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Study on α -relaxation of Permittivity in Biological Tissue Based on Bottcher-Bordewijk Model

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Abstract

Many studies show that low-frequency electromagnetic fields can produce biological effects more obviously. Dielectric properties are not stable in biological system, which changes with external conditions such as temperature, external electric frequency, etc. Understanding the dielectric characteristics of bio-tissues under electromagnetic field is very important for them. This paper introduce the frequency in the famous Bottcher-Bordewijk model to explore the α -relaxation of permittivity in biological tissue, it shows that the new model can meet the experiment result. The new model could lay foundation for studying the relaxation characteristics of other materials.

1. Introduction

The dielectric property of biological tissue is one of the important electrical properties, which vary with the internal structure, including a variety of electrolytes, macromolecules and different kinds of cells [1]. Prodan et al. theoretically calculated the effect of electromagnetic field on the polarization properties of the cell membrane, the results show that the electromagnetic characteristics of the cell membrane changed with the external field frequency [2]. Specifically, the dielectric constant of the cell membrane increased with the increase of the external field frequency, and the permittivity of the cell membrane is close to zero when the membrane in high frequency ($>10^5\text{Hz}$). On the contrary, Sonja et al. established a reliable model of cell organization, taking ion channels, proteins and organelles and other aspects of the factors into account when blood cell under electromagnetic fields of megahertz range. The results show that dielectric dispersion of blood mainly due to interface polarization of the cell membrane, and the external electromagnetic field can affect not only the characteristics of transmembrane molecules, but also the material exchange between internal and external [3]. Permittivity ϵ is measured in farads per meter (F/m), electric susceptibility χ is dimensionless. They are related to each other through the equation: $\epsilon = \epsilon_r \epsilon_0 = (1 + \chi) \epsilon_0$, where ϵ_r is the relative permittivity of the material, and ϵ_0 denote the vacuum permittivity. For the multi-phase biological system, the dielectric property is generally expressed via the multi-phase permittivity.

For the two-phase composite media, the famous classical mixing formulas are Maxwell-Garnett [4] formula for most condition, and Bruggeman [5] formula for the dilute composite media. Hanai formula is usually used for explaining higher concentrations of the composite medium. Also, Asami put forward a shell filler equivalent permittivity formula [6]. Bohren put forward the well-known formula for the equivalent

permittivity of the randomly oriented ellipsoidal packing composite [7]. Various works also revealed dispersion with a relaxation rate of tissue permittivity, which are named α -relaxation, β -relaxation and γ -dispersion.

The dielectric properties of living tissue has its special features, such as effective permittivity is calculated based on theory modeling, which can greatly reduce the workload of explore them [8-11]. Maxwell effective medium theory, MG theory, Debye equation are multi-phase permittivity model, among them, the most commonly used is the Debye equation [12-15]. The real part of the permittivity can reflect the ability of the charge stored ability under certain conditions in Debye equation. Many researches and implementation of resonant methods are used to study material at GHz or Terahertz frequency range [16-20]. Therefore, we will explore the effective permittivity focusing on the MHz range. When blood exposure to electromagnetic fields, it is very important to study the biological effects of electromagnetic fields [21-25]. This paper established a relative dielectric model of liver (can be seen as consisting of blood and fat) to explore the dielectric properties at low frequencies.

2. Theory and Method

Binary mixtures model is usually used in investigating equivalent permittivity of mixture system for its significant in understanding the intermolecular interactions. Many different mixing rules such as Bottcher-Bordewijk model and Mehrotra model, et al. have been suggested in the literature in order to predict the permittivity of a mixture. Here is Bottcher-Bordewijk model as follows [26]:

$$\frac{3\varepsilon_1}{2\varepsilon + \varepsilon_1}\varphi_1 + \frac{3\varepsilon_2}{2\varepsilon + \varepsilon_2}\varphi_2 = 1 \quad (1)$$

In eq. 1, it involves four variables, here ε_i denote the permittivity of the pure components, φ_i denote the volume fraction of each of the components. For a two-phase system, $\varphi_2 = 1 - \varphi_1$. In order to clearly express the volume fraction, from Eq. 1, we get

$$\varphi_1 = \frac{2\varepsilon^2 + \varepsilon_1\varepsilon - 2\varepsilon_2\varepsilon - \varepsilon_1\varepsilon_2}{3\varepsilon_1\varepsilon - 3\varepsilon_2\varepsilon} \quad (2)$$

Bottcher-Bordewijk model is a relatively simple model; it describes the composite system of volume fraction via weight ratio. The model calculates permittivity in two-phase system without considers frequency. So the result of the model could only reflect the static permittivity characteristics of the system. In order to investigate the relationship between permittivity and frequency, we measured the permittivity of the rat tissue and compared the applicability of the Bottcher-Bordewijk model via checking theoretical volume fraction (equation. 2) [27]. The result is shown as in table 1.

Table 1. Blood cell volume fraction of different frequency.

Frequency (Hz)	10k	100k	1M	10M
Permittivity of blood	167370	28674	2459	323
Permittivity of fat	8648	671	147	58
Permittivity of liver	82306	18899	1833	262
Theoretical volume fraction of blood cell in liver	0.6239	0.7632	0.8123	0.8296

From table 1, the theoretical volume fraction of blood cell in liver φ_1 can be replaced by the frequency if the relationship between the value of φ_1 and the frequency is established. Here we consider the relationship between the log (frequency) and the φ_1 . We get the relationship by fitting calculation result in table 1.

$$\varphi_1 = -0.0305[\log(\text{frequency})]^2 + 0.4021\log(\text{frequency}) - 0.4937 \quad (3)$$

From equation 2 and equation 3, we get

$$\varepsilon_{liver} = \varepsilon_{fat} + \frac{3}{2}(\varepsilon_{blood} - \varepsilon_{fat}) \left(-0.0305s^2 + 0.4021s - 0.4937 \right) \quad (4)$$

Here $s = \log(\text{frequency})$, is the function of frequency. So we obtain relationship of the permittivity varies with the frequency.

3. Results and Discussion

Cell structure is of a random nature with some predictable average properties such as cell size and density. It is can be modeled by an aggregate of randomly distributed spherical shells in the equation model. The model can be used to describe the system with two constituent materials. From the equation (4) and the table 1, we can get the relationship between the theoretical and the experiment, seen in figure 1.

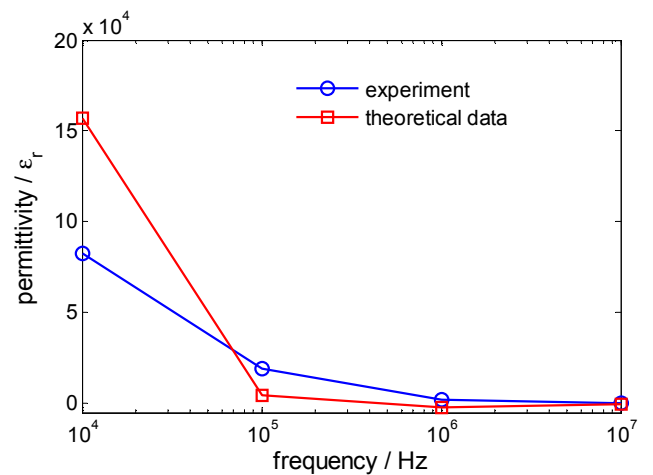


Figure 1. Theoretical and experiment result of the effective permittivity in two-phase system.

Since the blood cells in whole blood is accounted for 40%-50% by volume, which is a high concentration of a suspension, having dielectric characteristics similar to the

suspension system. But the volume fraction is more than 0.6, which is shown in table 1. Mainly reason for this phenomenon is that the permittivity of bio-tissue in high frequency is dominated by its water content. When regarding to the theory of strong ball dispersion, derivation of the equivalent permittivity, blood cell is wrapped in an insulating shell as conductive balls dispersed in the plasma. Except Debye model and the Bottcher-Bordewijk model mentioned before, many other mathematical models are established for exploring the permittivity of the material systems. All theoretical models can be deal with as this method and get the similar results. Such as Maxwell-Wagner model, here we introduce the Maxwell-Wagner equation for the multi-system: Maxwell calculated dielectric constant of the composite media in the electrostatic field. With the in-depth study of the dielectric of the composite material, researchers are constantly looking for ways to model accurately the relationship between the various components of the composite material.

Blood plasma contains large dielectric because its conductivity is much larger than pure blood cells. Because of the effect from the proteins in cell membrane, the fatty acids, other macromolecules, and the intracellular, blood can be mixed as a suspension system, here we consider without the interaction between different kinds of cells. When the dynamic electric field further considered, Wagner put out the dielectric constant model for complex biological tissue, which can be regarded as a small ball distributed in the continuum, the theory is often called W-M model:

$$\frac{\epsilon^* - \epsilon_1^*}{\epsilon^* + 2\epsilon_1^*} = \phi \frac{\epsilon_0^* - \epsilon_1^*}{2\epsilon_1^* + \epsilon_0^*} \quad (5)$$

The relationship between the equivalent permittivity of plasma and the blood cells also can be expressed as:

$$\frac{\epsilon^* - \epsilon_0^*}{\epsilon_1^* - \epsilon_0^*} \times \left(\frac{\epsilon_1^*}{\epsilon^*} \right)^{1/3} = 1 - f \quad (6)$$

Where ϵ^* is the multi-phase permittivity of blood, ϵ_1^* is the permittivity of plasma, ϵ_0^* is permittivity for whole blood, f is hematocrit values of blood cell in the system. On both sides of the relationship, cubic equation can be expressed by multi-phase model as follows, which is suitable for two-phase composite model system:

$$(\epsilon^*)^3 - 3\epsilon_0^*(\epsilon^*)^2 + \left\{ 3(\epsilon_0^*)^2 + \frac{[(f-1)(\epsilon_1^* - \epsilon_0^*)]}{\epsilon_1^*} \right\} \epsilon^* - (\epsilon_0^*)^3 = 0 \quad (7)$$

And the famous Skipetrov equation is as follows [28]:

$$\epsilon_{eff} = \epsilon_1 \frac{1 + 3f(\epsilon_2 - \epsilon_1)}{\epsilon_1(2 + f) + \epsilon_2(1 - f)} \quad (8)$$

This model has been derived based on the assumption that f here is much smaller than unity, and that either correlation

length or the particle diameter, is well below the wavelength of electromagnetic waves used. The Skipetrov equation is original and more transparent than others and is assumed to give more correct results under tough situations. All These mathematical expressions are more complex and can derive directly the dielectric constant with the frequency.

4. Conclusions

From the Bottcher-Bordewijk model and the frequency perspective, we build multi-phase model of biological tissue based on its identical function as a composite part of the dielectric material is processed in the same phase and each phase as a unified system. The new model of biological permittivity model study the dielectric α -relaxation of the bio-tissue, which can help us to simplify complex biological constitution, so the dielectric characteristic of bio-tissue can meet the new formula.

References

- [1] P. Zakharov, F. Dewarrat, A. Caduff, et, al. The effect of blood content on the optical and dielectric skin properties. *Physiological Measurement*, 2011, 32 (1): 131-149.
- [2] E. Prodan, C. Prodan, J. H. Miller. The Dielectric Response of Spherical Live Cells in Suspension: An Analytic Solution. *Biophysical Journal*, 2008, 95 (11): 4174-4182.
- [3] H. Sonja, E. Daniel, F. Jurg. Modeling effective dielectric properties of materials containing diverse types of biological cells. *Journal of Physics D: Applied Physics*, 2010, 43 (36): 365405.
- [4] Sergey G. Moiseev. Active Maxwell-Garnett composite with the unit refractive index [J]. *Physica B: Condensed Matter*, Volume 405, Issue 14, 15 July 2010, Pages 3042-3045.
- [5] Stanisław Koter, Anna Kujawska, Wojciech Kujawski. Modeling of transport and separation in a thermopervaporation process [J]. *Journal of Membrane Science*, Volume 480, 15 April 2015, Pages 129-138.
- [6] K. Asami. Effects of membrane disruption on dielectric properties of biological cells. *Journal of Physics D: Applied Physics*, 2006, 39 (21): 4656-4663.
- [7] Craig F Bohren, Daya P Gilra. Extinction by a spherical particle in an absorbing medium [J]. *Journal of Colloid and Interface Science*, Volume 72, Issue 2, November 1979, Pages 215-221.
- [8] Ruoyang Ma, Bin Xiao, Ning Ma, Piyi Du. Low-loss insulating-conductive ceramic composite with giant permittivity and high permeability using glass phase as separating layer. *Ceramics International* 42 (2016) 4126-4135.
- [9] Yvan Llave, Katsuya Mori, Daichi Kambayashi, Mika Fukuoka. Dielectric properties and model food application of tylose water pastes during microwave thawing and heating. *Journal of Food Engineering*, 178 (2016) 20-30.
- [10] Dylan Hickson, Shahroukh Sotodeh, Michael G. Daly, Rebecca Ghent, Michael C. Nolan. Improvements on effective permittivity measurements of powdered alumina: Implications for bulk permittivity properties of asteroid regoliths, *Advances in Space Research*, 59 (2017) 472-482.

- [11] Natalia A. Nikonorova, Marina Yu. Balakina, Olga D. Fominykh, Anastasiya V. Sharipova, Tatyana A. Vakhonina, Gulshat N. Nazmieva, Rene A. Castro, Alexander V. Yakimansky. Dielectric spectroscopy and molecular modeling of branched methacrylic (co)polymers containing nonlinear optical chromophores. *Materials Chemistry and Physics* 181 (2016) 217-226.
- [12] Wong Jee Keen Raymond, Chandan Kumar Chakrabarty, Goh Chin Hock, Ahmad Basri Ghani. Complex permittivity measurement using capacitance method from 300 kHz to 50 MHz. *Measurement* 46 (2013) 3796-3801.
- [13] M. Piralaee, A. Asgari, V. Siahpoush. Modeling and optimizing the performance of plasmonic solar cells using effective medium theory. *Physics Letters A* 381 (2017) 489-493.
- [14] P. G. Gallagher, Update on the clinical spectrum and genetics of red blood cell membrane disorders, *Current Hematology Reports* 3 (2004) 5-91.
- [15] T. I. Zohdi, F. A. Kuypers, W. C. Lee. Estimation of red blood cell volume fraction from overall permittivity measurements. *International Journal of Engineering Science* 48 (2010) 1681-1691.
- [16] Caroline B. Reid, George Reese, Adam P. Gibson and Vincent P. Wallace. Terahertz Time Domain Spectroscopy of Human Blood. *IEEE Transactions on Terahertz Science & Technology*, 2013, 3 (4): 363-367.
- [17] P. C. Ashworth et al., Terahertz pulsed spectroscopy of freshly excised human breast cancer, *Opt. Express*, vol. 17, no. 15, pp. 12444-12454, 2009.
- [18] P. Tewari et al., "In vivo terahertz imaging of rat skin burns," *Journal of biomedical optics*, vol. 17, no. 4, p. 040503, 2012.
- [19] H. Akimasa, K. Masami, K. Hiroki, et al. Acute Dosimetry and Estimation of Threshold-Inducing Behavioral Signs of Thermal Stress in Rabbits at 2.45-GHz Microwave Exposure. *IEEE Transactions on Biomedical Engineering*, 2010, 57 (5): 1234-1237.
- [20] K. K. Kesari, J. Behari. Microwave exposure affecting reproductive system in male rats. *Appl Biochem Biotechnol*, 2010, 162 (2): 416-428.
- [21] Sven Ansorge, Stéphane Lanthier, Julia Transfiguracion, Olivier Henry, Amine Kamen. Monitoring lentiviral vector production kinetics using online permittivity measurements. *Biochemical Engineering Journal* 54 (2011) 16-25.
- [22] Ashutosh Prasad, K. Prasad. Effective permittivity of random composite media: A comparative study. *Physica B* 396 (2007) 132-137.
- [23] Mark W. Coffey. On the generic behavior of the electrical permittivity at low frequencies. *Physics Letters A* 373 (2009) 2793-2795.
- [24] T. Sandu, D. Vrinceanu, E. Gheorghiu. Linear dielectric response of clustered living cells. *Physical Review E*, 2010, 81 (2): 021913.
- [25] C. Gabrie, A. Peyman, E. H. Grant. Electrical conductivity of tissue at frequencies below 1 MHz. *Physics in Medicine and Biology*, 2009, 54 (16): 4863-4878.
- [26] C. J. F. Böttcher and P. Bordewijk. *Dielectrics in time-dependent fields (Theory of electric polarization)*. Elsevier Scientific Pub. Co., 1978 2nd: 315-385.
- [27] Li Gun, Du Ning, Zhang Liang. Effective Permittivity of Biological Tissue: Comparison of Theoretical Model and Experiment, *Mathematical Problems in Engineering*, 2017 (2017): 7249672.
- [28] S. E. Skipetrov Effective dielectric function of a random medium *Phys. Rev. B* 60, 12705.