Radiation of a Photon, Length of a Photon and Photon in a Gravitational Field

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Citation  

Abstract  
It is shown that Schrodinger’s equation in space of the generalized coordinates equally as for a photon in free space without currents and charges, and for a photon radiated by a current. On a basis of the photon wave function normalization the dependence of the photon length on volumetric density of its energy is found. It is shown that length of the photon is quantized. The minimal photon length in space of the generalized coordinates and the maximal volumetric density of the photon energy is found. It is shown that in a constant gravitational field the Schrodinger’s equation for a photon in space of the generalized coordinates needs to be written down in a coordinate time dependent on size of a gravitational field. In a constant gravitational field the photon frequency and volumetric density of its energy increases but the photon length decreases.

1. Introduction  
The photon is a carrier of electromagnetic interaction \cite{1, 2, 3, 4}. It takes a special place among other elementary particles. The photon has a zero mass and zero charge. It has photon cyclic frequency $\delta$ the energy equal $\hbar\delta$ where $\hbar = 1.0545887 \, J \cdot s$ - Planck’s reduced constant, and also an impulse $k = \hbar \delta n$, where $c = 299792458 \, \frac{m}{s}$ - speed of a light (photon) in vacuum, $n$ - unit vector. The photon cannot be in a state of rest; for a photon there is no antiparticle. Spin of a photon (the moment of an impulse) it is equal to unit $\hbar$. The photon helicity there is $\pm 1$. If helicity is $+1$ there is polarization of a photon right-handed and spin it is directed on a direction of the photon movement. If there is helicity $-1$ there is polarization of a photon left-handed and spin it is directed against a direction of the photon movement. A photon it is absolutely stable particle.

The photon, being the gauge boson, submits to Bose-Einstein’s statistics \cite{5}.  
Set of photons submits to a principle of superposition in Euclidean space: the resulting characteristic of photons set is the linear sum of separate photons characteristics.

The purpose of article is the further research of photon characteristics on the basis of found before the Schrodinger’s equation for a photon \cite{6}, both during its radiation, and in free space at presence of a gravitational field.
2. Radiation of a Photon

In [6] the Schrodinger’s equation for a photon in space of the generalized coordinates has been found. This space represents actually the space of a vector-potential in which the principle of superposition is not carried out. It is connected to nonlinearity of the generalized coordinate concerning the magnetic field strength in Euclidean space. In [6] it is shown that nonlinearity of the generalized coordinate passes to nonlinearity of Schrodinger’s equation for a photon. In the Euclidean space the photon is described by the linear wave equation for vector-potential, hence the principle of superposition for set of photons in Euclidian space is correct.

First of all we research a problem of a photon radiation. Radiation of photons is carried out by the moving charged particles. The Lagrangian of system a quantum - charged particle plus field of the photon, there is Lagrangian of the radiated photon, where it is similar [6]:

\[ l = l^0 + l^a + l^{\text{int}} \]

(1)

where is Lagrangian of the charged particle, \( l^0 \) - Lagrangian of the charged particle, \( l^a \) - Lagrangian of the photon and the charged particle interactions, i.e. Lagrangian determining the mechanism of quantum radiation.

Let’s write down the Lagrangian of charged particle plus Lagrangian of the interactions as [7]:

\[ l^0 + l^a = \frac{mV^2}{2} + \frac{1}{c} Aj - e\phi \]

(2)

there is Lagrangian of the charged particle, \( l^0 \) - Lagrangian of the charged particle, \( l^a \) - Lagrangian of the photon and the charged particle interactions, i.e. Lagrangian determining the mechanism of quantum radiation.

Hence, the general Lagrangian of the photon - the charged particle system looks like:

\[ l = T - U = \frac{E^2 - H^2}{8\pi} + \frac{mV^2}{2} + \frac{1}{c} Aj - e\phi \]

(4)

The volumetric density of energy of this system is:

\[ w = T + U = \frac{E^2 + H^2}{8\pi} - \frac{mV^2}{2} - \frac{1}{c} Aj + e\phi \]

(5)

In formulas (4) and (5) it is accepted:

\[ T = \frac{E^2}{8\pi} \quad \text{and} \quad U = \frac{H^2}{8\pi} - \frac{mV^2}{2} - \frac{1}{c} Aj + e\phi \]

(6)

As well as in [6] for the generalized speed we use the strength of an electromagnetic field \( \dot{q} = E \). It is allowable, since the general formula is correct [8]:

\[ w = \dot{q} \frac{\partial l}{\partial \dot{q}} - l = E \frac{\partial l}{\partial E} - l \]

\[ = E \frac{2E}{8\pi} \left( \frac{E^2 - H^2}{8\pi} - \frac{mV^2}{2} \right) \]

(7)

\[ = \left( \frac{E^2 + H^2}{8\pi} - \frac{mV^2}{2} \right) - \frac{1}{c} Aj + e\phi \]

Let’s find the generalized coordinate. For this purpose we use the Euler-Lagrange’s equation [9]:

\[ \frac{d}{dt} \left( \frac{\partial l}{\partial \dot{q}} \right) = \frac{\partial l}{\partial q} \]

(8)

where \( q \) there is a vector of generalized coordinate.

The derivative \( \frac{\partial (mV^2)}{\partial q} = 0 \) since we assume \( V^2 = const \). The solitary quantum can be radiated owing to instant change of a sign on speed of the charged particle with \( V = V_o \) on \( V = -V_o \).

Having substituted in (8) the equation (4), we shall find:

\[ \frac{dE}{dt} = E \frac{\partial E}{\partial q} - H \frac{\partial H}{\partial q} + \frac{4\pi}{c} \frac{j}{\partial q} - 4\pi c \frac{\partial \phi}{\partial q} \]

(9)

Passing in (9) from full to partial derivatives, we have:

\[ \frac{\partial E}{\partial t} + \frac{\partial E}{\partial q} = E \frac{\partial E}{\partial q} - H \frac{\partial H}{\partial q} + \frac{4\pi}{c} \frac{j}{\partial q} - 4\pi c \frac{\partial \phi}{\partial q} \]

(10)

Reducing in (10) the identical terms, we shall find:

\[ \frac{\partial E}{\partial t} = \frac{4\pi}{c} \frac{j}{\partial q} - \frac{4\pi c}{c} \frac{\partial \phi}{\partial q} \]

(11)

Using the Maxwell’s equation \( \text{rot} H = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} j \) [7], we shall receive:

\[ \text{rot} H - 4\pi j = \frac{1}{c} \frac{\partial H}{\partial t} + \frac{4\pi}{c} j \frac{\partial A}{\partial t} - 4\pi c \frac{\partial \phi}{\partial t} \]

(12)

Applying the Maxwell equation \( \text{rot} H = -\frac{1}{c} \frac{\partial H}{\partial t} \) and \( q = E \), it is possible to write down \( \text{rot} \frac{\partial q}{\partial t} = -\frac{1}{c} \frac{\partial H}{\partial t} \) or \( \text{rot} q = -\frac{1}{c} H \). Taking into account \( H = \text{rot} A \), we shall find:

\[ q = -\frac{A}{c} \quad \text{or} \quad A = -c q \]. Thus, as the generalized coordinate
it is used the vector-potential $A$ (normalized on the light velocity in used system of units), taken with the opposite sign. This result corresponds to [6].

We use also the Maxwell equation $\nabla \mathbf{E} = 4\pi \mathbf{E}$. Taking into account $q = \mathbf{E}$, we shall find $\mathbf{div} \mathbf{A} = 4\pi e$. Taking into account interrelation between the generalized coordinate and vector-potential, we shall find $\frac{\partial \mathbf{div} \mathbf{A}}{\partial t} = 4\pi e$. Using Coulomb’s gauge $\mathbf{div} \mathbf{A} = 0$, we have $4\pi e = 0$. The received equality means that the motionless charge in the generalized coordinates (in the Euclidean also) does not radiate a photon and it can be not taken into account.

The last Maxwell equation $\nabla \mathbf{H} = 0$ is satisfied automatically as $\nabla \mathbf{rot} \mathbf{A} = 0$.

Substituting in (12) instead of the vector-potential $A$ its expression through the generalized coordinate $A = -e\mathbf{q}$ and reducing terms with density of the current $j$, we shall find:

$$\mathbf{rot} \mathbf{H} = -e \frac{\mathbf{H}}{\mathbf{q}}$$

(13)

that coincides with [6]. Thus, the generalized coordinate at radiation of a photon by currents coincides with the generalized coordinate of a free photon.

We also shall show that Euler-Lagrange’s equation (8) at the used the generalized velocity and the generalized coordinate passes to the wave equation for vector-potential $A$.

Let’s transform the right part of the equation (8):

$$\frac{\partial \mathbf{L}}{\partial \mathbf{q}} = \frac{\partial (T - U)}{\partial \mathbf{q}} = -\frac{\partial U}{\partial \mathbf{q}} = -\frac{e}{4\pi} \mathbf{H} + \frac{1}{c^2} \frac{\partial \mathbf{A}}{\partial \mathbf{t}} - e \frac{\partial \phi}{\partial \mathbf{q}}$$

(14)

Thus the Planck’s charge concerns not to the charge density of the particle generated the photon is not kept. It represents as though photon "memory" of what size was a photon and the Planck's charge reflecting principle of the photon occurrence, its genesis. "Memory" of what size was a charge of the particle generated the photon is not kept.

The solution of the equation (17) looks like [6]:

$$i\hbar \frac{\partial \Psi}{\partial \mathbf{t}} + 2\pi e^{\mu} \frac{\partial^2 \Psi}{\partial \mathbf{A}^2} + \ln \Psi \Psi = 0$$

(18)

There are used determination of the vector-potential $\mathbf{H} = \mathbf{rot} \mathbf{A}$, also the known formula of the vector analysis $\mathbf{rot} \mathbf{A} = \mathbf{grad} \mathbf{div} \mathbf{A} - \Delta \mathbf{A}$, and also Coulomb’s gauge $\mathbf{div} \mathbf{A} = 0$.

The transformations of the equation (8) left part with use $\mathbf{E} = \mathbf{grad} \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial \mathbf{t}}$ has result:

$$\frac{d}{dt} \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \right) = \frac{d}{dt} \left( \frac{\partial \mathbf{L}}{\partial \mathbf{q}} \right) = \frac{d}{dt} \left( \frac{\partial \mathbf{E}}{\partial \mathbf{q}} \right) = \frac{1}{4\pi} \frac{\partial \mathbf{E}}{\partial \mathbf{t}}$$

(15)

Equating (14) and (15) we shall find the wave equation for vector-potential [7] at presence of a radiation source - density of a current $j$:

$$\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial \mathbf{t}^2} - \frac{1}{c} \mathbf{grad} \frac{\partial \phi}{\partial \mathbf{t}} = - \frac{4\pi j}{c}$$

(16)

Thus, in a classical variant the radiation of a photon submits to the wave equation (16) following from Euler-Lagrange’s equation (8).

In connection with identity of the generalized coordinates for a free [6] and radiated photon the quantization of the radiated photon is carried out same as quantization of the free photon. In result we receive the Schrodinger’s equation as [6]:

$$i\hbar \frac{\partial \Psi}{\partial \mathbf{t}} + 2\pi e^{\mu} \frac{\partial^2 \Psi}{\partial \mathbf{A}^2} + \ln \Psi \Psi = 0$$

(17)

where $\Psi$ is the wave function of a photon.

It is simple to translate the equation (17) in the Euclidean space. For this purpose we use the formula connecting the generalized coordinate with vector-potential $q = \mathbf{A}/c$.

Substituting it in (17), we shall find:

$$i\hbar \frac{\partial \Psi}{\partial \mathbf{t}} + 2\pi e^{\mu} \frac{\partial^2 \Psi}{\partial \mathbf{A}^2} + \ln \Psi \Psi = 0$$

(18)

Let’s notice that the fourth degree so-called Planck’s charge (fundamental physical constant $e^{\mu} = \sqrt{\hbar c}$) enters into the equation (18).

Thus the Planck’s charge concerns not to the charged particle, and to a photon. The probability to find out a particle photon occurrence, its genesis. "Memory" of what size was a photon is satisfied.

The solution of the equation (17) looks like [6]:

$$\Psi = \exp \left[ 2\pi \left( \frac{1}{\hbar \mathbf{r}^2} - \hbar \delta + \frac{1}{2} \right) \exp \left[ \frac{\left( \mathbf{kq} - \omega \mathbf{t} \right)^2}{8\pi \left( \hbar \mathbf{k} \right)^2} \right] \exp \left[ i \left( \mathbf{rq} - \mathbf{t} \right) \right] \right]$$

(19)

As against [6] in (19) the factor in envelope exponent revised: instead of 4 correct size 8 is used.
3. Length of a Photon

In [6] it has been marked that \( \Psi \) can be interpreted as some density of probability of a quantum element presence in the given place of space of the generalized coordinates.

Therefore for the size \( \Psi \) correctly following normalizing equation:

\[
\int_{-\infty}^{\infty} |\Psi|^2 \, dq = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\Psi|^2 \, dq = 1
\]

where \( q \) there is quantum length in space of the generalized coordinates.

Substituting in (20) square of the module of the formula (19) we shall find the quantum length under condition of \( t = 0 \):

\[
q_0 = 2 \int_{0}^{\infty} |\Psi|^2 \, dq
= 2 \exp \left( 2\pi (\hbar r)^2 - \hbar \delta + \frac{1}{2} \right) \int_{0}^{\infty} \exp \left[ - \frac{c^2}{4\pi \hbar^2} \right] \, dq
\]

Hence, the module of a quantum length is equal:

\[
q_0 = 2\pi \hbar \exp \left( 2\pi (\hbar r)^2 - \hbar \delta + \frac{1}{2} \right)
\]

At finding (22) the formula \( \int_{0}^{\infty} \exp(-x^2) \, dx = \sqrt{\frac{\pi}{2}} \) is used.

Using the formula for the volumetric density of quantum energy \( w = \hbar \delta - \frac{1}{2} \) and also \( c = 4\pi \hbar \) we shall find the quantum length as:

\[
q_0 = 2\pi \hbar \exp \left( \frac{c^2}{8\pi} - w \right)
\]

From (23) follows that quantum length in space of the generalized coordinates quantized. Besides the quantum length the exponential falls with increase in volumetric density of the quantum energy \( w \). It apparently this dependence is correct and for Euclidian spaces.

On fig. 1 dependence of the relative quantum length on volumetric density of its energy.

4. Photon in Constant Homogeneous Gravitational Field

Let’s examine how characteristics of quantum of electromagnetic radiation in the constant homogeneous gravitational field will change. For this purpose it is used some elements of the constant gravitational field theory.

In inertial frame in the Cartesian system of coordinates the interval \( ds \) is determined by the formula [8]:

\[
ds^2 = c^2 \tau^2 - dX^2 - dY^2 - dZ^2 =
= g_{00} \left( dX^0 \right)^2 - g_{11} \left( dX^1 \right)^2 - g_{22} \left( dX^2 \right)^2 - g_{33} \left( dX^3 \right)^2
\]

where \( c \) - speed of light in vacuum, \( \tau \) - an interval coordinate time between events [8], \( g_{00} = 1, g_{11} = g_{22} = g_{33} = -1 \) - components of metric tensor:

\[
g_{\alpha} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

which signature is \( (+, -, -, -) \).

From (24) follows:
\[ d\tau = \frac{1}{c}\sqrt{g_{00}}dX^0 \] \hspace{1cm} (26)

As it will be shown further in a gravitational field \( g_{00} > 1 \). Therefore, coordinate time flows more slowly (\( d\tau \) increases) than it is more \( g_{00} \) in the given point of space.

Lagrangian of the gravitational field looks like [8]:

\[ l = T - U = -mc^2\sqrt{1 - \frac{V^2}{c^2}} - m\phi_g \]
\[ = -mc^2\left(1 - \frac{1}{2}\frac{V^2}{c^2}\right) - m\phi_g \] \hspace{1cm} (27)

where \( m \) there is mass of a particle in a field, \( V \) - its velocity, \( \phi_g \) - gravitational potential of the field, so the acceleration \( X = \frac{dX}{c} \) in the given point of space.

Thus, according to (24) components of metric tensor it is equal:

\[ g_{00} = \left(1 + \frac{\phi_g}{c^2}\right)^2 \] \hspace{1cm} (30)

For the further analysis we use concept of eikonal [8]. The eikonal it the phase of the periodic function describing a wave field:

\[ \varphi = \mathbf{r}q - \delta t \] \hspace{1cm} (31)

where \( \mathbf{r} \) there is a wave vector of the eikonal, \( q \) - coordinate vector of the eikonal (can be not Euclidian), \( \delta \) - cyclic frequency of the eikonal.

Action for the particle in the gravitational field is equal:

\[ S = \int k\mathbf{r}d\tau = -mc\int \left(c - \frac{V^2}{2c} + \frac{\phi_g}{c}\right)d\tau = -mc\int d\mathbf{s} \] \hspace{1cm} (28)

Hence, the square of interval is equal:

\[
\begin{align*}
 ds^2 &= \left(c - \frac{V^2}{2c} + \frac{\phi_g}{c}\right)^2 d\tau^2 \\
 &= \left(c^2 + \frac{V^4}{4c^2} + \frac{\phi_g^2}{c^2} - 2c\frac{V^2}{2c} + 2c\frac{\phi_g}{c} - 2\frac{V^2\phi_g}{2c^2}\right)d\tau^2 \\
 &= \left(c^2 - V^2 + \frac{\phi_g^2}{c^2} + 2\frac{\phi_g}{c}\right)d\tau^2 \\
 &= \left(c^2 + 2\frac{\phi_g}{c} + \frac{\phi_g^2}{c^2}\right)d\tau^2 - d\mathbf{s}^2 \\
 &= c^2\left(1 + \frac{\phi_g}{c} + \frac{\phi_g^2}{c^2}\right)d\tau^2 - d\mathbf{s}^2 \\
 &= c^2\left(1 + \frac{\phi_g}{c}\right)^2 d\tau^2 - d\mathbf{s}^2
\end{align*}
\]

where \( d\mathbf{s} = \mathbf{V}d\tau \).

Thus, according to (24) components of metric tensor it is equal:

\[ g_{00} = \left(1 + \frac{\phi_g}{c^2}\right)^2 \] \hspace{1cm} (30)

For the further analysis we use concept of eikonal [8]. The eikonal it the phase of the periodic function describing a wave field:

\[ \varphi = \mathbf{r}q - \delta t \] \hspace{1cm} (31)

where \( \mathbf{r} \) there is a wave vector of the eikonal, \( q \) - coordinate vector of the eikonal (can be not Euclidian), \( \delta \) - cyclic frequency of the eikonal.

Taking into account (26) and (31) it is possible to find frequency of eikonal (frequency of quantum in the given point in the coordinate time):

\[ \delta = -\frac{\partial \varphi}{\partial \tau} = -\frac{\partial \varphi}{\partial X^0} = -\frac{c}{\sqrt{g_{00}}}\frac{\partial \varphi}{\partial X^0} \] \hspace{1cm} (32)

If to use a spacetime \( t \) (outside of the gravitational field) so that \( t = \frac{X^0}{c} \), the cyclic frequency of quantum measured in spacetime is equal \( \delta_0 = -\frac{\partial \varphi}{\partial t} = -c\frac{\partial \varphi}{\partial X^0} \). Hence, according to (32) with the account (30) we shall find the frequency of quantum in coordinate time:

\[ \delta = \frac{\delta_0}{\sqrt{g_{00}}} = \frac{\delta_0}{1 + \frac{\phi_g}{c^2}} \] \hspace{1cm} (33)

where \( \delta_0 \) there is frequency of quantum at absence of the gravitational field (in spacetime).

Thus, frequency of quantum depends on size of the gravitational field potential. As the potential of the gravitational field is the size negative, at approach to the bodies creating a field the frequency of quantum \( \delta \) grows, and at distance falls (redshift). For example, for a body in mass \( m \) the potential of the field depends on radius \( R \) under the formula \( \phi_g = -\frac{km}{R} \), where \( k = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \) - gravitational constant [8].

Eikonal is the size quantized. The quantum of the eikonal is equal, see [6]:

\[ s_0 = h\varphi \] \hspace{1cm} (34)

For introduce the generalized coordinates in a constant homogeneous gravitational field we use the formal reception offered in [8]. We shall write down volumetric density of the electromagnetic field energy in substance [10]:

\[ w = \frac{DE + BH}{8\pi} \] \hspace{1cm} (35)

where \( D \) there is an induction electric and \( B \) - an induction of a magnetic field.

Both inductions we shall replace with formulas:

\[ D = \frac{E}{\sqrt{g_{00}}} \quad B = \frac{H}{\sqrt{g_{00}}} \] \hspace{1cm} (36)

In this case the volumetric density of the field energy (35) will become:
\( w = T + U = \frac{E^2 + H^2}{8\pi \sqrt{g_{00}}} \) \hspace{1cm} (37)

The Lagrangian for a free electromagnetic field (at absence of charges and currents) can be written down:

\[ l = T - U = \frac{E^2 - H^2}{8\pi \sqrt{g_{00}}} \] \hspace{1cm} (38)

As the quantum mass is equal to zero the Lagrangian of a gravitational field (27) also is equal to zero. Therefore full Lagrangian of quantum in the gravitational field looks like (38).

For Lagrangian \( l \) the equation in spacetime \( \partial_0 - \partial_\tau = 0 \)

\[
\frac{d}{dt} \left( \frac{\partial l}{\partial q_i} \right) = \frac{d}{d\tau} \left( \frac{\partial l}{\partial q_i} \right) \sqrt{g_{00}} = \frac{\partial l}{\partial q} \]

where according to (26) \( dt = \sqrt{g_{00}} d\tau \). I.e. transition from the spacetime \( t \) to the coordinate time \( \tau \) under the formula \( \frac{d}{dt} = \frac{d}{d\tau} \frac{d\tau}{dt} = \frac{d}{d\tau} \sqrt{g_{00}} \) is used. Interrelation of the generalized speed vector in spacetime and coordinate time looks like \( \frac{\partial q_i}{\partial t} = \frac{\partial q_i}{\partial \tau} \sqrt{g_{00}} = q_i \sqrt{g_{00}} \). We shall note that the generalized arguments of the equation (39) can be not connected to mechanical speeds and Euclidian coordinates.

Hence, Euler-Lagrange’s equation in the coordinate time will be transformed to standard kind [6]:

\[ \frac{d}{d\tau} \left( \frac{\partial l}{\partial q} \right) = \frac{\partial l}{\partial q} \] \hspace{1cm} (40)

As the generalized speed in coordinate time we shall accept the strength of an electromagnetic field \( q = \frac{\partial q}{\partial \tau} = E \).

Further transformations do not differ from [6] except that the coordinate time \( \tau \) is used.

The equations in the coordinate time \( \text{rot} E = \frac{1}{c^2 \sqrt{g_{00}}} \frac{\partial E}{\partial t} - \frac{1}{c} \frac{\partial E}{\partial \tau} \) and \( \text{rot} H = -\frac{1}{c} \frac{\partial H}{\partial \tau} \) [8] are used.

For example, using second Maxwell’s equation it is possible to write down \( \frac{\partial q}{\partial \tau} = -\frac{1}{c} \frac{\partial H}{\partial \tau} \) or \( \text{rot} q = -\frac{H}{c} \).

Taking into account determination of a vector-potential \( B = \frac{H}{\sqrt{g_{00}}} = \text{rot} A \) we shall find interrelation of the generalized coordinate and vector-potential \( q = -\sqrt{g_{00}} \frac{A}{c} \).

Also, as well as at absence of a gravitational field [6] the Euler-Lagrange’s equation (40) at the used dependences of the generalized speed and the generalized coordinate passes in the wave equation for vector-potential \( A \) in the coordinate time:

\[ \Delta A - \frac{1}{c^2} \frac{\partial^2 A}{\partial \tau^2} = 0 \] \hspace{1cm} (41)

Process of quantization of an electromagnetic field in the coordinate time differs nothing from quantization in spacetime [6]. In result we find the Schrodinger’s equation for a photon:

\[ i \hbar \frac{\partial \Psi}{\partial \tau} + \frac{2\pi \hbar^2}{c^2} \frac{\partial^2 \Psi}{\partial q^2} + \ln \left| \Psi \right| \Psi = 0 \] \hspace{1cm} (42)

and the equation of continuity:

\[ \frac{\partial \Psi}{\partial \tau} + \frac{\partial \Psi}{\partial q} = 0 \] \hspace{1cm} (43)

The solution of the Schrodinger’s equation (42) in the coordinate time looks like similar [6]:

\[ \Psi = \exp \left\{ \frac{c^2}{8\pi} - \frac{i \hbar \delta}{2} + \frac{1}{2} \right\} \exp \left\{ \frac{(q - \epsilon \tau)^2}{8\pi \hbar^2} \right\} \exp \left\{ \frac{c q}{4\pi \hbar} - \frac{\delta \tau}{1 + \frac{\phi}{c^2}} \right\} \] \hspace{1cm} (44)

The gravitational field according to (33) reduces the frequency of quantum. Therefore the formula (44) needs to be written down as:

\[ \Psi = \exp \left\{ \frac{c^2}{8\pi} - \frac{i \hbar \delta}{2} + \frac{1}{2} \right\} \exp \left\{ \frac{(q - \epsilon \tau)^2}{8\pi \hbar^2} \right\} \exp \left\{ \frac{c q}{4\pi \hbar} - \frac{\delta \tau}{1 + \frac{\phi}{c^2}} \right\} \] \hspace{1cm} (45)

In the gravitational field the volumetric density of quantum energy increases:
\[ w = \frac{\hbar \delta_0}{\frac{1}{\phi} + \frac{1}{c^2}} - \frac{1}{2} \]  

(46)

and also the length of quantum decreases in comparison with (23):

\[ q_0 = 2\pi \hbar \exp \left\{ 2\pi \left( h r \right)^2 - \frac{\hbar \delta}{\frac{1}{\phi} + \frac{1}{c^2}} + \frac{1}{2} \right\} = \frac{2\pi \hbar \exp \left( \frac{c^2}{8\pi} - \frac{\hbar \delta}{\frac{1}{\phi} + \frac{1}{c^2}} + \frac{1}{2} \right)}{8\pi} \]

(47)

5. Conclusion

It is shown, that Schrodinger’s equation in space of the generalized coordinates equally as for a photon in free space without currents and charges, and for a photon radiated by a current. The Schrodinger’s equation depends on two constants: Planck’s constant determining energy of a photon and Planck’s charge, representing photon “memory” that it has arisen due to charges and currents.

On a basis of the photon wave function normalization the dependence of the photon length on volumetric density of its energy is found. The photon length quantized and exponential falls with increase in volumetric density of the photon energy. The minimal photon length in space of the generalized coordinates and the maximal volumetric density of the photon energy is found. The minimal photon length in a space of the generalized coordinates is equal to Planck’s constant in this space.

It is shown that in a constant gravitational field the Schrodinger’s equation for a photon in space of the generalized coordinates needs to be written down in the coordinate time dependent on size of the gravitational field. In the constant gravitational field the photon frequency and volumetric density of its energy increases, but the photon length decreases.

References