Symmetrization and the Modification of the Equations of Induction and Material Equations of Maxwell

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Citation

Abstract
The equation of induction and Maxwell’s equations play very important role in the electrodynamics. However, is absent the complete system of equations, which is capable of solving entire spectrum of electrodynamic processes in the material media. In the article it is shown that Maxwell’s equations can be represented in the symmetrical form, which solve stated problem. Is introduced the new concept of kinetic capacity, which describes the energy processes, connected with the precessional motion of the magnetic moments of atoms in the magnetized media. The concepts of the electrokinetic and magnetopotential waves, which describe wave processes in the nonmagnetic and magnetized material media, are introduced. It is shown that the equations of electrodynamics can be recorded in a plural manner with the use of different potentials and currents.

1. Introduction
The laws of classical electrodynamics they reflect experimental facts they are phenomenological. The fundamental equations of contemporary classical electrodynamics are Maxwell’s equations. They are written as follows for the vacuum:

\[
\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t},
\]

\[
\text{rot } \vec{H} = \frac{\partial \vec{D}}{\partial t},
\]

\[
\text{div } \vec{D} = 0,
\]

\[
\text{div } \vec{B} = 0,
\]

where \( \vec{E} \), \( \vec{H} \) are tension of electrical and magnetic field, \( \vec{D} = \varepsilon_0 \vec{E} \), \( \vec{B} = \mu_0 \vec{H} \) are electrical and magnetic induction, \( \mu_0 \), \( \varepsilon_0 \) are magnetic and dielectric constant of vacuum. From Maxwell’s equations follow the wave equations

\[
\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2},
\]

\[
\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2},
\]
these equations show that in the vacuum can be extended the plane electromagnetic waves, the velocity of propagation of which is equal to the speed of light

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]  

(1.7)

For the material media Maxwell’s equation they take the following form:

\[ \text{rot} \, \vec{\varepsilon} = -\mu_0 \frac{\partial \vec{H}}{\partial t} = -\frac{\partial \vec{B}}{\partial t}, \]  

(1.8)

\[ \text{rot} \, \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial \vec{D}}{\partial t}, \]  

(1.9)

\[ \text{div} \, \vec{D} = n e, \]  

(1.10)

\[ \text{div} \, \vec{B} = 0, \]  

(1.11)

where \( \mu_0, \varepsilon_0 \) are the relative magnetic and dielectric constants of the medium and \( n, e, \dot{v} \) are density, value and charge rate.

Equations (1.8) and (1.9) indicate that Maxwell’s equations for the material media are asymmetrical.

### 2. Plasmo-Like Media

Let us write down Maxwell’s equations for the plasmo-like media, in which the ohmic losses can be disregarded. To such media in the first approximation, can be related the superconductors, free electrons or ions in the vacuum (subsequently conductors). In this case the equation of motion of electron takes the form:

\[ m \frac{d \vec{v}}{dt} = e \vec{E}, \]  

(2.1)

where \( m \) is mass electron, \( e \) is electron charge, \( \vec{E} \) is the tension of electric field, \( \dot{v} \) is speed of the motion of charge.

In the work [1] it is shown that this equation can be used also for describing the electron motion in the hot plasma.

Using an expression for the current density

\[ j = n e \dot{v}, \]  

(2.2)

from (2.1) we obtain the current density of the conductivity

\[ \vec{j}_L = \frac{n e^2}{m} \int \vec{E} \, dt. \]  

(2.3)

in relationship (2.2) and (2.3) the value of \( n \) represents electron density. After introducing the designation

\[ L_k = \frac{m}{n e^2}, \]  

(2.4)

we find

\[ \vec{j}_L = \frac{1}{L_k} \int \vec{E} \, dt \]  

(2.5)

In this case the value \( L_k \) presents the specific kinetic inductance of charge carriers [2]. Its existence connected with the fact that charge, having a mass, possesses inertia properties. Pour on \( \vec{E} = \vec{E}_0 \sin \omega \tau \) relationship (2.5) it will be written down for the case of harmonics:

\[ \vec{j}_L = -\frac{1}{\omega L_k} \vec{E}_0 \cos \omega \tau, \]  

(2.6)

from relationship (2.5) and (2.6) is evident that \( \vec{j}_L \) presents inductive current, since, its phase is late with respect to the tension of electric field to the angle \( \frac{\pi}{2} \).

If charges are located in the vacuum, then during the presence of summed current it is necessary to consider bias current

\[ j_z = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \vec{E}_0 \cos \omega \tau. \]

is evident that this current bears capacitive nature, since, its phase anticipates the phase of the tension of electrical to the angle \( \frac{\pi}{2} \). Thus, summary current density will be written down [3-6]:

\[ \vec{j}_z = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \int \vec{E} \, dt, \]

or

\[ \vec{j}_z = \left( \omega \varepsilon_0 - \frac{1}{\omega L_k} \right) \vec{E}_0 \cos \omega \tau. \]  

(2.7)

In relationship (2.7) the value, which stands in the brackets, presents summary susceptance of this medium \( \sigma_z \) and it consists of, in turn, of the capacitive \( \sigma_c \) and by the inductive \( \sigma_L \) of the conductivity

\[ \sigma_z = \sigma_c + \sigma_L = \omega \varepsilon_0 - \frac{1}{\omega L_k}. \]

Relationship (2.7) can be rewritten in other designations:

\[ \vec{j}_z = \omega \varepsilon_0 \left( 1 - \frac{1}{\omega L_k} \right) \vec{E}_0 \cos \omega \tau, \]

where \( \omega_b = \sqrt{\frac{1}{L_k \varepsilon_0}} \) is plasma frequency.
Value
\[ E^{*}(\omega) = \varepsilon_0 \left( 1 - \frac{\omega_0^2}{\omega^2} \right) = \varepsilon_0 - \frac{1}{\omega^2 \varepsilon} \]

it is accepted to call the dielectric constant of dielectric depending on the frequency. Into it enter two not depending on the frequency of the parameter: the dielectric constant of vacuum and the kinetic inductance of charges.

Current density for the medium in question to be determined by three components, which depend on the electric field:

\[ j = \sigma E + \varepsilon_0 \left( \frac{\partial E}{\partial t} + \frac{1}{L_k} \int E \, dt \right), \]

where \( \sigma \) is conductivity.

Maxwell's equations for this case take the form:

\[ \text{rot} \, E = -\mu_0 \frac{\partial H}{\partial t}, \]
\[ \text{rot} \, H = \varepsilon_0 \sigma E + \varepsilon_0 \left( \frac{\partial E}{\partial t} + \frac{1}{L_k} \int E \, dt \right). \]

The system of equations (2.8) completely describes all properties of the medium examined. The equations of this system are not symmetrical. In the case of the absence of ohmic losses from (2.8) follows the equation [7]

\[ \text{rot} \, H + \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} + \mu_0 \frac{L_k}{L_\Sigma} \frac{\partial H}{\partial t} = 0. \]  

For the case pour on, time-independent, equation (2.9) passes into the London equation

\[ \text{rot} \, \text{rot} \, H + \mu_0 \frac{L_k}{L_\Sigma} \frac{\partial H}{\partial t} = 0, \]

where \( L_\Sigma = \frac{L_k}{\mu_0} \) is London depth of penetration.

Thus, it is possible to conclude that the equations of London being a special case of equation (2.9), and do not consider bias currents on medium. Therefore they do not give the possibility to obtain the wave equations, which describe the processes of the propagation of electromagnetic waves in the superconductors.

3. Dielectrics

Let us examine the simplest case, when oscillating processes in atoms or molecules of dielectric obey the law of mechanical oscillator [5].

\[ \left( \frac{\beta}{m} - \omega^2 \right) \ddot{r}_m = \frac{e}{m} \dot{E}, \]  

where \( \ddot{r}_m \) is deviation of charges from the position of equilibrium, \( \beta \) is coefficient of elasticity, which characterizes the elastic electrical binding forces of charges in the atoms and the molecules. Introducing the resonance frequency of the bound charges

\[ \omega_0 = \frac{\beta}{m}, \]

we obtain from (3.1):

\[ r_m = \frac{e \, E}{m(\omega^2 - \omega_0^2)} \cdot \dot{E} \]  

Is evident that in relationship (3.2) as the parameter is present the natural vibration frequency, into which enters the mass of charge. This speaks, that the inertia properties of the being varied charges will influence oscillating processes in the atoms and the molecules.

Since the general current density on medium consists of the bias current and conduction current

\[ \text{rot} \, \dot{H} = j_{\Sigma} = \varepsilon_0 \frac{\partial \dot{E}}{\partial t} + ne \ddot{v}, \]

that, finding the speed of charge carriers in the dielectric as the derivative of their displacement through the coordinate

\[ \ddot{v} = \frac{\partial r}{\partial t} = -\frac{e}{m(\omega^2 - \omega_0^2)} \frac{\partial \dot{E}}{\partial t}, \]

from relationship (3.2) we find

\[ \text{rot} \, \dot{H} = j_{\Sigma} = \varepsilon_0 \frac{\partial \dot{E}}{\partial t} - \frac{1}{L_{kd}} \frac{1}{(\omega^2 - \omega_0^2)} \frac{\partial \dot{E}}{\partial t}. \]  

But the value

\[ L_{kd} = \frac{m}{ne^2} \]

presents the kinetic inductance of the charges, entering the constitution of atom or molecules of dielectrics, when to consider charges free. Therefore the relationship (3.3) it is possible to rewrite

\[ \text{rot} \, \dot{H} = j_{\Sigma} = \varepsilon_0 \left( 1 - \frac{1}{\varepsilon_0 L_{kd} (\omega^2 - \omega_0^2)} \right) \frac{\partial \dot{E}}{\partial t}. \]

Since the value

\[ \frac{1}{\varepsilon_0 L_{kd}} = \omega_{pd}^2 \]

it represents the plasma frequency of charges in atoms and molecules of dielectric, the relationship (3.4) takes the form:
\[ \text{rot}\vec{H} = j = \varepsilon_0 \left(1 - \frac{\omega^2_{pd}}{\omega^2 - \omega_0^2}\right) \frac{\partial \vec{E}}{\partial t}. \]  
(3.5)

Value

\[ \varepsilon^* (\omega) = \varepsilon_0 \left(1 - \frac{\omega^2_{pd}}{\omega^2 - \omega_0^2}\right), \]  
(3.6)

it is accepted to call the dielectric constant of dielectric depending on the frequency.

Maxwell's equations for this case take the form:

\[
\begin{align*}
\text{rot}\vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} \\
\text{rot}\vec{H} &= \varepsilon_0 \left(1 - \frac{\omega^2_{pd}}{\omega^2 - \omega_0^2}\right) \frac{\partial \vec{E}}{\partial t},
\end{align*}
\]  
(3.7)

from where we immediately find the wave equation:

\[
\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \left(1 - \frac{\omega^2_{pd}}{\omega^2 - \omega_0^2}\right) \frac{\partial^2 \vec{E}}{\partial t^2}.
\]

The Maxwell's equations (3.7) for the dielectrics are also asymmetrical.

4. Symmetrization of the Equations of the Induction

Maksvell in his famous treatise [1] during writing of the equations of electrodynamics used the substantive (total) derivative, which includes not only local time derivatives, but also contains convective component. Convective component considers the possibility of moving the frame of reference, in which are determined the fields with respect to the fixed frame of reference, in which the fields are assigned. Since during writing Maxwell's equations considered the rotary motion of frame of reference, he used a quaternion record of four-dimensional algebra above the real numbers. Hertz is later and Heaviside they excluded from the equations of induction convective component and wrote down them in particular derived [2]. In this form we use now these equations, calling their Maxwell's equations. Maxwell's equations do not give the possibility to write down fields in the moving coordinate systems, if fields in the fixed system are known. In general form this give the possibility to make Lorenz, however, these conversions from the classical electrodynamics they do not follow. Question arises, can the principles of classical electrodynamics give correct results regarding pour on in the moving coordinate systems at least in some approximation, and if yes, then as the equations of electromagnetic induction must appear in this case.

Writing Lorentz force

\[ \vec{F}^\prime = e \vec{E} + e \left[ \vec{V} \times \vec{B} \right], \]  
(4.1)

we will note fields and forces, which appear in the moving frame of reference, by prime.

Indication of how can be recorded fields in the moving coordinate system, if they are known in the fixed, there are already in the Faraday law, if we use ourselves the substantional derivative. For the study of this problem let us rewrite Faraday law in the precise form:

\[ \oint \vec{E} \cdot d\vec{l}^\prime = -\frac{d}{dt} \Phi_y, \]  
(4.2)

The refinement of law, is more accurate than its record, it concerns only that circumstance that if we determine contour integral in the moving (prime) coordinate system, then near \( \vec{E}, \ d\vec{l} \) must stand primes. But if circulation is determined in the fixed coordinate system, then primes near \( \vec{E}, \ d\vec{l} \) be absent, but in this case to the right in expression (4.2) must stand particular time derivative. Usually this circumstance in the literature on this question is not specified.

The substantional derivative in relationship (4.2) indicates the independence of the eventual result of appearance emf. in the outline from the method of changing the flow, i.e., flow can change both due to the local time derivative of the induction of and because the system, in which is measured \( \oint \vec{E} \cdot d\vec{l}^\prime \), it moves in the three-dimensional changing field \( \vec{B} \).

In relationship (4.2) the flow is determined from the following relationship

\[ \Phi_y = \int \vec{B} \cdot d\vec{S}^\prime, \]  
(4.3)

where the magnetic induction \( \vec{B} = \mu \vec{H} \) is determined in the fixed coordinate system, and the element \( d\vec{S}^\prime \) is determined in the moving system. Taking into account (4.3), from (4.2) we obtain

\[ \oint \vec{E} \cdot d\vec{l}^\prime = -\frac{d}{dt} \int \vec{B} \ d\vec{S}^\prime, \]  
(4.4)

and further, since \( \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \text{grad} \), let us write down

\[ \oint \vec{E} \cdot d\vec{l}^\prime = -\left[ \frac{\partial}{\partial t} - \vec{V} \times \vec{B} \right] \ d\vec{S} = \int \vec{B} \ d\vec{S} = \Phi_y \]  

(Let us immediately note that entire following presentation will be conducted under the assumption the validity of the Galileo conversions, i.e. \( d\vec{l} = d\vec{l}^\prime \ u \ d\vec{S} = d\vec{S}^\prime \). From (1.5) follows the well known result

\[ \vec{E}^\prime = \vec{E} + \left[ \vec{V} \times \vec{B} \right], \]  
(4.6)

from which follows that during the motion in the magnetic field the additional electric field, determined by last term of relationship appears (4.6). Let us note that this relationship we obtained not of the conversions of Lorenz, but altogether having only refined Faraday law. Thus, Lorentz force is the
consequence of this precise law.

From relationship (4.6) it follows that during the motion in the magnetic field to the charge acts the force perpendicular to direction of motion. However, physical nature of this force nowhere is examined. It should be noted that Lorentz force contradicts the existing laws of mechanics, since, in the mechanics is not known such force, which with the uniform and rectilinear motion of body is directed normal to direction of its motion.

For explaining physical nature of the appearance of last term in relationship (4.6) let us write down $\vec{B}$ and $\vec{E}$ through the magnetic vector potential $\vec{A}_g$:

$$
\vec{B} = \mathbf{rot} \vec{A}_g, \quad \vec{E} = -\frac{\partial \vec{A}_g}{\partial t}.
$$

(4.7)

Then relationship (4.6)) can be rewritten

$$
\vec{E}' = -\frac{\partial \vec{A}_g}{\partial t} + \left\{ \vec{V} \times \mathbf{rot} \vec{A}_g \right\},
$$

(4.8)

and further

$$
\vec{E}' = -\frac{\partial \vec{A}_g}{\partial t} + \left\{ \vec{V} \mathbf{grad} \vec{A}_g \right\}.
$$

(4.9)

The first two members of the right side of equality (1.9) can be gathered into the total derivative of vector potential on the time, namely:

$$
\vec{E}' = -\frac{d \vec{A}_g}{d t} + \mathbf{grad} \left\{ \vec{V} \vec{A}_g \right\}
$$

(4.10)

From relationship (4.9) it is evident that the field strength, and consequently also the force, which acts on the charge, consists of three parts.

The first of them is obliged by the local derivative of magnetic vector potential on the time. The sense of second term of the right side of relationship (4.9) is also intelligible. It is connected with a change in the vector potential, but already because charge moves in the three-dimensional changing field of this potential. Other nature of last term of the right side of relationship (4.9). It is connected with the presence of potential forces, since. potential energy of the charge, which moves in the potential field $\vec{A}_g$ with the speed $\vec{V}$, is equal $\mathbf{e} \left\{ \vec{V} \vec{A}_g \right\}$. The value $\mathbf{e} \mathbf{grad} \left\{ \vec{V} \vec{A}_g \right\}$ gives force, exactly as gives force the gradient of scalar potential.

The relationship (4.9) gives the possibility to physically explain all composing tensions electric fields, which appears in the fixed and that moving the coordinate systems. If the discussion deals with the appearance of electrical pour on out of the long solenoid, where there are no magnetic pour on, then in this case first term of the right side of equality works (4.9). In the case of unipolar generator in the formation of the force, which acts on the charge, two last addend right sides of equality (4.9) participate, introducing identical contributions.

Thus, to speak about the unipolar generator as about “an exception to the rule of flow” is impossible [4], since flow rule, as we see, this is the totality of all three components. Taking rotor from both parts of equality (1.10) and taking into account that $\mathbf{rot} \ = 0$, we obtain

$$
\mathbf{rot} \vec{E}' = -\frac{d \vec{B}}{d t}.
$$

(4.11)

If there is no motion, then relationship (4.11) is converted into the Maxwell first equation. Certainly, on its informativeness relationship (4.11) strongly is inferior to relationship (4.2), since. in connection with the fact that $\mathbf{rot} \ = 0$, in it there is no information about the potential forces, designated through $\mathbf{e} \mathbf{grad} \left\{ \vec{V} \vec{A}_g \right\}$. Therefore, if us interest all components of electrical pour on out that act on the charge both in the fixed and in that moving the coordinate systems, we must use relationship (4.2).

Consequently, we must conclude that the moving or fixed charge interacts not with the magnetic field, but with the field of magnetic vector potential, and only knowledge of this potential and its evolution they give the possibility to calculate all force components, which act on the charges. However, magnetic field appears altogether only of the gradient of such vectorial field.

From the aforesaid it follows that the record of Lorentz force in the terms of the magnetic vector potential:

$$
\vec{F}' = \mathbf{e} \vec{E} + \mathbf{e} (\vec{V} \times \mathbf{rot} \vec{A}_g) = \mathbf{e} \vec{E} - \mathbf{e} (\vec{V} \mathbf{grad} \vec{A}_g) + \mathbf{e} \mathbf{grad} (\vec{V} \vec{A}_g)
$$

(4.12)

is more preferable, since. the possibility to understand the complete structure of this force gives.

Faraday law (4.2) is called the law of electromagnetic induction in connection with the fact that it shows how a change in the magnetic pour on it leads to the appearance of electrical pour on. However, in the classical electrodynamics there is no law of magneto-electric induction, which would show, how a change in the electrical pour on, or motion in them, it leads to the appearance of magnetic pour on. The development of classical electrodynamics followed along another way. Was first recorded the Ampere law

$$
\oint \vec{H} \cdot d \vec{l} = I,
$$

(4.13)

where $I$ is current, which crosses the area, included by the outline of integration. In the differential form relationship (4.13) takes the form:

$$
\mathbf{rot} \vec{H} = \vec{j}_o.
$$

(4.14)

where $\vec{j}_o$ is current density of conductivity.

Maxwell supplemented relationship (4.14) with bias current

$$
\mathbf{rot} \vec{H} = \vec{j}_o + \frac{\partial \vec{D}}{\partial t}.
$$

(4.15)

But by analogy with the law of electromagnetic induction
(4.2) must exist the law of the magnetoelectric induction
\[ \oint H' d\vec{r} = \frac{d\Phi_o}{dt}, \]
(4.16)
where \( \Phi_o = \int \vec{D} d\vec{S}' \) is the flow of electrical induction.

\[ \oint H' d\vec{r} = \int \frac{\partial \vec{D}}{\partial t} d\vec{S} + \oint [\vec{D} \times \vec{V}] d\vec{r} + \oint \vec{V} \cdot \text{div} \vec{D} d\vec{S}'. \]
(4.17)

In contrast to the magnetic fields, when div \( \vec{B} = 0 \), for the electric fields div \( \vec{D} = \rho \) and last term in the right side of relationship (4.17) it gives the conduction current I, and from relationship (4.16) the Ampere law immediately follows. From relationship (4.17) follows also the equality:
\[ \vec{H} = [\vec{D} \times \vec{V}], \]
(4.18)
which earlier could be obtained only from the Lorenz conversions.

As shown in the work [4], from relationship (4.18) follows and Bio-Savara law, if for enumerating the magnetic pour on to take the electric fields of the moving charges. In this case the last member of the right side of relationship (4.17) can be simply omitted, and the laws of induction acquire the completely symmetrical form of
\[ \oint \vec{E}' d\vec{r} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{S} - \oint [\vec{B} \times \vec{V}] d\vec{r}, \]
\[ \oint \vec{H}' d\vec{r} = \int \frac{\partial \vec{D}}{\partial t} d\vec{S} + \oint [\vec{D} \times \vec{V}] d\vec{r}. \]
(4.19)

5. Symmetrization of Maxwell's Equations

So that Maxwell's equations would become symmetrical, the first equation must take following form [12]:
\[ \text{rot}\vec{E} = -\sigma_H \vec{H} - \mu \frac{\partial \vec{H}}{\partial t} - \frac{1}{C_k} \oint \vec{H} dt, \]
(5.1)
where \( \sigma_H \) is conductivity of magnetic currents.

In comparison with traditional writing of the first equation of Maxwell in the right side of the equation to be contained two additive terms. The first member of right side describes ohmic losses in the magnetic materials during the imposition on them of variable magnetic pour on. Let us examine the physical sense of the last member of the right side of equation (5.1), who earlier in the Maxwell first equation did not be present.

At the same time to us it is known that the atom, which possesses the magnetic moment \( \vec{m} \), placed into the magnetic field, and which accomplishes in it precessional motion, has potential energy \( U_m = -\mu \vec{m} \cdot \vec{H} \). Therefore potential energy can be accumulated not only in the electric fields, but also in the precessional motion of magnetic moments, which does not possess inertia. Similar case is located also in the mechanics, when the gyroscope, which precesses in the field of external gravitational forces, accumulates potential energy. Regarding mechanical precessional motion is also noninertial and immediately ceases after the removal of external forces. The same situation occurs also for the case of the precessing magnetic moment. Its precession is noninertial and ceases at the moment of removing the magnetic field.

Therefore it is possible to expect that with the description of the precessional motion of magnetic moment in the external magnetic field in the right side of relationship (5.3) can appear a term of the same type as in relationship
\[ \text{rot}\vec{H} = \sigma E + \sigma \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_k} \oint \vec{E} dt. \]

It will only stand \( L_k \), instead of the kinetic capacity, which characterizes that potential energy, which has the precessing magnetic moment in the magnetic field:

Resonance processes in the plasma and the dielectrics are characterized by the fact that in the process of fluctuations occurs the alternating conversion of electrostatic energy into the kinetic kinetic energy of charges and vice versa. This process can be named electrokinetic and all devices: lasers, masers, filters, etc, which use this process, can be named electrokinetic. At the same time there is another type of resonance - magnetic. If we use ourselves the existing ideas about the dependence of magnetic permeability on the frequency, then it is not difficult to show that this dependence is connected with the presence of magnetic resonance. In order to show this, let us examine the concrete example of ferromagnetic resonance. If we magnetize ferrite, after applying the stationary field \( H_0 \) in parallel to the axis \( z \), the like to relation to the external variable field medium will come out as anisotropic magnetic material with the complex permeability in the form of tensor [13]
\[ \mu = \begin{pmatrix} \mu_\parallel (\omega) & -i \alpha & 0 \\ i \alpha & \mu_\perp (\omega) & 0 \\ 0 & 0 & \mu_\perp \end{pmatrix}, \]
where
\[ \mu_\parallel (\omega) = 1 - \frac{\Omega}{\omega} \mu_0, \quad \mu_\perp = \mu_0, \quad \alpha = \frac{\omega}{\mu_0(\omega^2 - \Omega^2)}, \]
moreover
\[ \Omega = |\gamma| H_0 \]
(5.2)
is natural frequency of precession and
\[ M_0 = \mu_0(\mu - 1)H_0 \]
(5.3)
is a magnetization of medium. Taking into account (5.2) and
(5.3) for \( \mu_r^*(\omega) \), it is possible to write down

\[
\mu_r^*(\omega) = 1 - \frac{\Omega^2(\mu - 1)}{\omega^2 - \Omega^2} \tag{5.4}
\]

This value is conventionally designated as the depending on the frequency ignitnoy permeability of magnetic material. if we consider that the electromagnetic wave is propagated along the axis \( x \) and there are components pour on \( H_y, H_z \), then in this case the Maxwell first equation will be written down:

\[
\text{rot} \quad \vec{E} = \frac{\partial \vec{E}}{\partial x} = \mu_0 \mu_r \frac{\partial \vec{H}_x}{\partial t}.
\]

Taking into account (5.4), we will obtain

\[
\text{rot} \quad \vec{E} = \mu_0 \left[ 1 - \frac{\Omega^2(\mu - 1)}{\omega^2 - \Omega^2} \right] \frac{\partial \vec{H}_x}{\partial t}. \tag{5.5}
\]

for the case of \( \omega \gg \Omega \) we have

\[
\text{rot} \quad \vec{E} = \mu_0 \left[ 1 - \frac{\Omega^2(\mu - 1)}{\omega^2} \right] \frac{\partial \vec{H}_x}{\partial t}.
\]

assuming \( H_y = H_{0y} \sin \omega t \) and taking into account that in this case

\[
\frac{\partial \vec{H}_x}{\partial t} = -\omega^2 \int \vec{H}_y \, dt,
\]

we obtain from (5.5)

\[
\text{rot} \quad \vec{E} = \mu_0 \frac{\partial \vec{H}_x}{\partial t} + \mu_0 \Omega^2(\mu - 1) \int \vec{H}_y \, dt,
\]

or

\[
\text{rot} \quad \vec{E} = \mu_0 \frac{\partial \vec{H}_x}{\partial t} + \frac{1}{C_s} \int \vec{H}_y \, dt \tag{5.6}
\]

for the case \( \omega \ll \Omega \) we find

\[
\text{rot} \quad \vec{E} = \mu_0 \mu \frac{\partial \vec{H}_y}{\partial t}.
\]

Value

\[
C_s = \frac{1}{\mu_0 \Omega^2(\mu - 1)},
\]

which is introduced in relationship (5.6), let us name kinetic capacity [12].

Similarly can be described electron paramagnetic resonance.

With which is connected existence of kinetic capacity, and its which physical sense? If the direction of magnetic moment does not coincide with the direction of external magnetic field, then the vector of this moment begins to precess around the vector of magnetic field with the frequency \( \Omega \). The magnetic moment of \( \vec{m} \) possesses in this case potential energy \( U = -\vec{m} \cdot \vec{B} \). This energy similar to energy of the charged capacitor is potential, because precessional motion, although is mechanical, however, it not inertia and instantly it does cease during the removal of magnetic field. However, with the presence of magnetic field precessional motion continues until the accumulated potential energy is spent, and the vector of magnetic moment will not become parallel to the vector of magnetic field.

Wave processes and the waves, which are determined by equation (5.6) they can be named magnetopotential.

Idea of the Maxwell first equation by relationship (5.1) in combination with the second symmetrical Maxwell equation gives the possibility to present with the aid of these equations entire spectrum of electrodynamic processes in the material media.

6. Plurality of the Forms of the Writing of the Electrodynamic Laws

Magnetic and electric fields can be expressed through the vector potential of magnetic field and the vector potential of electric field [14]

\[
\vec{H} = \text{rot} \; \vec{A}_H \tag{6.1}
\]

\[
\vec{E} = \text{rot} \; \vec{A}_E \tag{6.2}
\]

Consequently, Maxwell’s equations can be written down with the aid of these potentials:

\[
\text{rot} \; \vec{A}_E = -\mu \frac{\partial \vec{A}_H}{\partial t} \tag{6.3}
\]

\[
\text{rot} \; \vec{A}_H = \varepsilon \frac{\partial \vec{A}_E}{\partial t} \tag{6.4}
\]

For each of these potentials it is possible to obtain wave equation, in particular

\[
\text{rot} \; \text{rot} \; \vec{A}_E = -\varepsilon \mu \frac{\partial^2 \vec{A}_E}{\partial t^2} \tag{6.5}
\]

and to consider that in the space are extended not the magnetic and electric fields, but the field of electrical vector potential.

In this case, as can easily be seen of the relationships (6.1 - 6.4), magnetic and electric field they will be determined through this potential by the relationships:

\[
\vec{H} = \varepsilon \frac{\partial \vec{A}_E}{\partial t} \tag{6.6}
\]

\[
\vec{E} = \text{rot} \; \vec{A}_E
\]
Spatial derivative \( \text{rot} \, \hat{A}_E \) and local time derivative \( \frac{\partial \hat{A}_E}{\partial t} \) are connected with wave equation (6.5).

Thus, the use only of one electrical vector potential makes it possible to completely solve the task about the propagation of electrical and magnetic currents. Taking into account (6.6), Poynting's vector can be written down only through the vector \( \hat{A}_E \):

\[
\hat{P} = \varepsilon \left[ \frac{\partial \hat{A}_E}{\partial t} \times \text{rot} \, \hat{A}_E \right].
\]

Characteristic is the fact that with this method of examination necessary condition is the presence at the particular point of space both the time derivatives, and the gradients of one and the same potential.

This task can be solved by another method, after writing down wave equation for the magnetic vector potential:

\[
\text{rot} \, \text{rot} \, \hat{A}_H = -\varepsilon \mu \frac{\partial^2 \hat{A}_H}{\partial t^2}.
\]

In this case magnetic and electric fields will be determined by the relationships

\[
\hat{H} = \text{rot} \, \hat{A}_H,
\]

\[
\hat{E} = -\mu \varepsilon \frac{\partial \hat{A}_H}{\partial t}.
\]

Poynting's vector in this case can be found from the following relationship:

\[
\hat{P} = -\mu \left[ \frac{\partial \hat{A}_H}{\partial t} \times \text{rot} \, \hat{A}_H \right].
\]

Spatial derivative \( \text{rot} \, \hat{A}_H \) and local time derivative of \( \frac{\partial \hat{A}_H}{\partial t} \) are connected with wave equation (6.7).

But it is possible to enter and differently, after introducing, for example, the electrical and magnetic currents

\[
\hat{j}_E = \text{rot} \, \hat{H},
\]

\[
\hat{j}_H = \text{rot} \, \hat{E}.
\]

The equations also can be recorded for these currents:

\[
\text{rot} \, \hat{j}_H = -\mu \frac{\partial \hat{j}_E}{\partial t},
\]

\[
\text{rot} \, \hat{j}_E = \varepsilon \frac{\partial \hat{j}_H}{\partial t}.
\]

This system in its form and information concluded in it differs in no way from Maxwell's equations, and it is possible to consider that in the space the magnetic or electric currents are extended. And the solution of the problem of propagation with the aid of this method will again include complete information about the processes of propagation.

The method of the introduction of new vector examined pour on it is possible to extend into both sides ad infinitum, introducing all new vectorial fields. Naturally in this case one should introduce and additional calibration, thus, there is an infinite set of possible writings of electrodynamic laws, but they all are equivalent according to the information concluded in them.

7. Conclusion

In the article it is shown that Maxwell's equations can be represented in the symmetrical form and such equations describe entire spectrum of electrodynamic processes in the material media. Is introduced the new concept of kinetic capacity, which describes the energy processes, connected with the precessional motion of the magnetic moments of atoms in the magnetized media. The concepts of the electrokinetic and magnetopotential waves, which describe wave processes in the nonmagnetic and magnetized material media, are introduced. It is shown that the equations of electrodynamics can be recorded in a plural manner with the use of different potentials and currents.

References

