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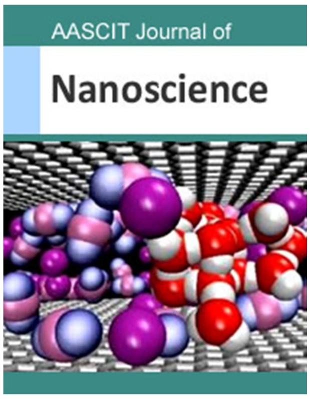

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Some Properties on Nano Topology Induced by Graphs

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Abstract
A nano topological space became a new type of modern topology in terms of rough sets. The paper aims to analyze some real life problems using nano topology. We point out that some examples and results as proposed by Lellis Thivagar et al. (2016) are not true. The corrections will improve further extensions of the results in [1]. Some new forms of topological structures on a simple directed graph and give more generalized nano topology induced by graphs will be established.

1. Introduction

Graph theory has recently emerged as a subject in its own right as well as being an important mathematical tool an such diverse subjects as operational research, chemistry, sociology and genetics.

A graph $G=(V,E)$ [2], is an ordered pair of disjoint sets (V,E) where V is a nonempty set and E is a subset of unordered pairs of V. The vertices and edges of a graph G are the elements $V=V(G)$ and $E=E(G)$, respectively. We say that a graph G is finite (resp. infinite) if the set $V(G)$ is finite (resp. infinite). The degree of a vertex $u\in V(G)$ is the number of edges containing u. If there is no edge in a graph G contains a vertex u, then u is called an isolated point, and so the degree of u is zero. An edge which has the same vertex to ends is called a loop, and the edge with distinct end is called a link. A graph is simple if it has no loop and no two of its links join the same pair of vertices. A graph which has no edge is called a null graph. A simple graph is called complete graph if any two distinct vertices are joined by an edge. If the vertex set of a graph G can be split into two disjoint sets A and B so that each edge of a graph G joins a vertex of A and a vertex of B, then a graph G is a bipartite graph. A complete bipartite graph is a bipartite graph in which each vertex in A is joined to each vertex in B by just one edge. Given a graph G, a walk in G is a finite sequence of edges of the form $v_0v_1, v_1v_2, \cdots, v_{n-1}v_n$. We call v_0 the initial vertex and v_n the final vertex of the walk. A walk in which all the edges are distinct is a trial. If, in addition the vertices $v_0, v_1, v_2, \cdots, v_n$ are distinct (except, possibly, $v_0=v_n$), then the trial is a path. A path or trial is closed if $v_0=v_n$) and a closed path containing at least one edge is a cycle. We can combine two graphs to make a larger graph. If the two graphs are $G_1=(V(G_1),E(G_1))$ and $G_2=(V(G_2),E(G_2))$, where $V(G_1)$ and $V(G_2)$ are disjoint, then their union $G_1\cup G_2$ is the graph with the vertex set $V(G_1)\cup V(G_2)$ and edge family $E(G_1)\cup E(G_2)$. A graph that is in one piece, so that any two vertices are connected by a path is a connected graph, and disconnected otherwise. Clearly any disconnected graph G can be expressed as the union of connected graphs, each of which is a component of G [2-4]. In many applications, it is necessary to associate with each edge of a graph an orientation or direction. In some situations, the orientation of the edges is a "true" orientation in the sense that the system represented by the graph exhibits some unilateral property. For example, the direction of the non-way streets of a city and the orientations representing the unilateral property of a communication network are true orientation of the physical system. In other situations, the orientations used a "pseudo"-orientation used in lieu of an elaborate reference system. For example, in electrical network theory the edges of a graph are assigned arbitrary orientations to represent the references of the branch currents and voltages. The study of directed graphs arises from making the roads into one way streets. For more details of a graph theory one can show [2-5].

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Lellis Thivagar and Richard proposed the theory of a nano topology [6, 7]. Also, Lellis Thivagar et al. [1] defined and studied nano topology induced by a direct simple graph.

The present work aims to correct some examples in [1]. Also some new forms of topological structures on a simple directed graph and give more generalized nano topology induced by graphs will be establish. Finally, the approximations of some structues by a relation which may have possible applications in quantum physics and superstring theory will be studied.

2. Preliminaries

In this section, we give some fundamental notions related to nano topology and graph theory.

Definition 2.1 [2, 4] If $G(V,E)$ is a directed graph and $u,v\in V(G)$, then

(i) u is invertex of v if $\overrightarrow{uv}\in E(G)$.

(ii) u is outvertex of v if $\overrightarrow{vu}\in E(G)$.

(iii) The indegree of a vertex v is the number of vertices u such that $\overrightarrow{uv}\in E(G)$.

(iv) The outdegree of a vertex v is the number of vertices u such that $\overrightarrow{uv}\in E(G)$.

Definition 2.2 [8] Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X\subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$, that is $L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$ where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $H_R(X)$, that is $U_R(X)=\bigcup_{x\in U}\{R(x):R(x)\cap X\neq\varphi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$, that is $B_R(X)=U_R(X)-L_R(X)$.

According to $Pawlak$'s definition X is called a rough set if $U_R(X)\neq L_R(X)$.

Definition 2.2 [6, 7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X)=\{U,\varphi,L_R(X),U_R(X),B_R(X)\}$, where $X\subseteq U$ and $\tau_R(X)$ satisfies the following axioms:

(i) $U\ and\ \varphi\in\tau_R(X)$;

(ii) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ in $\tau_R(X)$ That is $\tau_R(X)$ forms a topology on U. $(U,\tau_R(X))$ is called a nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

Definition 2.3 [1] Let $G(V,E)$ be a graph and $v\in V(G)$. The neighbourhood of v is $N(v)=\{v\}\bigcup\{u\in V(G):\overrightarrow{uv}\in E(G)\}$.

Definition 2.4 [1] Let $G(V,E)$ be a graph and H be a subgraph of G. Then

(i) The lower approximation $L:P(V(G))\to P(V(G))$ is $L_N(V(H))=\bigcup_{v\in V(G)}\{v:N(v)\subseteq V(H)\}$;

(ii) The upper approximation $U:P(V(G))\to P(V(G))$ is $U_N(V(H))=\{N(v):v\in V(H)\}$;

(iii) The boundary region is $B_N(V(H))=U_N(V(H))-L_N(V(H))$.

Definition 2.5 [1] Let G be a graph, $N(v)$ be a neighbourhood of v in V and H be a subgraph of G. $\tau_N(V(H)) = \{V(G), \varphi, L_N(V(H)), U_N(V(H)), B_N(V(H))\}$ forms a topology on $V(G)$ called the nano topology on $V(G)$ with respect to $V(H)$. $(V(G), \tau_N(V(H)))$ is a nano topological space induced by a graph G.

Proposition 2.6 ([7], [9]) If (U, R) is an approximation space and $X, Y \subseteq U$, then we have the following properties of *Pawlak's* rough sets:

(i) $L_R(X) \subseteq X \subseteq U_R(X)$ (Contraction and Extension).
(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ (Normality) and $L_R(U) = U_R(U) = U$ (Co-normality).
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$ (Addition).
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$ (Multiplication).
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$ (Monotone)
(viii) $U_R(X^c) = [U_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$ where X^c denotes the complement of X in U (Duality)
(ix) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$ (Idempotency).
(x) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$ (Idempotency).

3. Correction of Some Lellis Thivagar's Results

In this section, we correct some errors in [1] in the following observations.

Observation 3.1 Lellis Thivagar defined in Definition 3.2 in [7] the notion of upper approximation operation as follows: $U : P(V(G)) \to P(V(G))$ is $U_N(V(H)) = \{N(v) : v \in V(H)\}$. It is not suitable with the results in [1]. So, we correct it by the following $U_N(V(H)) = \bigcup_{v \in V(H)} \{v : N(v) \cap V(H) \neq \varphi\}$.

According to our definition in Observation 3.1, we write Definition 3.2 in [7] as follows:

Observation 3.2 Consider Figure 1,

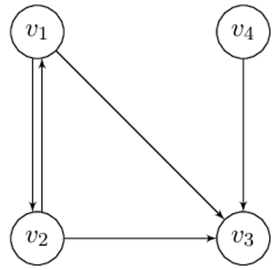

Figure 1. *Graph for Observation 3.2.*

the set of vertices of a direct simple graph G is $V(G) = \{v_1, v_2, v_3, v_4\}$ and $N(v_1) = N(v_2) = \{v_1, v_2, v_3\}$, $N(v_3) = \{v_3\}$ and $N(v_4) = \{v_3, v_4\}$. Let H be a subgraph of G, where $V(H) = \{v_1, v_2\}$. Then $L_N(V(H)) = \{v_3\}$, $U_N(V(H)) = V(G)$ and $B_N(V(H)) = \{v_1, v_2, v_4\}$. Therefore the nano topological space induced by a graph G is $\tau_N(V(H)) = \{V(G), \varphi, \{v_3\}, \{v_1, v_2, v_4\}\}$.

According to our definition in Observation 3.1, we modify Remark 3.8 in [7] as follows:

Observation 3.3 Consider Figure 2,

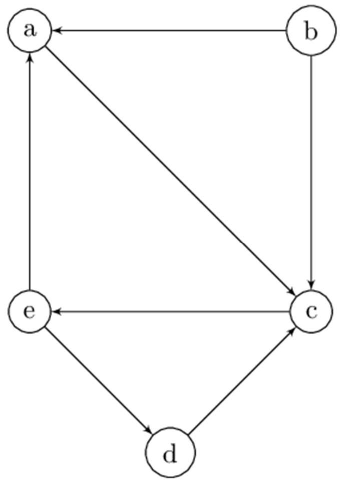

Figure 2. *Graph for Observation 3.3.*

the set of vertices of a direct simple graph G is $V(G) = \{a, b, c, d, e\}$ and $N(a) = \{a, c\}$, $N(b) = \{a, b, c\}$, $N(c) = \{c, e\}$, $N(d) = \{c, d\}$ and $N(e) = \{a, d, e\}$. Table 1, shows the nano topology of any subgraph H of a graph G.

Table 1. *Nano topology for possible subgraphs of* $G(V, E)$.

$V(G)$	$L_N(V(H))$	$U_N(V(H))$	$B_N(V(H))$	$\tau_N(V(H))$
$\{a\}$	φ	$\{a,c\}$	$\{a,c\}$	$\{V(G),\varphi,\{a,c\}\}$
$\{b\}$	φ	$\{a,b,c\}$	$\{a,b,c\}$	$\{V(G),\varphi,\{a,b,c\}\}$
$\{c\}$	φ	$\{c,e\}$	$\{c,e\}$	$\{V(G),\varphi,\{c,e\}\}$
$\{d\}$	φ	$\{c,d\}$	$\{c,d\}$	$\{V(G),\varphi,\{c,d\}\}$
$\{e\}$	φ	$\{a,d,e\}$	$\{a,d,e\}$	$\{V(G),\varphi,\{a,d,e\}\}$
$\{a,b\}$	φ	$\{a,b,c\}$	$\{a,b,c\}$	$\{V(G),\varphi,\{a,b,c\}\}$
$\{a,c\}$	$\{a\}$	$\{a,c,e\}$	$\{c,e\}$	$\{V(G),\varphi,\{a\},\{a,c,e\},\{c,e\}\}$
$\{a,d\}$	φ	$\{a,c,d\}$	$\{a,c,d\}$	$\{V(G),\varphi,\{a,c,d\}\}$
$\{a,e\}$	φ	$\{a,c,d,e\}$	$\{a,c,d,e\}$	$\{V(G),\varphi,\{a,c,d,e\}\}$

$V(G)$	$L_N(V(H))$	$U_N(V(H))$	$B_N(V(H))$	$\tau_N(V(H))$
$\{b,c\}$	φ	$\{a,b,c,e\}$	$\{a,b,c,e\}$	$\{V(G),\varphi,\{a,b,c,e\}\}$
$\{b,d\}$	φ	$\{a,b,c,d\}$	$\{a,b,c,d\}$	$\{V(G),\varphi,\{a,b,c,d\}\}$
$\{b,e\}$	φ	$V(G)$	$V(G)$	$\{V(G),\varphi\}$
$\{c,d\}$	$\{d\}$	$\{c,d,e\}$	$\{c,e\}$	$\{V(G),\varphi,\{d\},\{c,d,e\},\{c,e\}\}$
$\{c,e\}$	$\{c\}$	$\{a,c,d,e\}$	$\{a,d,e\}$	$\{V(G),\varphi,\{c\},\{a,c,d,e\},\{a,d,e\}\}$
$\{d,e\}$	φ	$\{a,c,d,e\}$	$\{a,c,d,e\}$	$\{V(G),\varphi,\{a,c,d,e\}\}$
$\{a,b,c\}$	$\{a,b\}$	$\{a,b,c,e\}$	$\{c,e\}$	$\{V(G),\varphi,\{a,b\},\{a,b,c,e\},\{c,e\}\}$
$\{a,b,d\}$	φ	$\{a,b,c,d\}$	$\{a,b,c,d\}$	$\{V(G),\varphi,\{a,b,c,d\}\}$
$\{a,b,e\}$	φ	$V(G)$	$V(G)$	$\{V(G),\varphi\}$
$\{a,c,d\}$	$\{a,d\}$	$\{a,c,d,e\}$	$\{c,e\}$	$\{V(G),\varphi,\{a,d\},\{a,c,d,e\},\{c,e\}\}$
$\{a,c,e\}$	$\{a,c\}$	$\{a,c,d,e\}$	$\{d,e\}$	$\{V(G),\varphi,\{a,c\},\{a,c,d,e\},\{d,e\}\}$
$\{a,d,e\}$	$\{e\}$	$\{a,c,d,e\}$	$\{a,c,d\}$	$\{V(G),\varphi,\{e\},\{a,c,d,e\},\{a,c,d\}\}$
$\{b,c,d\}$	$\{d\}$	$V(G)$	$\{a,b,c,e\}$	$\{V(G),\varphi,\{d\},\{a,b,c,e\}\}$
$\{b,c,e\}$	$\{c\}$	$V(G)$	$\{a,b,d,e\}$	$\{V(G),\varphi,\{c\},\{a,b,d,e\}\}$
$\{b,d,e\}$	φ	$V(G)$	$V(G)$	$\{V(G),\varphi\}$
$\{c,d,e\}$	$\{c,d\}$	$\{a,c,d,e\}$	$\{a,e\}$	$\{V(G),\varphi,\{c,d\},\{a,c,d,e\},\{a,e\}\}$
$\{a,b,c,d\}$	$\{a,b,d\}$	$V(G)$	$\{c,e\}$	$\{V(G),\varphi,\{a,b,d\},\{c,e\}\}$
$\{a,b,c,e\}$	$\{a,b,c\}$	$V(G)$	$\{d,e\}$	$\{V(G),\varphi,\{a,b,c\},\{d,e\}\}$
$\{a,b,d,e\}$	$\{e\}$	$V(G)$	$\{a,b,c,e\}$	$\{V(G),\varphi,\{e\},\{a,b,c,e\}\}$
$\{a,c,d,e\}$	$\{a,c,d,e\}$	$\{a,c,d,e\}$	φ	$\{V(G),\varphi,\{a,c,d,e\}\}$
$\{b,c,d,e\}$	$\{c,d\}$	$V(G)$	$\{a,b,e\}$	$\{V(G),\varphi,\{c,d\},\{a,b,e\}\}$
$V(G)$	$V(G)$	$V(G)$	φ	$\{V(G),\varphi\}$
φ	φ	φ	φ	$\{V(G),\varphi\}$

4. More Properties on Nano Topological Graphs

In this section, we deduce some new properties nano topological graphs.

Observation 4.1 Let $G(V,E)$ be a graph, S_1 and S_2 are two subgraphs of G. Then from Example 3.3, we state some properties as follows:

a $L_N[V(S_1)\cup V(S_2)] \not\subset L_N(V(S_1)\cup L_N(V(S_2))$. Because of in Observation 3.3, $V(S_1)\cup V(S_2)=\{a,b,c\}$, $L_N(\{a,b,c\})=\{a,b\}$. But $L_N(\{a,b\})=\varphi$ and $L_N(\{c\})=\varphi$. Then $L_N[(\{a,b\})\cup(\{c\})]=\varphi$.

b $U_N[V(S_1)]\cap U_N[V(S_2)]\not\subset U_N[V(S_1)\cap V(S_2)]$. Take $V(S_1)=\{c,e\}$ and $V(S_2)=\{d,e\}$. $U_N[V(S_1)\cap V(S_2)]=U_N(\{e\})=\{a,d,e\}$, and $U_N[V(S_1)]\cap U_N[V(S_2)]=\{a,c,d,e\}\cap\{a,c,d,e\}=\{a,c,d,e\}$.

c $L_N[V(S)]^c \neq [U_N(V(S))]^c$. Because of we take $V(S)=\{a\}$, then $[V(S)]^c=\{b,c,d,e\}$, and $L_N(\{b,c,d,e\})=\{c,d\}$. Also, $U_N(V(S))=\{a,c\}$ and $\{a,c\}^c=\{b,d,e\}$.

d $U_N[V(S)]^c \neq [L_N(V(S))]^c$. Because of we take $V(S)=\{a,b\}$, then $[V(S)]^c=\{c,d,e\}$, and $U_N(\{c,d,e\})=\{a,c,d,e\}$. Also, $L_N(V(S))=\varphi$ and $[\varphi]^c=\varphi$.

e $U_N[U_N(V(S))]\not\subset U_N(V(S))$. Take $V(S)=\{a\}$, then $U_N(V(S))=\{a,c\}$ and $U_N[U_N(V(S))]=\{a,c,e\}$.

f $U_N(V(S))\not\subset L_N[U_N(V(S))]$. Take $V(S)=\{a,c\}$, then $U_N(V(S))=\{a,c,e\}$ and $L_N[U_N(V(S))]=\{a,c\}$.

g $L_N[U_N(V(S))]\not\subset V(S)$. Take $V(S)=\{b\}$, then $U_N(V(S))=\{a,b,c\}$ and $L_N[U_N(V(S))]=\{a,b\}$.

h $V(S)\not\subset U_N[L_N(V(S))]$. Take $V(S)=\{a\}$, then $L_N(V(S))=\varphi$ and so $U_N[L_N(V(S))]=\varphi$.

i $U_N[L_N(V(S))]\not\subset L_N(V(S))$. Take $V(S)=\{a,c\}$, then $L_N(V(S))=\{a\}$ and so $U_N[L_N(V(S))]=\{a,c\}$.

j $L_N(V(S))\not\subset L_N[L_N(V(S))]$. Take $V(S)=\{a,c\}$, then $L_N(V(S))=\{a\}$ and so $L_N[L_N(V(S))]=\varphi$.

Theorem 3.9 Let $G(V,E)$ be a graph, S is a subgraphs of G. Then the following are hold:

(i) $U_N[L_N(V(S))]\subseteq V(S)\subseteq L_N[U_N(V(S))]$.

(ii) $L_N[U_N[L_N(V(S))]]=L_N(V(S))]$.

(iii) $U_N[L_N[U_N(V(S))]]=U_N(V(S))]$.

Proof. (i) Let $x\in U_N[L_N(V(S))]$, then there exists $v\in L_N(V(S))$ such that $\overrightarrow{vx}\in E(G)$. Therefore $x\in N(v)\subseteq V(S)$. Now, since $x\in V(S)$, then $N(v)\subseteq L_N(V(S))$ implies $x\in L_N[U_N(V(S))]$. (ii) Let $x\in L_N(V(S))$, then $N(x)\in U_N[L_N(V(S))]$. So $x\in L_N[U_N[L_N(V(S))]]$. We know that $U_N[L_N(V(S))]\subseteq V(S)$, implies $L_N[U_N[L_N(V(S))]]=L_N(V(S))]$.(iii) We know that $U_N[L_N(V(S))]\subseteq V(S)$, implies $U_N[L_N[U_N(V(S))]]=U_N(V(S))]$. Also, $V(S)\subseteq L_N[U_N(V(S))]$ implies $U_N[L_N[U_N(V(S))]]=U_N(V(S))]$.

5. Application

A tree [1] is a simple connected graph that contain no cycles. A fractal structure consists of pieces which are similar to each other and similar to the whole structure. Each of which is a compact metric space, say A, consists of pieces A_i which are similar to each other. In other words, $A = A_1 \cup A_2 \cup \cdots \cup A_m$ assigned with a homeomorphism function $f_i : A \to A_i = f_i(A)$. This setting leads to smaller and smaller parts, so the same maps can be applied for such smaller pieces $f_j : A_i \to f_j(A_i) = f_j f_i(A_i) = A_{ji}$, where $A_i \subseteq A$ and $A_{ji} \subseteq A_j$.

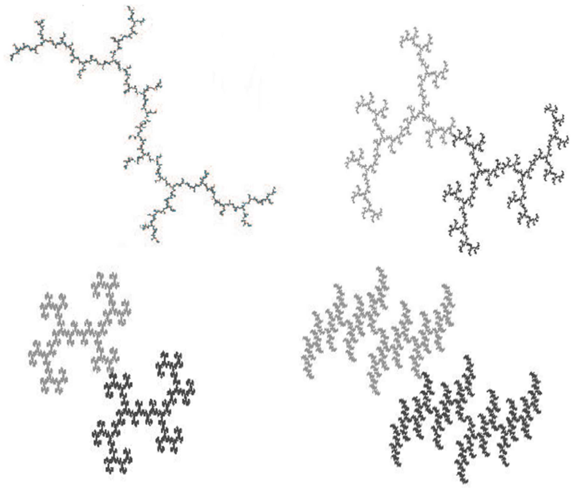

Figure 3. *Fractal structres with one connected point.*

There are some fractal structures with one connected point. In the following, we approximate the fractal structure as a tree and give the possible nano topological spaces for the tree.

6. Conclusion

The field of mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Therefore, the theory of nano topology is one of the most important subject in topology. On the other hand, topology plays a signi_cant rule in quantum physics, hight energy physics and superstring theory [10]. Thus, we study the approximations of some structues by a relation which may have possible applications in quantum physics and superstring theory.

References

[1] Lellis Thivagar, M.; Paul Manuel and Sutha Devi, V.: A detection for patent infringement suit via nanotopology induced by graph, Cogent Mathematics, 3, (2016), 1-10.

[2] Wilson, R. J.: Introduction to graph theory, Fourth Edition, Longmon Maleysia (1996).

[3] Bondy, J. A. and Murty, U. S. R.: Graph theory with applications, Elsevier Science Publishing Co. Inc. (1975).

[4] Diestel, R.: Graph theory II, Heidelberg, Springer-Verlag IV, (2010).

[5] Boninowski, Z.; Bryniarski, E. and Wybraniec, U.: Extensions and intensions in the rough set theory, Information Sciences, 107, (1998), 149-167.

[6] Lellis Thivagar, M. and Richard, C.: On nano forms of weakly open sets, International Journal of Mathematics and Statistics, 1, (2013), 31-37.

[7] Lellis Thivagar, M. and Richard, C.: On nano continuity, Journal of Mathematical Theory and Modelling, 3, (2013), 32-37.

[8] Pawlak, Z.: Rough sets, Theoretical Aspects of Reasoning About Data, Kluwer Acadmic Publishers Dordrecht, 1991.

[9] Bondy, J. A. and Murty, U. S. R.: Graph theory with applications, Elsevier Science Publishing Co. Inc. (1975).

[10] M. S. El Naschie, Topics in the mathematical physics of E-infinity theory, Chaos, Solitons, Fractals 30 (2006), 656-663.