction along which polarization was performed, is usually taken as the direction of axis 3. The equation of the inverse longitudinal piezoelectric effect [8, 12] has the form:

$$ S_3 = d_{33}E_3(t) + s_{33}^E T_3(x,t) \quad (6) $$

where $S_3 = \partial^2 \xi(x,t)/\partial t^2$ is the relative displacement of the cross section of the piezoactuator, $d_{33}$ is the piezomodule for the longitudinal piezoeffect, $E_3(t) = U(t)/\delta$ is the electric field strength, $U(t)$ is the voltage between the electrodes of actuator, $\delta$ is the thickness, $s_{33}^E$ is the elastic compliance along axis 3, and $T_3$ is the mechanical stress along axis 3.

The equation of the equilibrium for the forces acting on the piezoactuator (the piezoelectric plate) on Figure 1 can be written as

$$ T_3S_0 = F + M \frac{\partial^2 \xi(x,t)}{\partial t^2} \quad (7) $$

where $F$ is the external force applied to the piezoactuator, $S_0$ is the cross section area and $M$ is the displaced mass.

*Figure 1. Piezoactuator*
For constructing the structural parametric model of the voltage-controlled piezoactuator, let us solve simultaneously the Laplace transform of the wave equation, the equation of the inverse longitudinal piezoeffect, the equation of the forces acting on the faces of the piezoactuator. The calculations of the piezoactuators are performed using the wave equation [8, 11, 12] describing the wave propagation in the long line with damping but without distortions, which can be written in the form:

\[
\frac{1}{c^E} \frac{\partial^2 \xi(x,t)}{\partial t^2} + \frac{2\alpha}{c^E} \frac{\partial \xi(x,t)}{\partial t} + \alpha^2 \xi(x,t) = \frac{\partial^2 \xi(x,t)}{\partial x^2} \tag{8}
\]

where \(\xi(x,t)\) is the displacement of the section, \(x\) is the coordinate, \(t\) is time, \(c^E\) is the sound speed for \(\text{const} E\), \(\alpha\) is the damping coefficient of the wave.

Using Laplace transform, we can reduce the original problem for the partial differential hyperbolic equation of type (8) to the simpler problem for the linear ordinary differential equation [10, 11]. Applying the Laplace transform to the wave equation (5)

\[
\mathcal{L}\{\xi(x,t)\} = \int_0^\infty \xi(x,t)e^{-pt} dt \tag{9}
\]

and setting the zero initial conditions, we obtain the linear ordinary second-order differential equation with the parameter \(p\) in the form

\[
\frac{d^2 \Xi(x,p)}{dx^2} - \left[\frac{1}{(c^E)^2} p^2 + \frac{2\alpha}{c^E} p + \alpha^2\right] \Xi(x,p) = 0 \tag{10}
\]

or

\[
\frac{d^2 \Xi(x,p)}{dx^2} - p^2 \Xi(x,p) = 0
\]

with its solution being the function

\[
\Xi(x,p) = Ce^{-\gamma p} + Be^{\gamma p} \tag{11}
\]

where \(\Xi(x,p)\) is the Laplace transform of the displacement of the section of the piezoactuator, \(\gamma = \frac{p}{c^E} + \alpha\) is the propagation coefficient. We denote

\[
\Xi(0,p) = \Xi_1(p) \quad \text{for} \quad x = 0
\]

\[
\Xi(\delta,p) = \Xi_3(p) \quad \text{for} \quad x = \delta
\]

Then we get the coefficients \(C\) and \(B\)

\[
C = \frac{1}{\left[1 - \frac{\delta_2 e^{\delta\gamma}}{2\text{sh}\left(\delta\gamma\right)}\right]}
\]

\[
\Xi_1(p) = \left[1 - \frac{1}{\left(1/M_1 p^2\right)}\right] \left[-F_1(p) + \left(1/\chi_3^E\right)d_3 E_3(p) - \left[\gamma/\text{sh}\left(\delta\gamma\right)\right]\left[\text{ch}\left(\delta\gamma\right)\Xi_1(p) - \Xi_2(p)\right]\right]
\]

\[
\Xi_2(p) = \left[1 - \frac{1}{\left(1/M_2 p^2\right)}\right] \left[-F_2(p) + \left(1/\chi_3^E\right)d_3 E_3(p) - \left[\gamma/\text{sh}\left(\delta\gamma\right)\right]\left[\text{ch}\left(\delta\gamma\right)\Xi_2(p) - \Xi_1(p)\right]\right]
\]
where \( s_{ij}^F = s_{ij}^E / S_0 \).

From (2), (3), (17) we obtain the system of equations describing the generalized structural-parametric model of the piezoactuator for the nanoscience

\[
\begin{align*}
\Xi_1(p) &= \left[ \frac{1}{(M_1 p^2)} \right] \left[ -F_1(p) + \frac{1}{\sqrt{\gamma}} \chi_{ij}^E \left( d_{m} \Psi_m(p) - \left[ \gamma/\sinh(l \gamma) \right] \left[ \chi(l \gamma) \Xi_1(p) - \Xi_2(p) \right] \right) \right] \\
\Xi_2(p) &= \left[ \frac{1}{(M_2 p^2)} \right] \left[ -F_2(p) + \frac{1}{\sqrt{\gamma}} \chi_{ij}^E \left( d_{m} \Psi_m(p) - \left[ \gamma/\sinh(l \gamma) \right] \left[ \chi(l \gamma) \Xi_2(p) - \Xi_1(p) \right] \right) \right],
\end{align*}
\]  

(18)

where \( d_{m} = \{ d_{33}, d_{31}, d_{15} \} \), \( \Psi_m = \{ E_3, E_5, E_1 \} \), \( s_{ij}^E = \{ s_{13}^E, s_{11}^E, s_{55}^E \} \), \( l = \{ \delta, h, b \} \), \( c_{ij}^E = \{ c_{11}^E, c_{12}^E, c_{13}^E \} \), \( \gamma_{ij}^P = \{ \gamma_{ij}^F, \gamma_{ij}^D \} \).

\( \Xi_{ij}^E = s_{ij}^E / S_0 \), \( i = 1, 2, \ldots, 6, j = 1, 2, \ldots, 6, m = 1, 2, 3, \)

then parameter \( \Psi \) of the control parameter for the piezoactuator: \( E \) for voltage control, \( D \) for current control. On Figure 2 is shown the generalized parametric structural schematic diagram of the piezoactuator corresponding to the set (18) of equations.

### 3. Transfer Functions of Piezoactuator

From generalized structural-parametric model (18) of the piezoactuator after algebraic transformations we obtain the transfer functions in matrix form [11 – 14], where the transfer functions are the ratio of the Laplace transforms of the displacement of the face piezoactuator and the Laplace transform of the corresponding control parameter or force at zero initial conditions.

\[
\begin{align*}
\Xi_1(p) &= W_{11}(p) \Psi_m(p) + W_{12}(p) F_1(p) + W_{13}(p) F_2(p) \\
\Xi_2(p) &= W_{21}(p) \Psi_m(p) + W_{22}(p) F_1(p) + W_{23}(p) F_2(p),
\end{align*}
\]  

(19)

where the generalized transfer functions of the piezoactuator are the following equations

\[
\begin{align*}
W_{11}(p) &= \Xi_1(p) / \Psi_m(p) = d_{m} \left[ M_2 \chi_{ij}^E p^2 + \gamma \sinh(l \gamma) / 2 \right] / A_{ij} \\
W_{12}(p) &= \Xi_2(p) / F_1(p) = -\chi_{ij}^E \left[ M_2 \chi_{ij}^E p^2 + \gamma / \sinh(l \gamma) \right] / A_{ij} \\
W_{13}(p) &= \Xi_1(p) / F_2(p) = W_{22}(p), \\
W_{21}(p) &= \Xi_2(p) / F_1(p) = \left[ \chi_{ij}^E \gamma / \sinh(l \gamma) \right] / A_{ij} \\
W_{23}(p) &= \Xi_2(p) / F_2(p) = -\chi_{ij}^E \left[ M_4 \chi_{ij}^E p^2 + \gamma / \sinh(l \gamma) \right] / A_{ij}.
\end{align*}
\]

We obtain from equations (18) the generalized matrix equation for the piezoactuator

\[
\begin{pmatrix}
\Xi_1(p) \\
\Xi_2(p)
\end{pmatrix} =
\begin{pmatrix}
W_{11}(p) & W_{12}(p) & W_{13}(p) \\
W_{21}(p) & W_{22}(p) & W_{23}(p)
\end{pmatrix}
\begin{pmatrix}
\Psi_m(p) \\
F_1(p) \\
F_2(p)
\end{pmatrix}
\]  

(20)

Let us find the displacement of the faces piezoactuator in the stationary regime for inertial load at \( \Psi_m(t) = \Psi_{m0}. \mathcal{L} (t) \), \( F_1(t) = F_2(t) = 0 \).

Then we get the static displacement of the faces piezoactuator

\[
\bar{\xi}_1(\infty) = \lim_{t \to \infty} \bar{\xi}_1(t) = \lim_{p \to 0} p W_{11}(p) \Psi_{m0} / (p) = \frac{d_{m} \Psi_{m0} (M_2 + m/2) / (M_1 + M_2 + m)}{E_{11}(p) + 2 \alpha M_2 / c_{11}^E + \alpha^2}
\]

(21)

\[
\bar{\xi}_2(\infty) = \lim_{t \to \infty} \bar{\xi}_2(t) = \lim_{p \to 0} p W_{21}(p) \Psi_{m0} / (p) = \frac{d_{m} \Psi_{m0} (M_1 + m/2) / (M_1 + M_2 + m)}{E_{11}(p) + 2 \alpha M_2 / c_{11}^E + \alpha^2}
\]

(22)

\[
\bar{\xi}_1(\infty) + \bar{\xi}_2(\infty) = \lim_{t \to \infty} (\bar{\xi}_1(t) + \bar{\xi}_2(t)) = d_{m} \Psi_{m0} / (M_1 + M_2 + m)
\]

(23)

where \( m \) is the mass of the piezoactuator, \( M_1, M_2 \) are the load masses.

Let us consider a numerical example of the calculation of static characteristics of the piezoactuator from piezoceramics PZT under the longitudinal piezoelectric effect at \( m \ll M_1 \) and \( m \ll M_2 \). For \( d_{33} = 4 \cdot 10^{-10} \text{ m/V}, U = 200 \text{ V}, M_1 = 2 \text{ kg} \) and \( M_2 = 8 \text{ kg} \) we obtain the static displacement of the faces of the piezoactuator \( \bar{\xi}_1(\infty) = 64 \text{ nm}, \bar{\xi}_2(\infty) = 16 \text{ nm}, \bar{\xi}_1(\infty) + \bar{\xi}_2(\infty) = 80 \text{ nm} \).

The static displacement of the faces of the piezoactuator for the transverse piezoelectric effect and inertial load at \( U(t) = U_0 \cdot \mathcal{L} (t) \), \( E_3(t) = E_{30} \cdot \mathcal{L} (t) = (U_0 / \delta) \cdot \mathcal{L} (t) \) and \( F_1(t) = F_2(t) = 0 \) can be written in the following form...
\[ \xi_i(\infty) = \lim_{t \to \infty} \xi_i(t) = \lim_{p \to 0^+} \frac{pW_{11}(p)}{U_0/\delta}/p = \]

\[ d_{31}(h/\delta)U_0(M_2 + m/2)/\left(M_1 + M_2 + m\right) \]

\[ \xi_i(\infty) = \lim_{t \to \infty} \xi_i(t) = \lim_{p \to 0^+} \frac{pW_{21}(p)}{U_0/\delta}/p = \]

\[ d_{31}(h/\delta)U_0(M_1 + m/2)/\left(M_1 + M_2 + m\right) \]

\[ \xi_i(\infty) + \xi_1(\infty) = \lim_{t \to \infty}(\xi_1(t) + \xi_2(t)) = d_{31}(h/\delta)U_0 \]

From (24), (25) we obtain the static displacement of the faces of the piezoactuator for the transverse piezoeffect at \( m \ll M_1 \), \( m \ll M_2 \) in the form

\[ \xi_1(\infty) = \lim_{t \to \infty} \xi_1(t) = \lim_{p \to 0^+} \frac{pW_{11}(p)}{U_0/\delta}/p = \]

\[ d_{31}(h/\delta)U_0 M_2/(M_1 + M_2) \]

\[ \xi_2(\infty) = \lim_{t \to \infty} \xi_2(t) = \lim_{p \to 0^+} \frac{pW_{21}(p)}{U_0/\delta}/p = \]

\[ d_{31}(h/\delta)U_0 M_1/(M_1 + M_2) \]

Let us consider a numerical example of the calculation of static characteristics of the piezoactuator from piezoceramics PZT under the transverse piezoelectric effect at \( m \ll M_1 \) and \( m \ll M_2 \). For \( d_{31} = 2 \times 10^{-10} \) m/V, \( h = 4 \times 10^{-2} \) m, \( \delta = 2 \times 10^{-3} \) m, \( U = 100 \) V, \( M_1 = 2 \) kg and \( M_2 = 8 \) kg we obtain the static displacement of the faces of the piezoactuator \( \xi_1(\infty) = 320 \) nm, \( \xi_2(\infty) = 80 \) nm, \( \xi_1(\infty) + \xi_2(\infty) = 400 \) nm.

From (19) we obtain the transfer functions of the voltage-controlled piezoactuator for longitudinal piezoeffect with a fixed end and elastic inertial load so that \( M_1 \to \infty \) and \( m \ll M_2 \) in the following form

\[ W_2(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{33} h/\delta}{\left(1 + C_{\infty}/C_{33}^E\right)(T_t^2 p^2 + 2T_t\xi_3 p + 1)} \]

where the time constant \( T_t \) and the damping coefficient of the piezoactuator \( \xi_3 \) are determined by the following formulas

\[ T_t = \sqrt{M_2/(C_{\infty} + C_{33}^E)} \]

\[ \xi_3 = \alpha h^2 C_{33}^E/\left(3C_{\infty} M(C_{\infty} + C_{33}^E)\right) \]

Using the approximation of the hyperbolic cotangent by two terms of the power series in transfer functions (19) we obtain the transfer functions of the voltage-controlled piezoactuator for transverse piezoeffect with a fixed end and elastic inertial load so that \( M_1 \to \infty \) and \( m \ll M_2 \) in the following form

\[ W_2(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{33} h/\delta}{\left(1 + C_{\infty}/C_{33}^E\right)(T_t^2 p^2 + 2T_t\xi_3 p + 1)} \]

\[ T_t = \sqrt{M_2/(C_{\infty} + C_{33}^E)} \]

\[ \xi_3 = \alpha h^2 C_{33}^E/\left(3C_{\infty} M(C_{\infty} + C_{33}^E)\right) \]

\[ \xi_3 = \alpha h^2 C_{33}^E/\left(3C_{\infty} M(C_{\infty} + C_{33}^E)\right) \]

where \( \xi_3 \) is the steady state value of displacement of the piezoactuator, \( U_m \) is the amplitude of the voltage, \( \beta_t \) is the coefficient of the piezoactuator, \( \phi \) is the phase.

For the voltage-controlled piezoactuator for transverse piezoeffect from PZT with a fixed end and elastic inertial load so that \( M_1 \to \infty \), \( m \ll M_2 \), \( U_m = 100 \) V at \( d_{31} = 2.5 \times 10^{-10} \) m/V, \( h/\delta = 20 \), \( M_2 = 4 \) kg, \( C_{33}^E = 2 \times 10^7 \) N/m, \( C_{\infty} = 0.5 \times 10^{-7} \) H/m are obtained values \( \xi_3 = 400 \) nm, \( T_t = 0.4 \times 10^{-3} \) c. The calculated values dynamic characteristics for the voltage-controlled piezoactuator are in agreement with values [8] and experimental values to an accuracy of 5%.

### 4. Discussions

The structural-parametric model and the generalized parametric structural schematic diagram of piezoactuator for the nanoscience are obtained taking into account equation of generalized piezoeffects and Laplace transform of the wave equation.

The results of constructing the generalized structural-parametric model and the generalized parametric structural schematic diagram of piezoactuator for the longitudinal, transverse and shift deformations are shown in Figure 2. The parametric structural schematic diagrams of the piezoactuator for longitudinal, transverse, shift piezoelectric effects converts to the generalized parametric structural schematic diagram of the piezoactuator for the nanoscience Figure 2 with the replacement of the following parameters of the piezoactuator:

\[ \Psi_{\text{act}} = E_3, E_1, V_{\text{act}} = d_{33}, d_{31}, d_{45}, s_{\text{act}} = s_{33}, s_{11}, s_{55} \]
\[ l = \delta, h, b. \]

The generalized structural-parametric model and the generalized parametric structural schematic diagram of the piezoactuator after algebraic transformations provides the transfer functions of the piezoactuator for the nanoscience. The piezoelectric actuator with the transverse piezoelectric effect compared to the piezoactuator for the longitudinal piezoelectric effect provides the greater range of static displacement and the less working force.

It is possible to construct the generalized structural-parametric model, the generalized parametric structural schematic diagram and the transfer functions in matrix form of the piezoactuator for the nanoscience using the Laplace transform of the wave equation for the piezoactuator and taking into account the features of its deformations along the coordinate axes.

5. Results

The generalized structural-parametric model, the generalized parametric structural schematic diagram, the solution of wave equation, the generalized transfer functions of the piezoactuator are obtained.

The generalized structural-parametric model of the piezoactuator are constructed by using Laplace transform of the wave equation and the equation of piezoelectric effect, the boundary conditions on working surfaces of the piezoactuator, the strains along the coordinate axes.

The transfer functions in the matrix form for the piezoactuator are determined, taking into account the features for its deformations along the coordinate axes.

The investigations of the static and dynamic characteristics of a piezoactuator are performed in the calculation of control systems for the nanoscience. The effects of geometric and physical parameters of the piezoactuator and the external load on its static and dynamic characteristics are determined.

The transfer functions and the parametric structural schematic diagram of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are obtained from the structural-parametric model of the piezoactuator.

6. Conclusions

The systems of equations are determined for the structural-parametric models of the piezoactuator. The structural-parametric model, the decision of wave equation, the parametric structural schematic diagram, the transfer functions of the piezoactuator are obtained using the Laplace transform. The parametric structural schematic diagram and the transfer functions of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are determined from the structural-parametric model of the piezoactuator.

Using the Laplace transform of the wave equation and taking into account the features of the deformations along the coordinate axes, the generalized structural-parametric model and the parametric structural schematic diagram of the piezoactuator are constructed for the control systems in the nanoscience.

The transfer functions in the matrix form are described for the ratio of the Laplace transforms of the deformation to the control parameter during operation of the piezoactuator as the part of the mechatronic control systems.

From the Laplace transform of the wave equation and the features of the deformations along the coordinate axes we obtain the generalized structural-parametric model and the parametric structural schematic diagram and the dynamic and static characteristics of the piezoactuator for the nanoscience and the nanotechnology.

References


